

student
book
volume

1

Cultural, Social
and Technical

VISIONS

MATHEMATICS

Secondary
Cycle Two, Year Two

ANSWER KEY

Visions 1 to 3

LES ÉDITIONS
CEC
Une compagnie de Québecor Média

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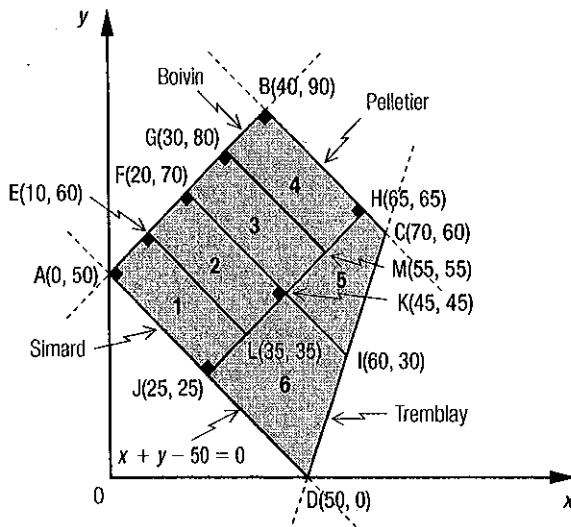
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LES 1

A new residential sector

Here is an approach to help you carry out a development plan that meets the overall restrictions proposed in the municipality's urban plan:

- Determine the equation of the line associated with each of the streets shown on the graph.
 - rue Simard: $y = -x + 50$
 - rue Boivin: $y = x + 50$
 - rue Pelletier: $y = -x + 130$
 - rue Tremblay: $y = 3x - 150$
- Determine the coordinates of the points corresponding to the fire hydrants.
 - Fire hydrant A: (0,50)
 - Fire hydrant B: (40,90)
 - Fire hydrant C: (70,60)
 - Fire hydrant D: (50,0)
- Produce a diagram of the sector including the exact boundaries of each lot.



- Calculate the exact area of each lot.

Lot 1

$$d(A, E) \approx 14.14 \text{ m}$$

$$d(A, J) \approx 35.36 \text{ m}$$

$$\text{Area of lot: } 14.14 \times 35.36 \approx 499.99 \text{ m}^2$$

Lots 1, 2, 3 and 4 have the same area.

Lot 5

$$d(K, I) \approx 21.21 \text{ m}$$

$$d(H, C) \approx 7.07 \text{ m}$$

$$d(H, K) \approx 28.28 \text{ m}$$

$$\text{Area of lot: } (21.21 + 7.07) \times 28.28 \div 2 \approx 399.88 \text{ m}^2$$

Lot 6

$$d(J, D) \approx 35.36 \text{ m}$$

$$d(K, I) \approx 21.21 \text{ m}$$

$$d(J, K) \approx 28.28 \text{ m}$$

$$\text{Area of lot: } (35.36 + 21.21) \times 28.28 \div 2 \approx 799.90 \text{ m}^2$$

- Conclusion
 - The development plan meets the constraints of this municipality's urban plan.
 - The sector is subdivided into six lots.
 - The area of four of these lots is approximately 500 m^2 — i.e. $\approx 499.99 \text{ m}^2$.
 - Fire hydrants are installed at points A, B, C and D.

LES 2

Zoning

Here is an approach that will allow you to solve this LES.

- Determine the equation of the line passing through points C and G.
 - Slope of the line: -2
 - Coordinates of point C: (1280, 3440)
 - Equation of the line passing through points C and G: $y = -2x + 6000$
- Establish the equation of the line passing through points B and D.
 - Slope of the line: 0.5
 - Coordinates of point C: (1280, 3440)
 - Equation of the line passing through points B and D: $y = 0.5x + 2800$
- Determine the coordinates of point D by solving the system of equations
 - $y = 0.5x + 2800$ and $y = -2x + 8000$, i.e. (2080, 3840).
- Determine the coordinates of points F and G.
 - The x-coordinate of point F corresponds to the x-intercept of the line passing through points D and F.

$$0 = -2x + 8000$$

$$-8000 = -2x$$

$$x = 4000$$
 The coordinates of point F are (4000, 0).
 - The x-coordinate of point G corresponds to the x-intercept of the line passing through points C and G.

$$0 = -2x + 6000$$

$$-6000 = -2x$$

$$x = 3000$$
 The coordinates of point G are (3000, 0).
- Determine the coordinates of point E.
 - Since the y-coordinate of points B and E is the same, students could find the x-coordinate of point E by substituting 2800

for the y -variable in the equation of the line passing through points C and G, as follows:

$$\begin{aligned} 2800 &= -2x + 6000 \\ -3200 &= -2x \\ x &= 1600 \end{aligned}$$

The coordinates of point E are (1600, 2800).

- Calculate the area of each lot.

Lot	Polygon	Measurements of sides necessary to calculate area	Lot Area (m ²)
Industrial zone	Right triangle	$m_{\overline{BC}} \approx 1431.08$ $m_{\overline{CE}} \approx 715.54$	$\approx 511\,997.49$
Residential and commercial zone	Right trapezoid	$m_{\overline{DF}} \approx 4293.25$ $m_{\overline{CG}} \approx 3846.04$ $m_{\overline{CD}} \approx 894.43$	$\approx 3\,640\,012.58$
Agricultural zone	Right trapezoid	$m_{\overline{AG}} = 3000$ $m_{\overline{BE}} = 1600$ $m_{\overline{AB}} = 2800$	$= 6\,440\,000$

- Determine the area of the territory occupied by the municipality, which is $\approx 10\,592\,010.07$ m².
- Calculate the percentage of the territory occupied by each of the zones.

Industrial zone

$$\frac{511\,997.49}{10\,592\,010.07} \approx 4.83\%$$

Agricultural zone

$$\frac{6\,440\,000}{10\,592\,010.07} \approx 60.8\%$$

Residential and commercial zone

$$\frac{3\,640\,012.58}{10\,592\,010.07} \approx 34.37\%$$

- Conclusion

The industrial zone occupies 3% to 5% of the municipal territory, approximately 4.83%. The agricultural zone occupies less than 62% of the municipal territory, approximately 60.8%. However, the residential and commercial zone does not occupy more than 35% of the municipality's territory. It occupies approximately 34.37% of this territory. Thus, the zoning plan does not meet the municipal zoning regulations.

Here are the inequalities that allow each of the municipality's three zones to be defined graphically.

Industrial zone

$$\begin{aligned} y &\leq 0.5x + 2800 \\ y &\leq -2x + 6000 \\ y &\geq 2800 \end{aligned}$$

Residential and commercial zone

$$\begin{aligned} y &\leq 0.5x + 2800 \\ y &\leq -2x + 8000 \\ y &\geq -2x + 6000 \\ y &\geq 0 \end{aligned}$$

Agricultural zone

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ y &\leq 2800 \\ y &\leq -2x + 6000 \end{aligned}$$

REVISION 1

Page 4

Prior learning 1

- a. 1) $y = 147x + 1500$ 2) $y = 150x$
- b. This company must manufacture more than 500 bicycles to make a profit.

Page 5

Prior learning 2

a.

Day	Algebraic expression
First	d
Second	$2d - 55$
Third	$d + 10$
Fourth	$2d - 55$
Fifth	d
Sixth	$d + 5$

b.

Day	Minimum distance travelled (km)
First	50
Second	45
Third	60
Fourth	45
Fifth	50
Sixth	55

Page 8

Knowledge in action

1. a) $x \leq 2$ b) $x \geq 14$ c) $x < 9$
d) $x \geq 6$ e) $x < y$ f) $x \leq y$
2. a) (20, -5) b) (-6, 9) c) (-15, 130) d) (-32, 72)
3. A 2, B 3, C 1, D 4

Page 9

Knowledge in action (continued)

4. a) $x > -2$ b) $a \leq 8$ c) $t \leq 19$
d) $b < 110$ e) $m \geq -4.2$ f) $c \geq -5$
g) $n > \frac{19}{5}$ h) $x > 2$

5. a) (14, 16) b) $(\frac{5}{2}, -\frac{15}{2})$ c) $(-\frac{1}{2}, \frac{1}{2})$
 d) (10, -13) e) $(\frac{8}{7}, \frac{4}{7})$ f) (21, 1)
6. a) $x > 3$ b)]3, 5] c)]6, 5, +\infty[
7. a) x : time elapsed since the start of the trek (in hours).
 y : quantity of remaining water (in mL).
 b) $y = -500x + 1800$ and $y = -100x + 1000$.
 c) Two hours after the start of the trek.

