

3° We calculate the first remainder by subtracting the product obtained in the 2nd step ($6x^2 - 4x$) from the dividend. We then get ($9x - 4$).

We repeat the process...

We stop the division when the degree of the remainder is less than the degree of the divisor.

We therefore get the quotient $Q(x) = 2x + 3$ and the remainder $R(x) = 2$.

$$\begin{array}{r}
 2x \\
 3x - 2 \overline{) 6x^2 + 5x - 4} \\
 \underline{6x^2 - 4x} \\
 9x - 4 \\
 2x + 3 \leftarrow \text{quotient} \\
 3x - 2 \overline{) 6x^2 + 5x - 4} \\
 \underline{6x^2 - 4x} \\
 9x - 4 \\
 \underline{- 9x - 6} \\
 2 \\
 \text{remainder} \longrightarrow
 \end{array}$$

- The dividend $A(x)$, the divisor $B(x)$, the quotient $Q(x)$ and the remainder $R(x)$ verify the following Euclidean relation:

$$A(x) = B(x) \cdot Q(x) + R(x) \quad \text{where} \quad \deg R(x) < \deg B(x).$$

In fact, $6x^2 + 5x - 4 = (3x - 2)(2x + 3) + 2$

21. Determine the quotient $Q(x)$ and the remainder $R(x)$ in the division of $A(x) = 2x^2 + 5x - 3$ by $B(x) = x - 1$.
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22. In each of the following cases, determine the quotient $Q(x)$ and the remainder $R(x)$ in the division of $A(x)$ by $B(x)$.

a) $A(x) = 2x^2 - x - 6$; $B(x) = 2x + 3$ _____

b) $A(x) = 3x^2 - 2x + 1$; $B(x) = x - 2$ _____

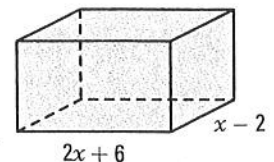
c) $A(x) = 2x^3 + 3x^2 + 2x + 4$; $B(x) = x + 1$ _____

d) $A(x) = x^3 - 2x + 1$; $B(x) = x - 1$ _____

23. The area of a parallelogram is $A(x) = 10x^2 + 19x - 15$. The height is represented by the binomial $5x - 3$. Use a polynomial to express the parallelogram's base. _____

24. The area of a rectangle is given by the polynomial $A(x) = 6x^2 - 13x - 5$. The width is represented by the binomial $3x + 1$. Use a polynomial to express the perimeter of this rectangle. _____

25. The following right prism has the volume $V(x) = 2x^3 + 4x^2 - 10x - 12$. The dimensions of the prism's base are $(2x + 6)$ and $(x - 2)$.



a) Determine the height of the prism. _____

b) Use a polynomial to express the total area of the prism. _____

26. The area of a triangle is $A(x) = x^2 + x - 6$. The base is represented by the binomial $2x + 6$. Use a polynomial to express the height of this triangle. _____

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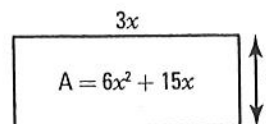
Factoring a polynomial

ACTIVITY 1 Removing the common factor

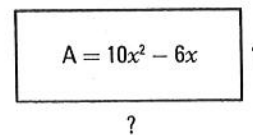
a) The sum $ab + ac$ has two terms ab and ac . Each term is a product of factors.

1. What is the common factor to both terms? _____
2. Write the sum $ab + ac$ as a product of two factors. _____

b) The rectangle on the right has the area $A = 6x^2 + 15x$. If one of the dimensions is $3x$, find the other dimension. _____



c) The rectangle on the right has the area $A = 10x^2 - 6x$. What could the dimensions of this rectangle be? _____

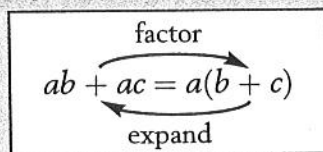


d) Given the polynomial $P(x) = 6x^3 - 15x^2$. In each of the following statements, a common factor to both terms of the polynomial $P(x)$ has been removed. Complete the writing of $P(x)$ as a product of two factors.

1. $6x^3 - 15x^2 = 3$ _____
2. $6x^3 - 15x^2 = x$ _____
3. $6x^3 - 15x^2 = x^2$ _____
4. $6x^3 - 15x^2 = 3x$ _____
5. $6x^3 - 15x^2 = 3x^2$ _____

REMOVING THE COMMON FACTOR

- Factoring a polynomial means writing the polynomial as a product of factors.
- Removing a common factor is a method which can be used to factor a polynomial composed of monomials which all have a common factor. To factor, you need to apply the distributive property of multiplication over addition.



Ex.: Factor: $P(x) = 6x^4 + 15x^3 - 18x^2$
 $= 3x^2(2x^2 + 5x - 6)$

Notice that the **greatest common factor** ($3x^2$) to all three terms is factored out.