SECTION 1.1

Points and segments in the Cartesian plane

Problem

Page 10

The itineraries ABDFECA and ACEFDBA are the shortest, at ≈ 6962.88 km.

Activity 1

Page 11

- a. 9
 b. -120
 c. 7,5 %
 d. (132, 2)
 e. A right triangle.
 f. 1) 9 m 2) 120 m 3) ≈ 120.34 m

Activity 2

Page 12

- a. (30, -10)
 b. 1) (-30, -10) 2) (30, 40) 3) (-30, 40)
 c. The survey marker P will be situated closer to point C. If the marker is situated at $\frac{2}{3}$ of \overline{DC} , its distance to point D corresponds to $\frac{2}{3}$ of $m \overline{DC}$, whereas its distance to point C corresponds to $\frac{1}{3}$ of $m \overline{DC}$.
 d. 2:1
 e. 1) (30, -40) 2) (-90, -50) 3) (30, -50)

Activity 3

Page 13

- a. You must prove that $m \overline{AM} = m \overline{BM} = m \overline{CM}$.
 b. A(0, b), B(a, 0), C(0, 0)
 c. $(\frac{a}{2}, \frac{b}{2})$
 d. 1) $\frac{\sqrt{a^2 + b^2}}{2}$ 2) $\frac{\sqrt{a^2 + b^2}}{2}$ 3) $\frac{\sqrt{a^2 + b^2}}{2}$
 e. $m \overline{AM} = m \overline{BM} = m \overline{CM}$

- f. This representation makes it possible to work more easily with the coordinates of the triangle's vertices, because these include a minimal number of variables.

Technomath

Page 14

- a. 1) The right triangle has been moved.
 2) The dimensions of the right triangle have been changed.
 3) The dimensions of the right triangle and the slope of segment AB have been changed.
- b. 1) For example, in Screen 3: $\frac{1.7 + 6.7}{2} = 4.2$.
 2) For example, in Screen 3: $\frac{1.5 + 4.7}{2} = 3.1$.
- c. 1) They are identical or nearly identical.
 2) Screen 4: ≈ 5.94 cm Screen 5: ≈ 10.57 cm
 Screen 6: ≈ 10.8 cm
- d. 1) No, there is no relation between the slope of a segment and its length.
 2) The slope is 0.
 3) The slope is undefined.

Practice 1.1

Page 17

1. a) Several answers possible. Example: Star B can appear larger than Star A.
 b) $(15, 5\sqrt{3})$, $(-15, 5\sqrt{3})$, $(15, 15\sqrt{3})$, $(-15, 15\sqrt{3})$ and $(0, 20\sqrt{3})$.
2. a) 8°C b) -92°C
 c) -92°C d) 8°C or -92°C .
3. Several answers possible. Example:
 Because $(5 - 2)^2 = (2 - 5)^2$ since $3^2 = (-3)^2$. The same explanation applies for $(7 - 3)^2 = (3 - 7)^2$.
4. a) $2\sqrt{13}$ u or ≈ 7.21 u. b) $\sqrt{349}$ u or ≈ 18.68 u.
 c) $20\sqrt{58}$ u or ≈ 152.32 u.
5. a) A parallelogram. b) A parallelogram.
 c) A rectangle. d) A trapezoid.

Practice 1.1 (continued)

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6. a) 1:3 b) 1:3 c) $\frac{1}{2}$
7. a) Right triangle. b) Equilateral triangle.
 c) Isosceles triangle. d) Scalene triangle.
8. a) Lesser slope. b) Lesser slope.
 c) Greater slope. d) Equal slope.
9. a) 4 b) 3 c) 0 d) $\frac{11}{4}$
10. a) (3, -4) b) (0, 8) c) $(\frac{7}{2}, 1)$ d) $(0, 1)$
 $(\frac{5}{3}, -2)$ $(1, 0)$

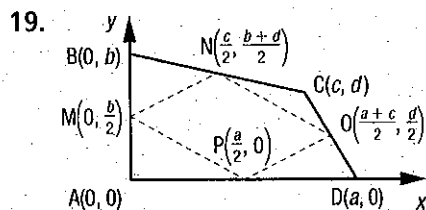
Practice 1.1 (continued)

11. $2\pi\sqrt{53} u$ or $\approx 45,74 u$
12. a) $M_1(0, a)$ $M_2(b + c, a)$
 b) 1) 0 2) 0 3) 0
 c) 1) $2b$ 2) $b + c$ 3) $2c$
 d) Yes, since the slopes of these three segments are equal and since $b + c$ is equivalent to half of the sum of $2b$ and $2c$.
13. (7, 13)
14. The coordinates of City B are (0, 14) ou (0, -2).

Practice 1.1 (continued)

15. a) $\approx 138.77 \$$ b) $(-\frac{3}{2}, \frac{5}{2})$ c) $(\frac{22}{7}, 1)$
16. a) $(\frac{11}{2}, 0)$ b) 1.5 m
 c) $\frac{3}{2}\sqrt{17} m$ or $\approx 6.18 m$.
17. 164 882 students
18. a) 90° b) 0 c) -10
 d) This ratio involves a division by 0.
 e) The slope is undefined.

Practice 1.1 (continued)



Consider quadrilateral ABCD and points M, N, O and P being the respective midpoints of sides AB, BC, CD and AD. Calculate the slope of segments MN, OP, NO and MP.

Slope of \overline{MN}	Slope of \overline{OP}
$\frac{\frac{b+d}{2} - \frac{b}{2}}{\frac{c}{2}} = \frac{d}{c}$	$\frac{\frac{d}{2}}{\frac{a+c}{2} - \frac{a}{2}} = \frac{d}{c}$

Slope of \overline{NO}	Slope of \overline{MP}
$\frac{\frac{d}{2} - \frac{b+d}{2}}{\frac{a+c}{2} - \frac{c}{2}} = \frac{-b}{a}$	$\frac{\frac{b}{2}}{\frac{a}{2}} = \frac{-b}{a}$

Quadrilateral MNOP is a parallelogram.

20. a) $(-\frac{3}{2}, \frac{1}{2})$ b) $\frac{1}{2}\sqrt{170} km$ or $\approx 6.52 km$.
21. a) $\frac{2}{9}$ b) $10\sqrt{85} m$ or $\approx 92.20 m$.
 c) $\frac{20\sqrt{85}}{7} m$ or $\approx 26.34 m$.

SECTION 1.2

Lines in the Cartesian plane

Problem

The water bomber will dump 36 822 L of water during this operation.

Activity 1

- a. $y = -\frac{12}{5}x + 118$
- b. Slope: $-\frac{12}{5}$, intercept: 118.
- c. 1) Variable y should be replaced by 0, then the equation can be solved.
 2) All the terms should be grouped together on the same side of the equals sign.
- d. 1) 1 2) -2 3) 120
- e. Variable y , usually situated to the left of the equals sign, should be isolated.
- f. 1) The slope is equivalent to $-\frac{A}{B}$.
 2) The y -intercept is equivalent to $-\frac{C}{B}$.
 3) The x -intercept is equivalent to $-\frac{C}{A}$.

Activity 2

- a. $m_{\overline{AC}} = \sqrt{(3 - -6)^2 + (7 - -1)^2} = 3\sqrt{13}$
 $m_{\overline{BC}} = \sqrt{(0 - -6)^2 + (-8 - -1)^2} = 3\sqrt{13}$
 $m_{\overline{AB}} = \sqrt{(3 - 0)^2 + (7 - -8)^2} = 3\sqrt{26}$
 Since $(m_{\overline{AC}})^2 + (m_{\overline{BC}})^2 = (m_{\overline{AB}})^2$, triangle ABC is a right triangle.
- b. 1) $-\frac{3}{2}$ 2) $\frac{2}{3}$
- c. $-\frac{3}{2} \times \frac{2}{3} = -1$
- d. 1) 1 2) 1
- e. The slopes are equal.
- f. Lines l_4 and l_5 are parallel.
- g. Quadrilateral DEFG is a right trapezoid.

Technomath

- a. Several answers possible. Example: (-2, 2.93), (0, 3.43), (3.6, 4.33), (4.8, 4.63)
- b. The coefficient of x is the same.
- c. $0.25 \times -4 = -1$
- d. $y = -\frac{4}{5}x - \frac{11}{30}$
- e. The y -intercept of line $l_1 = 3.47$.
 The y -intercept of line $l_2 = -1.5$.
 The y -intercept of line $l_3 \approx 2.23$.