1. Find the greatest common factor of the following algebraic expressions.

- a) $18x^4; 24x^3; 12x^5$ _____
- b) $18x^3y^2z^4; 24x^4y^3z^4; 36x^2y^4z^3$ _____
- c) $15x^2(a + b)^3; 18x^3(a + b)^2$ _____
- d) $24x^3y^2(a - b)^3; 36x^2y^4(a - b)^2$ _____

2. Factor the following polynomials.

- a) $5x - 10$ _____
- b) $18x + 24y - 12z$ _____
- c) $4x^2 + 6x$ _____
- d) $12x + x^2 - 5x^3$ _____
- e) $12a^2b + 18a^2b^2$ _____
- f) $-3x^4 + 6x^3 - 9x^2$ _____
- g) $a^2 + ab + a$ _____
- h) $x^4 - x^3y - x^2$ _____

3. Factor the following polynomials.

- a) $12x^3 - 16x^2$ _____ b) $6x^2y^3 + 4x^3y^2$ _____
 c) $12x^2y^3 - 18x^3y^2 + 24x^2y^2$ _____ d) $-14x^3 + 21x^2 - 7x$ _____
 e) $2x(x + 1) + 3y(x + 1)$ _____ f) $2(x - 3) - x(x - 3)$ _____

4. Factor the following polynomials.

- a) $x(x + 2) + 5(x + 2)$ _____ b) $3(x - 2) - x(x - 2)$ _____
 c) $a(b + c) - d(b + c)$ _____ d) $x(3 - y) + y(3 - y)$ _____
 e) $(x + 3)(x + 2) + (x + 3)(x - 1)$ _____ f) $(x + y)(x - 2) - (x + y)(2x - 3)$ _____
 g) $(x + y)^2 + x(x + y)$ _____ h) $(x - y)^2 + (x - y)(x + y)$ _____

ACTIVITY 2 Factoring by grouping

a) The sum $ac + ad + bc + bd$ is composed of 4 terms. Each term is the product of two factors. Can we find a common factor to all 4 of these terms? _____

b) Justify the steps which enable you to factor the sum $ac + ad + bc + bd$.

$$\begin{aligned} ac + ad + bc + bd &= (ac + ad) + (bc + bd) \\ &= a(c + d) + b(c + d) \\ &= (c + d)(a + b) \end{aligned}$$

c) The rectangle on the right has an area of $A = 6x^2 + 4x + 9xy + 6y$. What could the dimensions of this rectangle be?

$A = 6x^2 + 4x + 9xy + 6y$

?

FACTORING BY GROUPING

- Factoring by grouping is a method which enables you to factor polynomials by grouping the terms which contain a common factor.

You then remove the common factor in each of the groupings:

$$\begin{aligned} ac + ad + bc + bd &= (ac + ad) + (bc + bd) \\ &= a(c + d) + b(c + d) \\ &= (c + d)(a + b) \end{aligned}$$

Ex.: Factor the following expression using factoring by grouping.

$$\begin{aligned} P(x) &= \underline{9x^2 - 12xy^2} + \underline{6xy - 8y^2} \quad \leftarrow \text{Group the terms containing a common factor.} \\ &= 3x(3x - 4y^2) + 2y(3x - 4y) \quad \leftarrow \text{Remove the common factor in each grouping.} \\ &= (3x - 4y^2)(3x + 2y) \quad \leftarrow \text{Remove the common factor a 2nd time.} \end{aligned}$$

5. Factor the following polynomials.

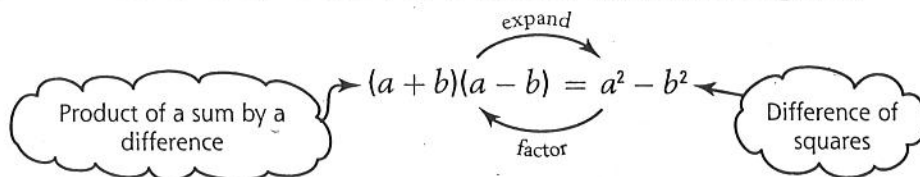
- a) $x^2 + 5xy + 3x + 15y$ _____ b) $2x^2 + 3xy - 10x - 15y$ _____
 c) $6a^2 - 15a + 2ab - 5b$ _____ d) $6x^2 - 8x - 9xy + 12y^2$ _____
 e) $10xy + 2x + 15y + 3$ _____ f) $x^3 - x^2 + x - 1$ _____

6. Factor the following polynomials.

- a) $2x^2y + 3x^2 + 10y + 15$ _____
 b) $15x^4y^2 + 35x^2y^2 - 9x^2 - 21$ _____
 c) $2x^3 + 4x^2y - 2x^2 - 4xy$ _____
 d) $3x^3y - 9x^3 + 6x^2y - 18x^2$ _____

ACTIVITY 3 Difference of squares

The following remarkable identity enables you to factor a difference of squares.



a) Factor the following difference of squares.

1. $x^2 - 25$ _____ 2. $4x^2 - 9y^2$ _____ 3. $x^2 - 7$ _____
 4. $(3x + 1)^2 - 4x^2$ _____

b) The rectangle on the right has an area of $A = 16x^2 - 9$. What polynomial could represent its perimeter?

$A = 16x^2 - 9$?
 ?

DIFFERENCE OF SQUARES

- A difference of squares is an algebraic expression of the form $a^2 - b^2$.
- Every difference of squares is factorable. You simply need to apply the remarkable identity:

$$a^2 - b^2 = (a + b)(a - b)$$

Ex.: Factor:

- $9x^2 - 4y^2 = (3x)^2 - (2y)^2$ ← Write in the form $a^2 - b^2$.
 $= (3x + 2y)(3x - 2y)$ ← Apply the remarkable identity.
- $(2x + 1)^2 - 36 = (2x + 1)^2 - 6^2$
 $= [(2x + 1) + 6][(2x + 1) - 6]$
 $= (2x + 7)(2x - 5)$
- $(3x + 5)^2 - (2x + 1)^2 = [(3x + 5) + (2x + 1)][(3x + 5) - (2x + 1)]$
 $= (5x + 6)(x + 4)$

- A sum of squares is not factorable.

7. Factor the following differences of squares.

a) $x^2 - 25$ _____

b) $16x^2 - 9$ _____

c) $49x^2 - 36y^2$ _____

d) $36x^4 - 25y^6$ _____

e) $100 - x^2$ _____

f) $\frac{x^4}{16} - \frac{y^4}{9}$ _____

g) $x^2 - 3$ _____

h) $x^2 - 1$ _____

8. Factor the following differences of squares.

a) $(3x - 1)^2 - 9$ _____

b) $(x + 1)^2 - 4$ _____

c) $(2x + 5)^2 - 16x^2$ _____

d) $25x^2 - (2x - 5)^2$ _____

e) $16x^2 - (3x + 2)^2$ _____

f) $36x^2 - (2 - x)^2$ _____

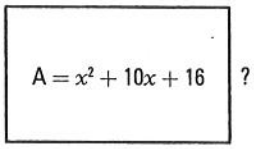
g) $(x + 3)^2 - (2x + 5)^2$ _____

h) $(3x - 5y)^2 - (2x - 3y)^2$ _____

ACTIVITY 4 Second degree trinomials $ax^2 + bx + c$

a) The rectangle on the right has an area of $A = x^2 + 10x + 16$.

Find a method for factoring this trinomial. What could be the possible dimensions of this rectangle?



b) Find a method for factoring the trinomial $2x^2 + 9x + 20$ and factor it.

SECOND DEGREE TRINOMIALS: $ax^2 + bx + c$

• The “product and sum” method enables you to factor a second degree trinomial.

Let us illustrate this method by factoring $P(x) = 2x^2 + 7x + 6$.

1. Identify the coefficients a , b and c :	1. $a = 2; b = 7; c = 6$
2. Find two integers m and n such that $\begin{cases} m \cdot n = ac \leftarrow \text{product of the end coefficients} \\ m + n = b \leftarrow \text{middle coefficient} \end{cases}$	2. $\begin{cases} mn = 12 \\ m + n = 7 \end{cases}$ $m = 4, n = 3$
3. We write: $ax^2 + bx + c = ax^2 + mx + nx + c$ and we factor by grouping.	3. $\begin{aligned} 2x^2 + 7x + 6 &= 2x^2 + 4x + 3x + 6 \\ &= 2x(x + 2) + 3(x + 2) \\ &= (x + 2)(2x + 3) \end{aligned}$

9. Factor the following trinomials using the "product and sum" method.

- a) $2x^2 + 9x + 4$ _____ b) $6x^2 - 19x + 10$ _____
 c) $4x^2 - 5x - 21$ _____ d) $5x^2 - 32x - 21$ _____
 e) $12x^2 + 13x + 3$ _____ f) $16x^2 - 26x + 3$ _____
 g) $6x^2 + 11x - 10$ _____ h) $8x^2 + 2x - 15$ _____

10. Factor the following trinomials.

- a) $x^2 - 10x + 21$ _____ b) $x^2 - 5x - 14$ _____
 c) $x^2 - 7x + 12$ _____ d) $x^2 - 9x + 20$ _____
 e) $2x^2 + 7x + 3$ _____ f) $3x^2 + 5x - 2$ _____
 g) $6x^2 + x - 2$ _____ h) $10x^2 - 19x + 6$ _____

11. Factor the following trinomials.

- a) $x^2 + 8x + 15$ _____ b) $x^2 - 8x + 15$ _____
 c) $x^2 + 5x - 14$ _____ d) $x^2 + 9x + 14$ _____
 e) $6x^2 + 19x + 15$ _____ f) $2x^2 - 7x - 15$ _____
 g) $3x^2 - x - 4$ _____ h) $5x^2 - 17x + 6$ _____

ACTIVITY 5 Multi-step factoring

Explain the steps in factoring the following polynomials.

- a) $2x^3 - 18x = 2x(x^2 - 9)$ _____
 $= 2x(x + 3)(x - 3)$ _____
 b) $4x^3 - 4x^2 - 8x = 4x(x^2 - x - 2)$ _____
 $= 4x(x + 1)(x - 2)$ _____
 c) $x^4 - 16 = (x^2 + 4)(x^2 - 4)$ _____
 $= (x^2 + 4)(x + 2)(x - 2)$ _____

MULTI-STEP FACTORING

Many steps are sometimes necessary to completely factor a polynomial.