- f. Line l_2 is parallel to line l_1 and it is perpendicular to line l_3 .
- g. 1) The slope is zero. The equation is of the form $y = k$.
 2) The equation is no longer written in function form. It is written under the form $x = h$.

Practice 1.2

Page 28

1. a) $y = \frac{11}{14}x + \frac{5}{7}$ b) $y = \cancel{x} + 3$
 c) $x + \cancel{\frac{8}{7}} = 0$ d) $y = -\frac{2}{3}x + 6$
2. a) 1) $y = 4x - 13$ 2) $4x - y - 13 = 0$
 b) 1) $y = 5x - 5$ 2) $5x - y - 5 = 0$
 c) 1) $y = \frac{4}{5}x + \frac{13}{5}$ 2) $4x - 5y + 13 = 0$
 d) 1) $y = -8x + 17$ 2) $8x + y - 17 = 0$

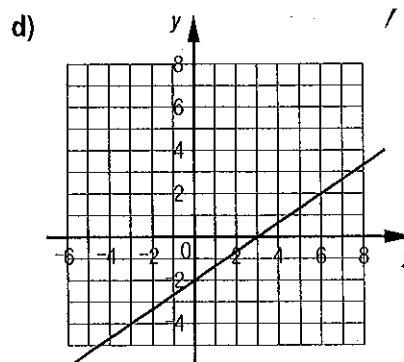
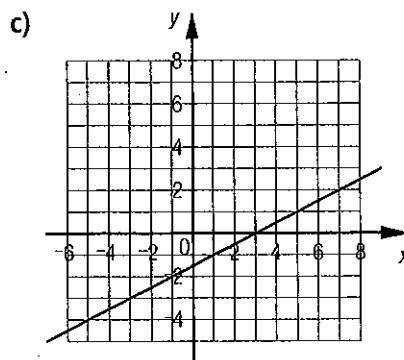
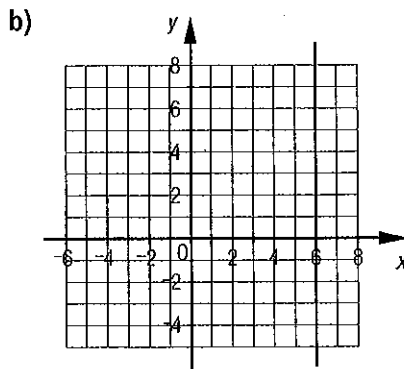
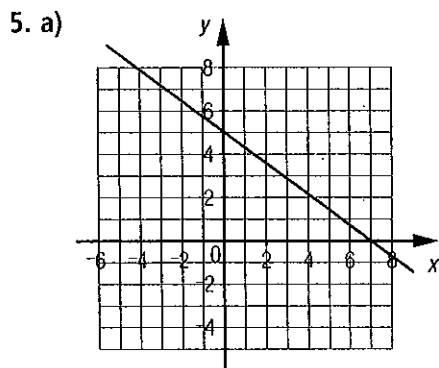
Practice 1.2 (continued)

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3.

	Slope	y intercept	x intercept
a)	-1	23	23
b)	-12	5	$\frac{5}{12}$
c)	-1	15	15
d)	3	$-\frac{19}{2}$	$\frac{19}{6}$
e)	-1	7	7
f)	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{5}{3}$

4. a) $y = 2x + 3$ or $2x - y + 3 = 0$.
 b) $y = 4$
 c) $x = -2$
 d) $y = -\frac{1}{2}x + 4$ or $x + 2y - 8 = 0$.
 e) $y = -\frac{5}{4}x + 17$ or $5x + 4y - 68 = 0$.



6. a) 1) $2x + y + 1 = 0$ 2) $x - 7y + 14 = 0$
 3) $4x + 5y + 165 = 0$
 b) 1) $y = x + 15$ 2) $y = -x + 112$
 3) $y = 5x + \frac{5}{2}$
7. $y = -\frac{1}{4}x$
8. Tables of values ① and ②.

Practice 1.2 (continued)

Page 30

9. a) Lines ①, ② and ④ are parallel to each other, and Lines ③ and ⑤ are parallel to each other.
 b) There are no lines perpendicular to each other in this list.
10. a) $5x - 3y - 14 = 0$ b) $y = -\frac{3}{5}x + 1$
 c) Triangle ABC is a right triangle, since the product of the slopes of line AB and line AC is equal to -1.
11. $k = 4$

12. 3750 cm^2

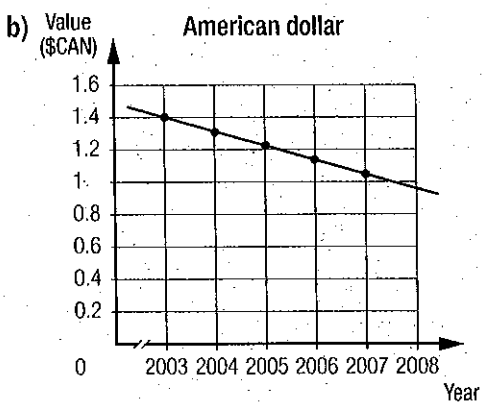
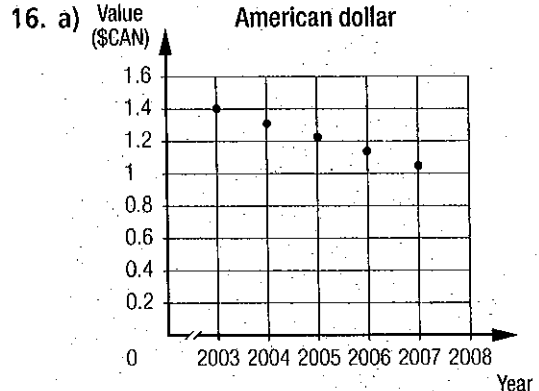
13. a) 30
 b) $2.5x - y + 30 = 0$
 c) Several answers possible. Example : $y = -0.4x + 1$.

- b) 1) $(-\frac{8}{3}, 0)$
 2) $(\frac{8}{3}, 0)$
 3) $(-\frac{16}{3}, \frac{8}{3})$
 4) $(\frac{16}{3}, \frac{8}{3})$

20. a) (11, 9) b) $\approx 2.77 \text{ min}$ or $\approx 2 \text{ min } 46 \text{ s}$.

Practice 1.2 (continued) Page 31

14. $8x + 10y - 91 = 0$
 15. a) -0.25 b) $0.25x + y - 550 = 0$



- c) Several possible answers.
 Example: $y \approx -0.08x + 167.64$
 d) $\approx \$1.47$

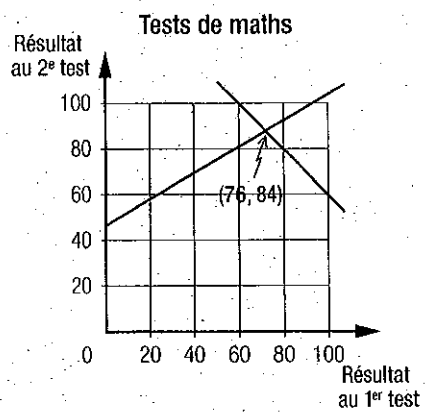
SECTION 1.3 Systems of equations

Problem Page 34

The two plants will have reached the same height in 40 days.

Activity 1 Page 35

- a. $\frac{a+b}{2} = 80$
 $a = 2b - 92$
 b. 1) The two variables are situated on the same side of the equals sign.
 2) One of the variables is situated to the left of the equals sign and the other, to the right.
 c. $\frac{3b-92}{2} = 80$
 d. His result in the 2nd test is 84.
 e. This student obtained 76 on the 1st test.
 f. The coordinates of the point of intersection correspond to the marks of the two tests.



Practice 1.2 (continued) Page 32

17. a) 1) $y = -\frac{1}{2}x + 45$ 2) $y = 2x - 30$
 3) $y = 0$ 4) $y = 2x$
 b) (15, 0)
 c) 1) $\approx 102.21 \text{ km}$ 2) 495 km^2
 18. a) $y = -\frac{1}{3}x + \frac{9}{2}$ b) $\approx 6.32 \text{ m}$ on 9 m .
 c) 1.73 m

Activity 2 Page 36

- a. $30x + 60y = 1275$
 $90x + 30y = 1950$
 b. 1) The two variables are situated on the same side of the equals sign.
 2) The two variables are situated on the same side of the equals sign.

Practice 1.2 (continued) Page 33

19. a) 1) $y = x + 8$ 2) $y = -x + 8$
 3) $y = -x - \frac{8}{3}$ 4) $y = x - \frac{8}{3}$

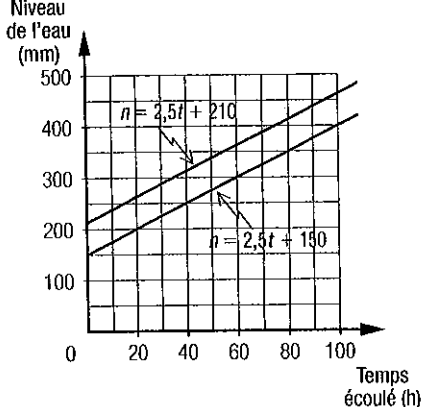
- c. 1) $30x + 60y = 1275$
 $180x + 60y = 3900$
 2) No, since two systems of equivalent equations have the same solution.
- d. It is the same coefficient, 90.
- e. $150y = 1875$
- f. 1) 12.5 kcal/min 2) 17.5 kcal/min

Activity 3

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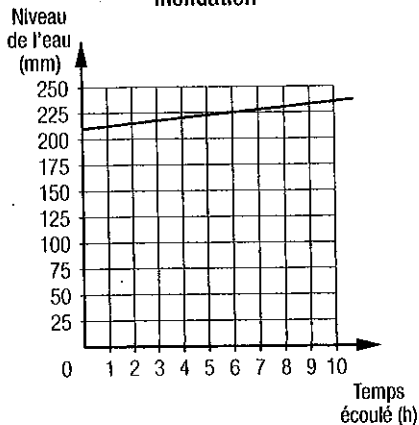
a. $n = 2.5t + 150$
 $n = 2.5t + 210$

b. Inondation



- c. The two lines are parallel and non-coinciding.
- d. The water level of the two rivers will never be the same. The solutions of the equation $n = 2.5t + 150$ are totally different from the solutions of the equation $n = 2.5t + 210$ since the two lines corresponding to these equations are parallel and non-coinciding.
- e. $n = 2.5t + 210$
 $n = 2.5t + 210$

f. Inondation



- g. The two lines are parallel and coinciding.
- h. The water level of the two rivers is always the same. The solutions to the two equations $n = 2.5t + 210$ are the same since the two lines corresponding to these equations are parallel and coinciding.

Technomath

Page 38

- a. (-4.8, 3.2)
- b. The increment of scale. The x-axis is graduated in increments of 1. The y-axis is graduated in increments of 1.
- c. (1, 3), (-5, -15), (7, -27)

Practice 1.3

Page 41

1. a) (0, 2) b) (40, 18)
 c) An infinite number of solutions.
 d) No solution.
2. a) (7, 3) b) (8, 7) c) (-7, -3)
 d) $(\frac{1}{2}, \frac{1}{2})$ e) (1, -6) f) (21, 1)
3. Several answers possible.
 Example: $-5x + 2y - 10 = 0$.
4. $y = \frac{4x + 5}{3}$

Practice 1.3 (continued)

Page 42

5.	1) Identifying unknowns	2) System of equations
a)	x: number of documents y: =total cost (\$)	$y = 0.06x + 25$ $y = 0.31x$
b)	x: price of a sweater (\$) y: price of a pair of pants (\$)	$3x + 2y = 125$ $4x + 3y = 180$
c)	x: length of the field (m) y: width of the field (m)	$2x + 2y = 248$ $x = 3y$
d)	x: mass of a bottle (g) y: mass of a glass (g)	$2x + 5y = 440$ $3x + 3y = 534$

3) Solution

- a) (100, 31): For 100 documents, the total cost is the same and a pair of pants costs \$40.
- b) (15, 40): A sweater costs \$15 and a pair of pants costs \$40.
- c) (93, 31): The field measures 93 m by 31 m.
- d) (150, 28): The mass of a bottle is 150 g and the mass of a glass is 28 g.

6. (24, 31)
7. $m \angle A \approx 44.67^\circ$ and $m \angle C \approx 25.33^\circ$.
8. 4880 km

Practice 1.3 (continued)

9. a) $k = 4$ b) Every value of k other than 4.
 10. $y = \frac{1}{2}x + 8$ and $y = \frac{7}{2}x + 23$.

11.

	1) System of equations	2) Mass of objects
a)	x : mass of a marble (g) y : mass of a cube (g) $5x + 4y = 92$ $2x + 2y = 40$	(12, 8): The mass of a marble is 12 g and the mass of a cube is 8 g.
b)	x : mass of a marble (g) y : mass of a cube (g) $4x + 3y = 4000$ $y = 250 + 3x$	(250, 1000): The mass of a marble is 250 g and the mass of a brick is 1000 g.
c)	x : mass of a marble (g) y : mass of a cube (g) $x = 41 + 7y$ $x = 77 + 4y$	(125, 12): The mass of a marble is 125 g and the mass of a cube is 12 g.

12. A croissant costs \$1.35 and a coffee costs \$1.15.
 13. The piggybank contains 28 \$1 coins and 31 \$2 coins.

Practice 1.3 (continued)

14. a) $m \overline{AD} = 55$ m and $m \overline{BC} = 11$ m.
 b) 3289 m² c) $\approx 16\,265.11$ m³
 15. Company ③, because, after eight months, its assets will have surpassed the assets of the other two companies.
 16. a) Automatic sprinkler at B: (-4, 8). Automatic sprinkler at C: (6, 3).
 b) ≈ 15.63 m³
 17. The temperature was hotter yesterday, at 25°C or 77°F.

SECTION 1.4

Half-planes in the Cartesian plane

Problem

Several answers possible. Example: When considering that the electricity company rounds off the amount of the bill to the nearest hundredth, the Tremblay family spent \$55 on electricity consumption during the month of July. Since they had planned to spend less than \$55 in electricity for this month — i.e. \$54.99 or less — they did not, meet their budget.

Activity 1

- a. No. If the distance covered by cycling is 15 km, it will be equal to and not greater than three times the 5 km travelled by running.

- b. There can be any quantity greater than or equal to 13.2 g.
 c. 30 800 001 people.
 d. No. If there are 8 births and 2 deaths, the difference between these two numbers is 6, which is not less than or equal to 3.
 e. Each of these situations has an infinite number of solutions.

Activity 2

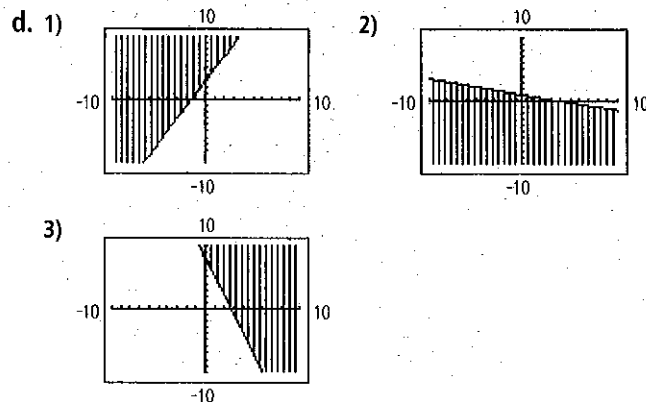
- a. $y > \frac{4}{3}x$
 b. 1) No. 2) No. 3) Yes. 4) Yes.
 c. The building's possible height is less than 9 m.
 d. 1) When the symbol of inequality is ">," the shaded half-plane is located above the boundary line.
 2) The line is a dotted line since the symbol of inequality ">" means "greater than."
 e. 1) No. Any point located below the dotted line will not validate the inequality $y > \frac{4}{3}x$.
 2) Yes. Any point located above the dotted line will validate the inequality $y > \frac{4}{3}x$.
 3) No. Any point located on the dotted line will not validate the inequality $y > \frac{4}{3}x$.
 f. 1) The inequality becomes $y \geq \frac{4}{3}x$.
 2) The boundary line of equation $y = \frac{4}{3}x$ corresponds to a solid line instead of a dotted line.
 3) The coordinates of the points located on the boundary line are part of the solution set.