Ex.: $2x^3 - 18x = 2x(x^2 - 9)$ ← Remove the common factor
 $= 2x(x + 3)(x - 3)$ ← Difference of squares

$$4x(2x + 3) + 4x^2 - 9 = 4x(2x + 3) + (2x + 3)(2x - 3) \quad \leftarrow \text{Difference of squares}$$

$$= (2x + 3)[4x + (2x - 3)] \quad \leftarrow \text{Remove the common factor}$$

$$= (2x + 3)(6x - 3) \quad \leftarrow \text{Reduce}$$

$$= (2x + 3) \cdot 3(2x - 1) \quad \leftarrow \text{Remove the common factor}$$

$$= 3(2x + 3)(2x - 1) \quad \leftarrow \text{Commutative property of multiplication}$$

12. Factor the following polynomials completely .

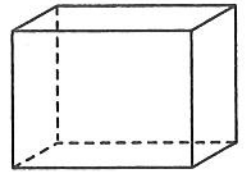
a) $x^3 + 3x^2 + 2x =$ _____ b) $x^3 - 2x^2 + x =$ _____

c) $x^3 - 16x =$ _____ d) $x^4 - 1 =$ _____

e) $6x^4 + 9x^3 + 3x^2 =$ _____ f) $3x^3 - 12x =$ _____

13. The prism on the right has a volume of $V = 2x^3 + 6x^2 + 4x$.

What could the dimensions of this prism be?



14. The area of a rectangle is expressed by the polynomial $A(x) = 6x^2 + 17x + 12$. What could be the perimeter of this rectangle?

15. The area of a square is expressed by the polynomial $9x^2 + 12x + 4$. What is the perimeter of this square?

16. The volume of a right rectangular prism is expressed by $V(x) = x^3 + 2x^2 - x - 2$. What could the dimensions of the prism be?

17. The total area of a cube is expressed by $A(x) = 24x^2 + 24x + 6$. What is the volume of this cube?

18. The volume of a right rectangular prism is expressed by $V(x) = x^3 + 4x^2 + x - 6$. If $(x + 3)$ represents the height of the prism, find two binomials that could express the dimensions of the prism's base.

1. For each of the following rational expressions, indicate the restrictions and simplify if possible.

a) $\frac{5x^2}{20x^3}$ _____

b) $\frac{4x^2 - 6x}{3x^2 + 6x}$ _____

c) $\frac{5x + 10y}{5x - 10y}$ _____

d) $\frac{2x^2 + 6x}{6x^2 + 10x}$ _____

e) $\frac{6x^3 + 4x^2}{9x^2 + 6x}$ _____

f) $\frac{x^2 + 3x + 2}{x^2 + x - 2}$ _____

g) $\frac{2x^2 - x - 6}{2x^2 + 5x + 3}$ _____

h) $\frac{x^2 - 8x + 15}{x^2 - 7x + 10}$ _____

ACTIVITY 2 Multiplying and dividing rational expressions

Explain the steps in the following operations.

In each case, the variable does not take values which make the denominators zero.

a) $\frac{2x+6}{x-2} \cdot \frac{x^2-3x+2}{x+3} = \frac{2(x+3)}{(x-2)} \cdot \frac{(x-1)(x-2)}{(x+3)}$ _____
 $= \frac{2(x+3)(x-1)(x-2)}{(x-2)(x+3)}$ _____
 $= 2(x-1)$ _____

b) $\frac{5x-5}{x^2-3x} \div \frac{x-1}{x-3} = \frac{5x-5}{x^2-3x} \cdot \frac{x-3}{x-1}$ _____
 $= \frac{5(x-1)}{x(x-3)} \cdot \frac{(x-3)}{(x-1)}$ _____
 $= \frac{5(x-1)(x-3)}{x(x-3)(x-1)}$ _____
 $= \frac{5}{x}$ _____

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Apply the rules: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ and $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$.

In each of the following operations, the variable does not take values which make the denominators zero.

Ex.: $\frac{x^2-x}{x+3} \cdot \frac{x^2+5x+6}{x^2-1} = \frac{x(x-1)}{(x+3)} \cdot \frac{(x+2)(x+3)}{(x+1)(x-1)}$
 $= \frac{x(x-1)(x+2)(x+3)}{(x+3)(x+1)(x-1)}$
 $= \frac{x(x+2)}{x+1}$

Ex.: $\frac{x^2-25}{x^2-3x} \div \frac{x-5}{x-3} = \frac{x^2-25}{x^2-3x} \cdot \frac{x-3}{x-5}$
 $= \frac{(x+5)(x-5)}{x(x-3)} \cdot \frac{(x-3)}{(x-5)}$
 $= \frac{(x+5)(x-5)(x-3)}{x(x-3)(x-5)}$
 $= \frac{x+5}{x}$

2. Perform the following multiplications, given that the variable does not take values which make the denominators zero.

a) $\frac{2x-4}{x+3} \times \frac{x^2+6x+9}{x^2-4}$ _____

b) $\frac{x^2-1}{x+3} \times \frac{x-3}{x^2-4x+3}$ _____

c) $\frac{2x+3}{x-1} \times \frac{x^2+2x-3}{2x^2-x-6}$ _____

d) $\frac{x^2-1}{x+2} \div \frac{x-1}{3x+6}$ _____

e) $\frac{x^2-x-2}{x^2-x-6} \div \frac{x+1}{x+2}$ _____

f) $\frac{3x^2+8x-3}{x^2+x-6} \div \frac{2x+1}{x-2}$ _____

ACTIVITY 3 Addition and subtraction of rational expressions

a) 1. Justify the steps in the following operations.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd}$$

$$= \frac{ad+bc}{bd}$$

2. Perform the following addition, given that $x \neq -3$ and $x \neq 1$.

$$\frac{2}{x+3} + \frac{3}{x-1} =$$

=

=

b) 1. Justify the steps in the following operations.