Technomath

- a. 1) $y \geq x + 14$ 2) $y \leq -0.5x - 13$
 b. 1) $y \geq x + 14 \Rightarrow -10 \geq 16 + 14 \Rightarrow -10 \geq 30$ is false, so the coordinates (16, -10) do not belong to the solution set for the inequality.
 2) $y \leq -0.5x - 13 \Rightarrow -15 \leq -0.5 \times -23 - 13 \Rightarrow -15 \leq -1.5$ is true, so the coordinates (-23, -15) belong to the solution set for the inequality.

c. *Several answers possible. Example:*

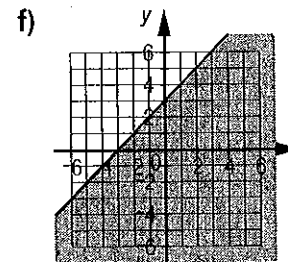
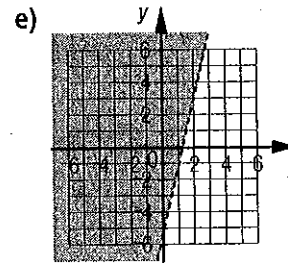
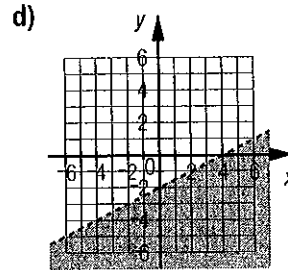
- 1) (-10, -10) 2) (-40, 1)



Practice 1.4

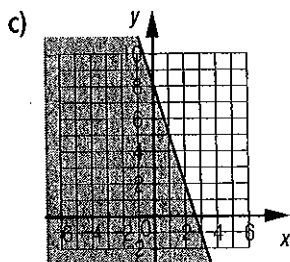
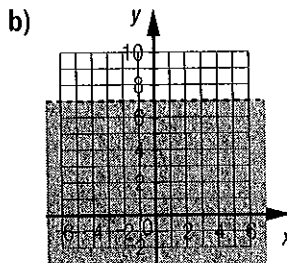
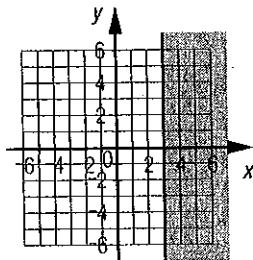
1. a) $L - l \geq 250$ b) $f > 2g$
 c) $100x + 50y \leq 6000$ d) $x + y < 500$
 e) $\frac{v}{p} > 12$
2. a) $y \geq -\frac{4}{3}x + 2$ b) $y \leq \frac{1}{2}x + \frac{1}{5}$
 c) $y \geq 7x - 77$ d) $y \leq -\frac{3}{2}x + 3$
 e) $y \leq -\frac{1}{10}x - \frac{1}{10}$ f) $y \geq -\frac{5}{2}x + 3$

3.	1) Identification of variables	2) Inequality
a)	x: quantity of wheat harvested y: quantity of barley harvested	$x \geq 2y$
b)	x: Simon's age y: his father's age	$3x + y < 86$
c)	x: surface area of Canada y: surface area of France	$x \leq 14y$
d)	x: surface area of Lake Superior (km ²) y: surface area of Lake Ontario (km ²)	$x \geq 4y + 5000$



Practice 1.4 (continued)

4. A and 2, B and 4, C and 1, D and 3
5. a)



6. Screen 1: $y < -2.5x - 1.5$
 Screen 2: $y < -0.25x + 1$

Practice 1.4 (continued)

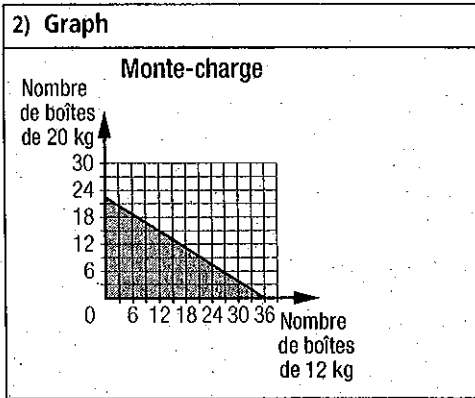
7. a) $y \leq -\frac{3}{2}x + 1$ b) $y < -\frac{1}{5}x + 40$
 c) $y \geq \frac{1}{2}x - 5$ d) $y > 5x - 25$
8. a) $0.05a + 0.06b \leq 207$
 b) $0.05a + 0.06b = 207$
 c) Yes, since the inequality sign is \leq .
 d) Region A.

Practice 1.4 (continued)

9. a) $\frac{(B + b) \times 3}{2} < 18$
 b) Several answers possible. Example: 3.5 cm and 8 cm, 8.5 cm and 3 cm, 9 cm and 2 cm.
 c) No. With these dimensions, the area of the right trapezoid would be equal to 18 cm² instead of being less than 18 cm².
10. A maximum of 54 camper vans can be accommodated in this campground.

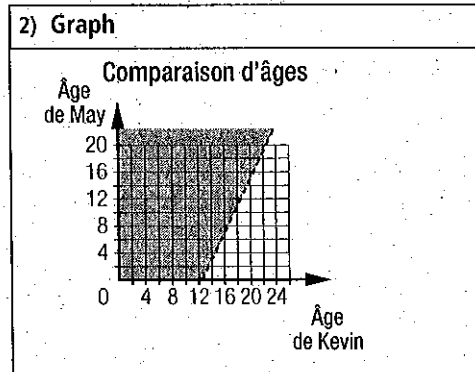
11. a) 1) Inequality

x : number of 12-kg boxes
 y : number of 20-kg boxes

$$12x + 20y \leq 440$$


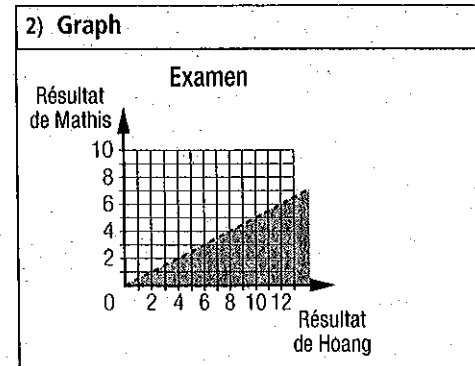
b) 1) Inequality

x : Kevin's age
 y : May's age

$$2x - y < 25$$


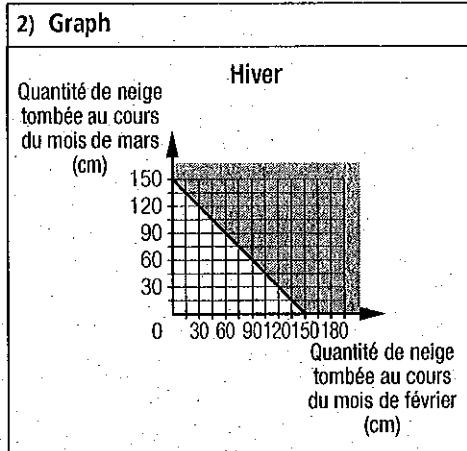
c) 1) Inequality

x : mark earned by Hoang
 y : mark earned by Matthew

$$x > 2y$$


d) 1) Inequality

x : amount of snowfall during the month of February (cm)
 y : amount of snowfall during the month of March (cm)

$$x + y \geq 150$$


12. 57 solutions.

13. $\frac{44}{3}$

Practice 1.4 (continued)

14. The operation will take 225 min to 300 min.

15. 23.2 dam²

16. Inequality \textcircled{D} .

17. Several answers possible. Example:

- a) $y \leq x$ b) $y \leq \frac{3}{4}x + 6$
c) $y \geq -\frac{1}{5}x + 5$ d) $y \leq x + 10$

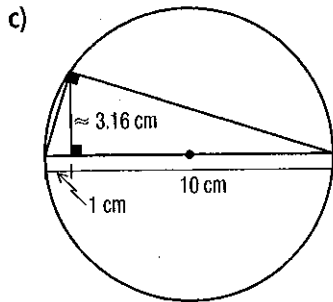
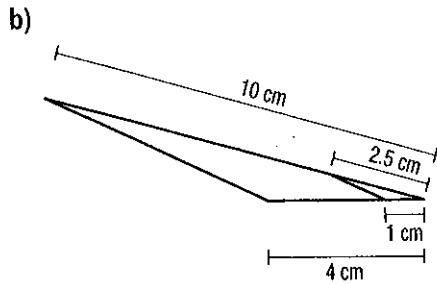
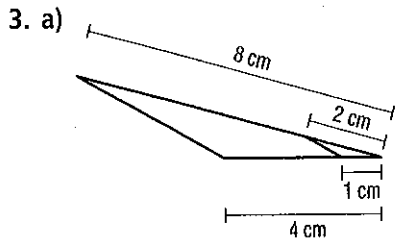
18. a) x : time (in s) to cover the distance between the departure point and point A
 y : time (in s) to cover the distance between point A and the destination point
- b) During a maximum of 120 s.
c) During a maximum of 180 s.
d) The object took a maximum of 100 s.