$$\frac{a}{b} + c = \frac{a}{b} + \frac{bc}{b}$$

$$= \frac{a+bc}{b}$$

2. Perform the following operation, given that $x \neq -2$.

$$\frac{5}{(x+2)} - 2 =$$

=

=

ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS

- The denominators have no common factor.

$$\frac{A(x)}{B(x)} \pm \frac{C(x)}{D(x)} = \frac{A(x)D(x) \pm B(x)C(x)}{B(x)D(x)}$$

Ex.: $\frac{2}{x-1} + \frac{3}{x+3} = \frac{2(x+3) + 3(x-1)}{(x-1)(x+3)}$

The product of the denominators is the common denominator.

$$= \frac{2x+6+3x-3}{(x-1)(x+3)} = \frac{5x+3}{(x-1)(x+3)}$$

Ex.: $\frac{2x}{x+1} - \frac{3x}{x-1} = \frac{2x(x-1) - 3x(x+1)}{(x+1)(x-1)}$

$$= \frac{2x^2 - 2x - 3x^2 - 3x}{(x+1)(x-1)} = \frac{-x^2 - 5x}{(x+1)(x-1)}$$

- The denominators have a common factor.

Ex.: $\frac{4}{x^2-9} - \frac{2}{(x+3)^2} = \frac{4}{(x+3)(x-3)} - \frac{2}{(x+3)^2}$

(x+3)²(x-3) is the lowest common denominator.

$$= \frac{4(x+3)}{(x+3)^2(x-3)} - \frac{2(x-3)}{(x+3)^2(x-3)}$$

$$= \frac{4(x+3) - 2(x-3)}{(x+3)^2(x-3)} = \frac{2x+18}{(x+3)^2(x-3)}$$

Factor the denominators.

Find the common denominator using the least number of factors.

Reduce to a common denominator.

Simplify.

In each of the following operations, the variable does not take values which make the denominators zero.

- 3.** Perform the following operations.

a) $\frac{2}{x} + \frac{5}{y}$ _____

b) $\frac{3}{x} - \frac{2}{x^2}$ _____

c) $\frac{x+y}{x} - \frac{x+y}{y}$ _____

d) $\frac{2x+3y}{9x} + \frac{2x-3y}{6y}$ _____

- 4.** Perform the following operations.

a) $\frac{2}{x+1} + \frac{3}{x-2}$ _____

b) $\frac{3}{x+2} - \frac{2}{x-2}$ _____

c) $\frac{3x}{x-1} + \frac{2x}{x+1}$ _____

d) $\frac{x-2}{x+3} - \frac{x-3}{x+2}$ _____

- 5.** Perform the following operations and simplify your answer.

a) $\frac{x}{x^2-9} - \frac{1}{2x-6}$ _____

b) $\frac{1}{x+5} + \frac{1}{x-5}$ _____

c) $\frac{1}{x^2-2x+1} + \frac{1}{x-1}$ _____

d) $\frac{2a}{3a-15} + \frac{4a}{2a-10}$ _____

- 6.** Perform the following additions and subtractions.

a) $\frac{2x+1}{2} - \frac{3x-1}{5}$ _____

b) $\frac{5}{x-2} + \frac{3}{x+3}$ _____

c) $\frac{x+1}{x-2} - \frac{x+2}{x-1}$ _____

d) $\frac{x+3}{x-3} - \frac{x-3}{x+3}$ _____

p24

3a) $4x^2(3x - 4)$

c) $6x^2y^2(2y - 3x + 4)$

e) $(2x + 3y)(x + 1)$

f) $(x - 3)(2 - x)$

4a) $(x + 2)(x + 5)$

c) $(b + c)(a - d)$

e) $(x + 3)(x + 2 + x - 1)$
 $(x + 3)(2x + 1)$

g) $(x + y)(x + y + x)$
 $(x + y)(2x + y)$

b) $2x^2y^2(3y + 2x)$

d) $-7x(2x^2 - 3x + 1)$
 $-7x(2x^2 - 2x - 1x + 1)$
 $-7x(2x(x - 1) - 1(x - 1))$
 $-7x(2x - 1)(x - 1)$

b) $(x - 2)(3 - x)$

d) $(x + y)(3 - y)$

f) $(x + y)(x - 2 - (2x - 3))$
 $(x + y)(x - 2 - 2x + 3)$
 $(x + y)(-x + 1)$

h) $(x - y)(x - y + x + y)$
 $(x - y)(2x)$

p25

$$\begin{aligned} 5a) \quad & x(x+5y) + 3(x+5y) \\ & (x+5y)(x+3) \end{aligned}$$

$$\begin{aligned} b) \quad & x(2x+3y) - 5(2x+3y) \\ & (x-5)(2x+3y) \end{aligned}$$

$$\begin{aligned} c) \quad & 3a(2a-5) + b(2a-5) \\ & (2a-5)(3a+b) \end{aligned}$$

$$\begin{aligned} d) \quad & 2x(3x-4) - 3y(3x-4) \\ & (3x-4)(2x-3y) \end{aligned}$$

$$\begin{aligned} e) \quad & 2x(5y+1) + 3(5y+1) \\ & (5y+1)(2x+3) \end{aligned}$$

$$\begin{aligned} f) \quad & x^2(x-1) + 1(x-1) \\ & (x-1)(x^2+1) \end{aligned}$$

$$\begin{aligned} 6a) \quad & x^2(2y+3) + 5(2y+3) \\ & (2y+3)(x^2+5) \end{aligned}$$

$$\begin{aligned} b) \quad & 5x^2y^2(3x^2+7) - 3(3x^2+7) \\ & (3x^2+7)(5x^2y^2-3) \end{aligned}$$

$$\begin{aligned} c) \quad & 2x^2(x+2y) - 2x(x+2y) \\ & 2x(x-1)(x+2y) \end{aligned}$$

$$\begin{aligned} d) \quad & 3x^2(xy-3x+2y-6) \\ & 3x^2(x(y-3)+2(y-3)) \\ & 3x^2(x+2)(y-3) \end{aligned}$$

p 26

7a) $(x-5)(x+5)$

b) $(4x-3)(4x+3)$

c) $(7x-6y)(7x+6y)$

d) $(6x^2-5y^3)(6x^2+5y^3)$

e) $(10-x)(10+x)$

f) $\left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x}{4} + \frac{y}{3}\right)$

g) $(x-\sqrt{3})(x+\sqrt{3})$

h) $(x-1)(x+1)$

8a) $(3x-1-3)(3x-1+3)$
 $(3x-4)(3x+2)$

b) $(x+1-2)(x+1+2)$
 $(x-1)(x+3)$

c) $(2x+5-4x)(2x+5+4x)$
 $(-2x+5)(6x+5)$

d) $(5x-(2x-5))(5x+(2x-5))$
 $(3x+5)(7x-5)$

e) $(4x-(3x+2))(4x+(3x+2))$
 $(x-2)(7x+2)$

f) $(6x-(2-x))(6x+(2-x))$
 $(7x-2)(5x+2)$

g) $(x+3-2x-5)(x+3+2x+5)$
 $(-x-2)(3x+8)$

h) $(3x-5y-2x+3y)(3x-5y+2x-3y)$
 $(x-2y)(5x-8y)$

p 27

9a) $(x+4)(2x+1)$

b) $(3x-2)(2x-5)$

c) $(x-3)(4x+7)$

d) $(x-7)(5x+3)$

e) $(3x+1)(4x+3)$

f) $(8x-1)(2x-3)$

g) $(3x-2)(2x+5)$

h) $(4x-5)(2x+3)$

10a) $(x-7)(x-3)$

b) $(x-7)(x+2)$

c) $(x-4)(x-3)$

d) $(x-5)(x-4)$

e) $(x+3)(2x+1)$

f) $(x+2)(3x-1)$

g) $(2x-1)(3x+2)$

h) $(5x-2)(2x-3)$

11a) $(x+3)(x+5)$

b) $(x-3)(x-5)$

c) $(x+7)(x-2)$

d) $(x+7)(x+2)$

e) $(2x+3)(3x+5)$

f) $(x-5)(2x+3)$

g) $(x+1)(3x-4)$

h) $(x-3)(5x-2)$

P 28

$$12a) \quad x(x^2 + 3x + 2) \\ x(x+2)(x+1)$$

$$b) \quad x(x^2 - 2x + 1) \\ x(x-1)^2$$

$$c) \quad x(x^2 - 16) \\ x(x-4)(x+4)$$

$$d) \quad (x^2 - 1)(x^2 + 1) \\ (x-1)(x+1)(x^2 + 1)$$

$$e) \quad 3x^2(2x^2 + 3x + 1) \\ 3x^2(x+1)(2x+1)$$

$$f) \quad 3x(x^2 - 4) \\ 3x(x-2)(x+2)$$

$$13) \quad 2x(x^2 + 3x + 2) \\ 2x(x+2)(x+1)$$

$$14) \quad (3x+4)(2x+3) \\ 2(3x+4) + 2(2x+3)$$

$$6x+8 + 4x+6$$

$$10x+14 = \text{Perimeter}$$

$$15) \quad 9x^2 + 12x + 4$$

$$(3x+2)^2$$

$$4(3x+2)$$

$$= 12x + 8 \rightarrow \text{perimeter}$$

$$16) \quad x^2(x+2) - 1(x+2)$$

$$(x+2)(x-1)(x+1)$$

17) Don't worry

18)

$$\begin{array}{r} x^2 + x - 2 \\ x+3 \overline{) x^3 + 4x^2 + x - 6} \\ \underline{- x^3 + 3x^2} \\ x^2 + x - 6 \\ \underline{- x^2 + 3x} \\ -2x - 6 \\ \underline{-2x - 6} \\ \phi \end{array}$$