SPECIAL FEATURES 1

Chronicle of the past

1. $2x^2 + 2x + 4 = 0$

2. Several answers possible. Example: $9cc + 5c.5$.



In the workplace

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- Since Airplane B will arrive first by approximately 21.73 s, there will not be a collision if Airplane B measures less than ≈ 86.94 m.
- In ≈ 1.62 h or ≈ 1 h 37 min.
- Approximately 23.17 km from the airport.

Overview

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1.

	1) Length	2) Slope	3) Coordinates of the midpoint
a)	$3\sqrt{13} u$ or $\approx 10.82 u$	$\frac{2}{3}$	$(\frac{3}{2}, 1)$
b)	$\sqrt{89} u$ or $\approx 9.43 u$	$-\frac{8}{5}$	$(-\frac{3}{2}, 1)$

2. a) (12, 16)
c) $(-4, \frac{9}{2})$

- b) (3, 12)
d) (59, 40)

3. Slope $\overline{AD} = \frac{2b-b}{a-0} = \frac{b}{a}$ Slope $\overline{BC} = \frac{b-0}{2a-a} = \frac{b}{a}$
 Slope $\overline{AD} = \text{Slope } \overline{BC}$ Slope $\overline{AB} = \frac{2b-b}{a-2a} = -\frac{b}{a}$
 Slope $\overline{DC} = \frac{b-0}{0-a} = -\frac{b}{a}$ Slope $\overline{AB} = \text{Slope } \overline{DC}$

4. a) (4, -9) b) $(\frac{9}{2}, -\frac{9}{2})$ c) $(\frac{2}{3}, \frac{7}{3})$
 d) (-2, -1) e) (-12, -4) f) $(-\frac{127}{50}, \frac{13}{5})$

Overview (continued)

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5. a) $y \geq -3x$ b) $y > \frac{3}{5}x - 20$
 c) $y < -\frac{1}{4}x - 1$ d) $y \leq 12x + 2$
6. a) $2x + y < 80$
 b) 1) No. 2) Yes. 3) No.
 4) Yes. 5) No. 6) No.
7. a) Lines l_1 and l_2 are parallel and non-coinciding.
 b) Lines l_3 and l_4 are perpendicular.
 c) Lines l_5 and l_6 intersect.

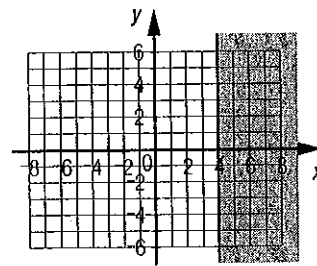
Overview (continued)

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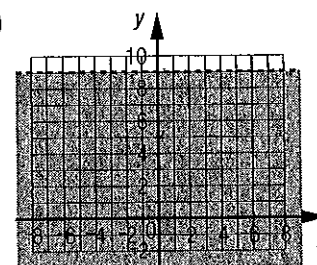
8. a) 1) e : height (in m) of Mount Everest
 i : height (in m) of Mount Iberville
 2) $e \geq 4i + 300$
- b) 1) c : number of gold medals collected by China
 g : number of gold medals collected by Greece
 2) $c > 3g$
- c) 1) u : population of the United States
 c : population of Canada
 2) $u \geq 9c$
- d) 1) v : mass (in kg) of the 10-wheel vehicle
 c : mass (in kg) of the cargo
 2) $v + c \leq 25\ 250$
9. a) $y = 3x - 5$ b) $y = x + 8$

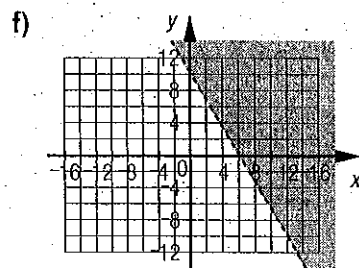
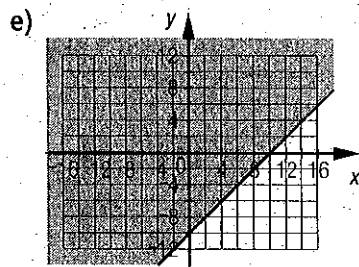
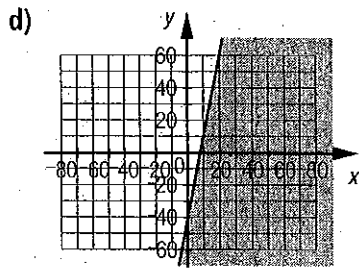
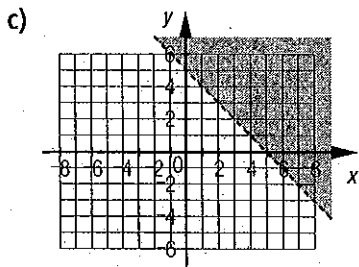
10. $20 u^2$

11. a)



b)





Overview (continued)

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12. a) $s_A = 0.04v + 200$
 $s_B = 0.05v$
 where v corresponds to the total amount of sales
 and s corresponds to the weekly salary.
- b) (20,000, 1,000).
- c) For weekly sales under \$20,000, Dealership A's offer is more attractive.
 For weekly sales over \$20,000, the offer from Dealership B is more attractive.
 For weekly sales of \$20,000, the two dealers offer the same salary, which is \$1,000.
13. 784 m²
14. 155 mL of pineapple juice.
15. \$59.95/m²

Overview (continued)

Page 64

16. a) ≈ 3316.46 km b) (50, 450)
 c) (12, 70) d) $y = -8x + 166$
17. $y = -14$
18. a) ≈ 12 h 31 min b) ≈ 16 h 29 min
 c) ≈ 17 h 56 min

Overview (continued)

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19. There are 9 blue marbles, 10 yellow marbles and 6 green marbles in the jar.
20. a) $(1, -\frac{4}{3})$ b) $(\frac{4}{3}, 0)$
 c) $(\frac{251}{226}, \frac{56}{113})$ d) $(-\frac{7}{2}, -\frac{3}{2})$
21. You need to mix 5 mL of 5% hydrochloric acid and 5 mL of 20% hydrochloric acid.
22. The singer stands at the point (0.625, 6.75) or $(\frac{5}{8}, \frac{27}{4})$.

Bank of problems

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23. The width of the green space is approximately 46.21 m.
24. 135 residences must be evacuated.

Bank of problems (continued)

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25. Driver A's strategy is the most efficient with a time of approximately 3.15 h or ≈ 3 h 8 min 53 s. Driver B's time is approximately 3.15 h or ≈ 3 h 9 min 14 s.
26. The possible dimensions of the rectangular garden are 19 m by 29 m, 23 m by 34 m, 27 m by 39 m or 31 m by 44 m.

LES 3 Carbon exchange

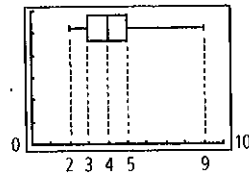
Below is an approach that will allow you to analyze all of the data concerning greenhouse gas emissions by the companies in the four countries registered in the carbon exchange.

- Use diagrams, such as the stem-and-leaf plot, to represent the quantities of CO₂ released by the companies in a given country.

Country 1: Quantities of CO₂ released by the companies (tons)

0	4	5	5	5	6	6	7	7	9
1	0	1	1	1	2	4	8		

Country 2: Quantities of CO₂ released by the companies (tons)



Country 3: Quantities of CO₂ released by the companies (tons)

0	5	5	7	8	9	9
1	0	1	2	3	4	9

Country 4: Quantities of CO₂ released by the companies (tons)

0	4	5	5	7	8	9
1	1	2	4	5	5	
2	1	1	2			

- Use a table to represent the difference between the CO₂ emissions and the CO₂ quotas for the companies of each country.