$$(x+2)(x-1)$$

p 30

1a) $x \neq 0$

$$\frac{1}{4x}$$

b) $\frac{4x^2 - 6x}{3x^2 + 6x}$

$$\frac{2x(2x - 3)}{3x(x + 2)}$$

$x \neq 0$
 $x \neq -2$

$$\frac{2(2x - 3)}{3(x + 2)}$$

c) $\frac{5x + 10y}{5x - 10y}$

$$\frac{\cancel{5}(x + 2y)}{\cancel{5}(x - 2y)}$$

$x - 2y \neq 0$
 $x \neq 2y$
 $x \neq y$

$$\frac{x + 2y}{x - 2y}$$

d) $\frac{2x^2 + 6x}{6x^2 + 10x}$

$$\frac{\cancel{2}x(x + 3)}{\cancel{2}x(3x + 5)}$$

$x \neq 0$
 $x \neq -\frac{5}{3}$

$$\frac{x + 3}{3x + 5}$$

e) $\frac{2x^2(\cancel{3x + 2})}{3x(\cancel{3x + 2})}$

$x \neq 0$
 $x \neq -\frac{2}{3}$

$$\frac{2x}{3}$$

$$f) \frac{x^2 + 3x + 2}{x^2 + x - 2}$$

$$\frac{(x+2)(x+1)}{(x+2)(x-1)}$$

$$\frac{x+1}{x-1}$$

$$x \neq -2$$

$$x \neq 1$$

$$g) \frac{2x^2 - x - 6}{2x^2 + 5x + 3}$$

$$\frac{(2x+3)(x-2)}{(2x+3)(x+1)}$$

$$\frac{x-2}{x+1}$$

$$\frac{x-2}{x+1}$$

$$P = -12 \quad S = -1 \quad \left. \vphantom{\begin{matrix} P \\ S \end{matrix}} \right\} -4, 3$$

$$2x^2 - 4x + 3x - 6$$

$$2x(x-2) + 3(x-2)$$

$$(2x+3)(x-2)$$

$$P = 6 \quad S = 5 \quad \left. \vphantom{\begin{matrix} P \\ S \end{matrix}} \right\} 3, 2$$

$$2x^2 + 2x + 3x + 3$$

$$2x(x+1) + 3(x+1)$$

$$h) \frac{x^2 - 8x + 15}{x^2 - 7x + 10}$$

$$\frac{(x-5)(x-3)}{(x-5)(x-2)}$$

$$\frac{x-3}{x-2}$$

$$\frac{x-3}{x-2}$$

$$x \neq 5$$

$$x \neq 2$$

P 31

#2a) $\frac{2x-4}{x+3} \cdot \frac{x^2+6x+9}{x^2-4}$

$$\frac{2(\cancel{x-2})(\cancel{x+3})(x+3)}{(\cancel{x+3})(\cancel{x-2})(x+2)}$$

$x \neq -3, 2, -2$

$$\frac{2(x+3)}{(x+2)}$$

b) $\frac{(\cancel{x-1})(x+1)(\cancel{x-3})}{(x+3)(\cancel{x-3})(\cancel{x-1})}$

$x \neq -3, 3, 1$

$$\frac{x+1}{x+3}$$

c) $\frac{\cancel{2x+3}}{\cancel{x-1}} \cdot \frac{(x+3)(\cancel{x-1})}{(\cancel{2x+3})(x-2)}$

$$2x^2 - x - 6$$

$$P = -12$$

$$S = -1$$

$$\frac{x+3}{x-2}$$

$x \neq 1, -\frac{3}{2}, 2$

$$2x^2 - 4x + 3x - 6$$

$$2x(x-2) + 3(x-2)$$

$$(x-2)(2x+3)$$

d) $\frac{x^2-1}{x+2} \div \frac{x-1}{3x+6}$

* Restrict num + den.

$$\frac{(\cancel{x-1})(x+1)}{\cancel{x+2}} \cdot \frac{3(\cancel{x+2})}{(\cancel{x-1})}$$

$x-1 \neq 0 \quad x \neq 1$
 $3x+6 \neq 0 \quad x \neq -2$

$$3(x+1)$$

$$e) \frac{(x-2)(x+1)}{(x-3)(x+2)} \div \frac{x+1}{x+2}$$

$$x \neq -1$$

$$x \neq -2$$

$$x \neq 3$$

$$\frac{(x-2)\cancel{(x+1)}}{(x-3)\cancel{(x+2)}} \cdot \frac{\cancel{x+2}}{\cancel{x+1}}$$

$$\frac{x-2}{x-3}$$

$$f) \frac{3x^2 + 8x - 3}{(x+3)(x-2)} \div \frac{2x+1}{x-2}$$

$$x \neq 2 \quad x \neq -\frac{1}{2}$$

$$x \neq -3$$

$$\frac{(3x-1)\cancel{(x+3)}\cancel{(x-2)}}{\cancel{(x+3)}\cancel{(x-2)}(2x+1)}$$

$$\frac{3x-1}{2x+1}$$

$$3x^2 + 8x - 3$$

$$P = -9 \quad S = 8 \quad \left. \begin{array}{l} P = -9 \\ S = 8 \end{array} \right\} 9, -1$$

$$3x^2 + 9x - 1x - 3$$

$$3x(x+3) - 1(x+3)$$

p 32

$$3a) \frac{2}{x} + \frac{5}{y}$$

$$\frac{2y + 5x}{xy}$$

$$b) \frac{3}{x} - \frac{2}{x^2}$$

$$\frac{3x - 2}{x^2}$$

$$c) \frac{x+y}{x} - \frac{x+y}{y}$$

$$\frac{y(x+y) - x(x+y)}{xy}$$

* notice there's a common factor.