Difference between the CO₂ emissions and the CO₂ quotas for the companies of four countries

	Company															
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Country 1	3	-1	1	1	0	1	-3	-4	1	2	2	-6	5	0	-3	1
Country 2	0	0	2	1	-1	2	1	0	-3	1	2	0	1	1	0	
Country 3	-3	-1	0	-1	-2	-1	-5	-4	-3	1	2	0				
Country 4	1	3	0	2	3	-1	2	-2	3	4	-1	5	1	-1		

- Determine the mean of the differences calculated above in relation to each country.
 Country 1: 0 tons Country 2: ≈ 0.47 tons
 Country 3: ≈ -1.42 tons Country 4: ≈ 1.36 tons

- Determine the mean deviation of the differences calculated above in relation to each country.
 Country 1: 2.125 tons Country 2: ≈ 0.97 tons
 Country 3: ≈ 1.65 tons Country 4: 1.79 tons

- Calculate the percentile of company D in each country based on the data shown in the table in step 2.
 Country 1: 41 Country 2: 37 Country 3: 46 Country 4: 43

- Conclusion

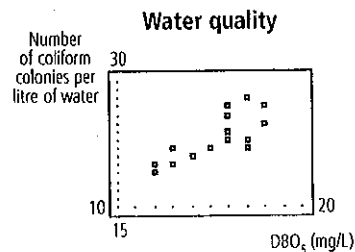
- Among the four countries, Country 3 is the one that most closely meets the quotas since, on average, it is the only country whose companies release less CO₂ than the quotas they are entitled to.
- If, in each country, one compares the performance of Company D with that of the other companies, one can see that Country 3 is the one where Company D has the best performance.

LES 4

Water quality

Below is an approach that will allow you to solve this LES.

- Graphically represent the two-variable distribution.



2. Qualitatively and quantitatively determine the correlation between the two variables.

The contingency table and the scatter plot show a correlation between the two variables that is linear, positive and moderate. Using technology, you can determine that the correlation coefficient is ≈ 0.78 .

3. Determine the equation of a regression line corresponding to this situation using two different methods.

4. Conclusion

The study shows that as the BOD_5 of a river increases, so does the number of coliform colonies, meaning that the correlation is positive. Since the data in the contingency table is arranged along a diagonal of the table and the points in the scatter plot are located along a line, the correlation between the two variables is linear. The linear correlation coefficient ≈ 0.78 indicates that the strength of the correlation between the two variables is moderate.

Below are some examples of recommendations regarding better water management:

- no more refuse disposal in fresh water bodies
- no disposal of fertilizers in fresh water bodies
- using PCB-, PPC-, and phosphate-free products
- filtering domestic waste water
- using an alternative to coal to fuel factories, in order to reduce acid rains that pollute fresh water.

Method 1: The Mayer line method

BOD_5 (mg/L)	Number of coliform colonies per litre of water	BOD_5 (mg/L)	Number of coliform colonies per litre of water
12	19	16	24
12	20	16	24
12	20	16	26
13	20	16	26
13	22	16	27
13	22	17	22
13	22	17	23
14	21	17	23
15	22	17	28
15	22	17	28
16	23	18	25
16	24	18	27

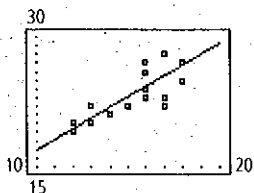
- a) Organize the ordered pairs of the distribution based on their x -coordinates.
- b) Divide all of the ordered pairs into two groups and calculate the mean of the x -coordinates and the mean of the y -coordinates of each group to form ordered pairs of the means.

Group 1: $P_1(13.67, 21.42)$

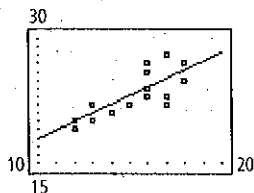
Group 2: $P_2(16.75, 25.25)$

- c) Determine the equation of the line passing through the points found above; this is the regression line.

$$y \approx 1.24x + 4.43$$



Method 2: Using technology



```

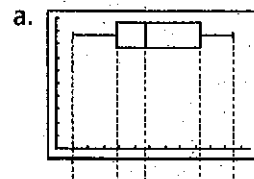
RegLin
y=ax+b
a=1.038370441
b=7.54144935
r^2=.6027828405
r=.776390907
  
```

$$y \approx 1.04x + 7.54$$

REVISION 2

Prior learning 1

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- b. 1) $\approx \$14,500$ 2) $\approx \$14,500$ 3) $\$7,000$
 c. $\approx \$2,060.30$

Knowledge in action

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1. a) 101 mm b) 12 mm
 2. B

3.

Mode	Median	Mean	Range
8	14	15	27
5 and 9	6.5	≈ 7.06	5
44	33.5	≈ 31.07	32
None	20	≈ 20.54	31

4. a) 636 graduates.
 b) [25, 30] years.
 c) 30 years.
 d) [30, 35] years.
 e) ≈ 31.67 years.

5.

Class	Frequency
[10, 20[4
[20, 30[7
[30, 40[5
[40, 50[7
[50, 60[2
[60, 70[7
[70, 80[3
[80, 90[4
[90, 100[5
Total	44

c) Group ① since in this group, for example, there are four data values greater than those in Group ②.

12. The student in Group ①.

SECTION 2.1

Diagrams and statistics

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Problem

Several answers possible. Example:
Yes, Anthony will be selected. He is in the 20th group.

Knowledge in action (continued)

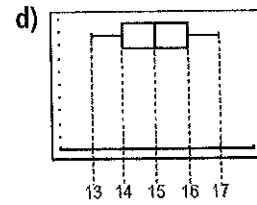
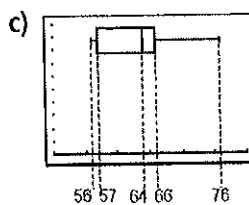
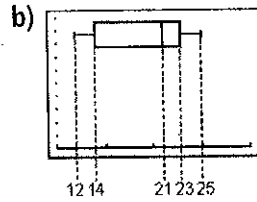
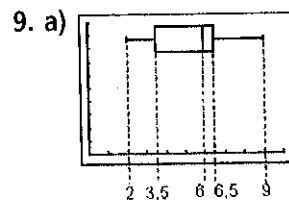
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6.

	1) Mode	2) Median	3) Average
a)	2 children	1.5 children	1.45 children
b)	1 coffee	1 coffee	≈ 1.37 coffee
c)	≈ 200 cm	180 cm	≈ 174.29 cm

7. ≈ 90.33%

8. A



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Activity 1

a. ≈ 314.61 points.

b. 1) Dunnichay 2) Espinosa

c.

Name	Deviation between the result and the mean of the results
Espinosa	≈ 66.34
Ishimatsu	≈ 49.99
Veloso	≈ 41.64
Heymans	≈ 33.59
Ortiz Galicia	≈ 26.29
Marleau	≈ 11.99
Dunnichay	≈ 5.01
Mena Yaima	≈ 28.86
Pineda Zuleta	≈ 44.51
Sae	≈ 47.21
Ortiz	≈ 51.46
Buelvas	≈ 52.81

d. ≈ 38.31 points.

e. The women's final.

f. Guerra, since the deviation between his result and the mean is 84.16 points, while, in Espinosa's case, it is ≈ 66.34 points.

Knowledge in action (continued)

Page 75

10. a) The time in seconds clocked by the 15 swimmers in the competition.

b) Continuous quantitative.

c) Approximately 26.67 to 46.67%.

d) In the 1st quarter.

e) 4 s

11. a)

Measure	Group ①	Group ②
Median	42,5	40
Range	12	21
Interquartile range	7	8

b) We don't know the value of each data value in the distribution.

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Activity 2

a. 1) These numbers represent the number in the tens position in each of the diameters of the oaks listed.

2) 3 data values.

3) 4th line.

4) 6th line.

5) The number in the tens position is the same.

b. 1) 55 cm

2) 68 cm

c. *Several answers possible. Example:* The stem-and-leaf plot allows you to quickly find the mode of the distribution as well as certain trends. In addition, this type of diagram allows you to see all the data in the distribution, in increasing order.

- d. 1) $\approx 38.71\%$ 2) $\approx 9.68\%$
 3) $\approx 74.19\%$ 4) $\approx 12.9\%$

e. **Red oak: diameter of the trunk (cm)**

4	0	5							
5	3	5	5	5	5	9			
6	0	0	3	4	4	5	7	7	8
7	0	0	0	1	2	5	5	9	
8	1	2	3	3					
9	1	6	6	7					
10	2	9							
11	3								

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- a. 1) 18 2) E3
 b. The data values appearing in the formulas come from the range of cells A1 to J5, i.e. columns A to J and lines 1 to 5.
 c. 1) 19 2) 41 3) 85
 d. 1) 46 2) 31.5 3) 31.92
 e. 1) ≈ 11.84 2) 50

Practice 2.1

Page 83

1. a) 11 b) 142 c) 0
 d) 5 e) 16.4 f) 24.78
 2. a) ≈ 3.67 b) ≈ 9.43 c) 0.5 d) 0
 3. At the bars, since she has a better percentile in this event ($86 > 83$) and a better mean deviation ($1.05 > 0.65$).
 4. a) 6 children. b) 4 children.

Practice 2.1 (continued)

Page 84

5. 89
 6. a) 40 b) 52 c) 36
 7.