$$\frac{(x+y)(y-x)}{xy}$$

$$d) \frac{2x+3y}{9x} + \frac{2x-3y}{6y}$$

$$\frac{6y(2x+3y) + 9x(2x-3y)}{54xy}$$

$$\frac{12xy + 18y^2 + 18x^2 - 27xy}{54xy}$$

$$\frac{18x^2 + 18y^2 - 15xy}{54xy}$$

GCF
 $\boxed{\div 3}$

$$\frac{6x^2 + 6y^2 - 5xy}{18xy}$$

$$4a) \quad \frac{2}{x+1} + \frac{3}{x-2}$$

$$\frac{2(x-2) + 3(x+1)}{(x+1)(x-2)}$$

$$\frac{2x-4 + 3x+3}{(x+1)(x-2)}$$

$$= \frac{5x-1}{x+1}$$

$$b) \quad \frac{3}{x+2} - \frac{2}{x-2}$$

$$\frac{3(x-2) - 2(x+2)}{(x+2)(x-2)}$$

$$\frac{3x-6 - 2x-4}{(x+2)(x-2)}$$

$$\frac{x-10}{(x+2)(x-2)}$$

$$c) \quad \frac{3x}{x-1} + \frac{2x}{x+1}$$

$$3x(x+1) + 2x(x-1)$$

$$\frac{3x^2+3x+2x^2-2x}{(x-1)(x+1)}$$

$$\frac{5x^2+x}{(x-1)(x+1)}$$

$$6a) \frac{(2x+1)}{2} - \frac{3x-1}{5}$$

$$\frac{5(2x+1) - 2(3x-1)}{10}$$

$$\frac{10x+5 - 6x+2}{10}$$

$$\frac{4x+7}{10}$$

$$b) \frac{5}{x-2} + \frac{3}{x+3}$$

$$\frac{5(x+3) + 3(x-2)}{(x-2)(x+3)}$$

$$\frac{5x+15 + 3x-6}{(x-2)(x+3)}$$

$$\frac{8x-9}{(x-2)(x+3)}$$

$$c) \frac{x+1}{x-2} - \frac{x+2}{x-1}$$

$$\frac{(x+1)(x-1) - (x+2)(x-2)}{(x-1)(x-2)}$$

$$\frac{x^2-1 - (x^2-3x+2)}{(x-1)(x-2)}$$

$$\frac{x^2-1 - x^2+3x-2}{(x-1)(x-2)}$$

$$\frac{3x-3}{(x-1)(x-2)}$$

$$\frac{3(x-1)}{(x-1)(x-2)}$$

$$\frac{3}{x-2}$$

$$d) \frac{x+3}{x-3} - \frac{x-3}{x+3}$$

$$\frac{(x+3)(x+3) - (x-3)(x-3)}{(x-3)(x+3)}$$

$$\frac{x^2+6x+9 - (x^2-6x+9)}{(x-3)(x+3)}$$

$$x^2+6x+9 - x^2+6x-9$$

$$\frac{12x}{(x-3)(x+3)}$$

$$d) \frac{x-2}{x+3} - \frac{x-3}{x+2}$$

$$\frac{(x-2)(x+2) - (x-3)(x+3)}{(x+3)(x+2)}$$

$$\frac{x^2 - 4 - (x^2 - 9)}{(x+3)(x+2)}$$

$$\frac{5}{(x+3)(x+2)}$$

$$\frac{5}{(x+3)(x+2)}$$

$$5a) \frac{x}{(x-3)(x+3)} - \frac{1}{2(x-3)}$$

$$\frac{2x - 1(x+3)}{2(x-3)(x+3)}$$

$$\frac{2x - x - 3}{2(x-3)(x+3)}$$

$$\frac{x-3}{2(x-3)(x+3)}$$

$$\frac{1}{2(x+3)}$$

$$\frac{1}{2(x+3)}$$

$$\frac{1}{2(x+3)}$$

$$c) \frac{1}{(x-1)(x-1)} + \frac{1}{x-1}$$

$$\frac{1 + 1(x-1)}{(x-1)(x-1)}$$

$$\frac{1 + x - 1}{(x-1)(x-1)}$$

$$\frac{x}{(x-1)(x-1)}$$

$$\frac{x}{(x-1)(x-1)}$$

$$b) \frac{x+3}{x+5} + \frac{1}{x-5}$$

$$\frac{1(x-5) + 1(x+5)}{(x+5)(x-5)}$$

$$\frac{x-5 + x+5}{(x+5)(x-5)}$$

$$\frac{2x}{(x+5)(x-5)}$$

$$\frac{2x}{(x+5)(x-5)}$$

$$\frac{2x}{(x+5)(x-5)}$$

$$d) \frac{2a}{3(a-5)} + \frac{4a}{2(a-5)}$$

$$\frac{2(2a) + 4a(3)}{6(a-5)}$$

$$\frac{4a + 12a}{6(a-5)}$$

$$\frac{16a}{6(a-5)}$$

$$\frac{16a}{6(a-5)} \rightarrow \frac{8a}{3(a-5)}$$