	Table 1	Table 2
a)	≈ 12.84	≈ 9.47
b)	41	68
c)	99	90

 8. a) 67 b) 35 c) 65 d) 59

Practice 2.1 (continued)

Page 85

9. a) **Mean mass of adult dogs**

0	2	3	4	4	6	6	8	8	9
1	0	3	3	3	4	8			
2	1	4	4	5	6	6	7	9	9
3	4	4	4	6	6	9			
4	7								
5	7	9	9						
6	0	3	4						
7	1								
8	2	7							

- b) 1) 70 2) 13 3) 2
 c) 59 kg
 d) 94

Practice 2.1 (continued)

Page 86

10. a) 40, 60 b) 20, 40 c) 30, 70
 d) *Several answers possible. Example:* 35, 40, 45, 55, 60, 65
 11. It is preferable to get a result of 77% and a percentile of 90, since only 10% of the players are better than we are. In the second instance, 23% of the players are better than we are.
 12. In the 4th quarter.
 13. 50
 14. In Group ②, since, in this group, the mean deviation is much greater for 86%.
 15. In Distribution 1.
 16. $\approx \$1,425.42$

Practice 2.1 (continued)

Page 87

17. a) The run time with cheese.
 b) The run time without cheese.
 c) $\approx 32.49\%$
 18. a) 1) 75% 2) 75%
 b) The range is 30 at the two first stages, and 22 at the final stage. As for the mean deviation, it is ≈ 5.45 at the first stage, ≈ 7.01 at the second stage, and ≈ 6.78 at the final stage.
 c) 1) 29 2) 57 3) 87
 d) 2 people.

19. a) Maximum speed reached during a qualifying run (km/h) Maximum speed reached during the final run (km/h)

	596	170																			
		171																			
		172																			
		173	858																		
964	942	855	172	174	317																
		785	175																		
		270	176	124	140	574	738	996													
745	689	555	048	177	152	167	179	186	199	566	584	617	819								
		851	749	411	178																
			072	179																	
			859	180																	

- b) In the distribution representing the maximum speed reached during the final run. The mean deviation in this distribution is approximately 0.81 km/h, while the mean deviation in the distribution representing the maximum speed reached during a qualifying run is approximately 1.93 km/h.
- c) In the distribution representing the maximum speed reached during a qualifying run.

20. No. In Mexico, temperatures vary less than in Canada.

21. 98th

SECTION 2.2

Qualitative interpretation of correlation

Problem

Page 89

Analyst C.

Activity 1

Page 90

- a. 1) 140 people 2) 95 people
- 3) 31 people 3) 8 people
- 5) At most 9 people
- b. $\approx 28.57\%$
- c. 1) Yes. Based on the data in the table, the two variables seem to vary in the same direction.
- 2) Yes. Based on the data in the table, the greater the dosage, the greater the drop in attacks.

Activity 2

Page 91

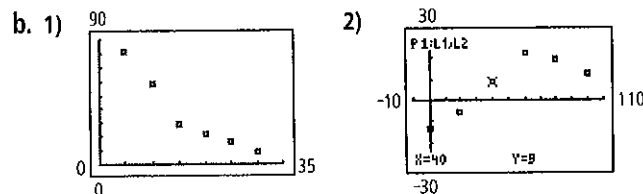
- a. Venus.
- b. Serena.

- c. Yes. It appears that, when Serena's world ranking is high, so is Venus'.
- d. Yes. The points in the scatter plot seem to be arranged along a line with a negative slope.
- e. Yes. It appears that the more games Serena wins, the better her world ranking.
- f. A strong relation. The points are fairly well aligned.

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- a. The x-coordinate and y-coordinate of the point on which the cursor is pointed.



1. Legal age to marry

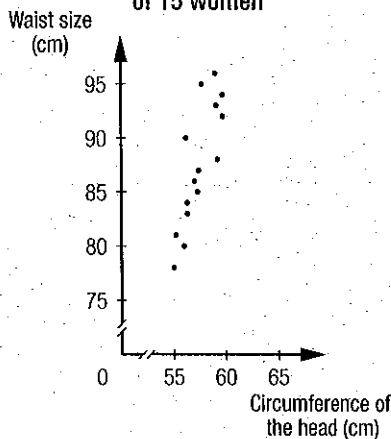
Age of the man \ Age of the woman	18	19	20	21	Total
16	12	0	2	3	17
17	7	0	5	6	18
18	15	0	9	15	39
19	0	0	8	13	21
Total	34	0	24	37	95

2. a) 99 students. b) $\approx 37.37\%$
 c) $\approx 30.3\%$ d) $\approx 33.33\%$

3. Several answers possible. Example:

- a) The greater the distance between where the sweater was bought and where it was sold, the more expensive it is.
 b) It appears that the sweater is more expensive when it was made close to where it was sold.
 c) There is no relation between the distance between where the sweater was made and where it was sold, and the price of the sweater.

4. a) Measurements of 15 women




- b) The correlation between the two variables is linear, positive and strong.
 5. a) Negative correlation.
 b) Positive correlation.
 c) Positive correlation.
 d) Zero correlation.

6. a) Positive and strong correlation.
 b) Zero correlation.
 c) Positive and moderate correlation.
 d) Positive and moderate correlation.


7. a) Start of the 2007-2008 season

Points scored \ Number of games played	[0, 10[[10, 20[[20, 30[[30, 40[[40, 50[Total
[0, 10[1	0	0	0	0	1
[10, 20[4	0	0	0	0	4
[20, 30[3	0	0	0	0	3
[30, 40[2	3	0	0	0	5
[40, 50[0	4	2	4	1	11
Total	10	7	2	4	1	24

- b) 14 players. c) 1 player.
 d) 10 player. e) $\approx 31.58\%$ $\times 25\%$
 f) Yes, since the numbers in the contingency table are relatively aligned along one of the tables' diagonals.

8. 
 9. a) 1) Positive correlation. 2) Moderate correlation.
 b) 1) Positive correlation. 2) Weak correlation.
 c) 1) Positive correlation. 2) Moderate correlation.
 10. No, since no line can be draw from the scatter plot.

11. Yes, the correlation is linear, positive and strong.
 12. a) Several possible answers.
 b) Yes, the correlation is linear, strong and positive.
 c) The quotient comes close to 1.62.
 13. a) Perfect correlation. b) Zero correlation.
 c) Strong correlation.

14. 
 15. Yes, but this relation is very weak.

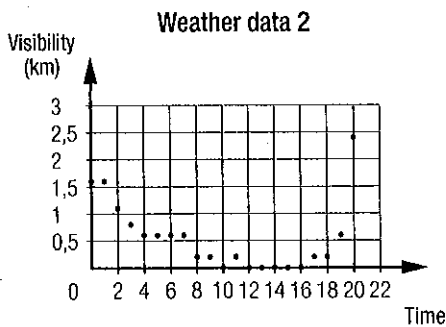
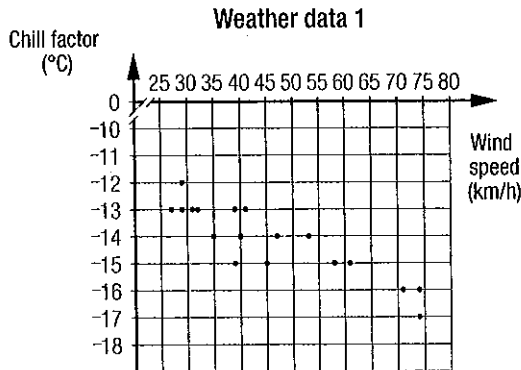
16. a) Several answers possible. Example:

Cegep

Number of hours of class time \ Number of hours of study	Number of hours of class time						Total
	[0, 2[[2, 4[[4, 6[[6, 8[[8, 10[[10, 12[
[0, 2[1	0	0	0	0	0	1
[2, 4[1	3	0	0	0	0	4
[4, 6[0	1	3	0	0	0	4
[6, 8[0	0	1	3	0	0	4
[8, 10[0	0	0	1	3	0	4
[10, 12[0	0	0	0	2	1	3
[12, 14[0	0	0	0	0	1	1
Total	2	4	4	4	5	2	21

b) Linear, positive and strong correlation.

17. Several answers possible. Example:
Wind speed and chill factor. Time and visibility.



18. No. Though the table shows numbers that are aligned, there is no relation between a student's age and his or her favourite colour.
19. No. The scatter plot formed by the two variables does not appear to represent a particular type, nor to have any particular direction or strength.

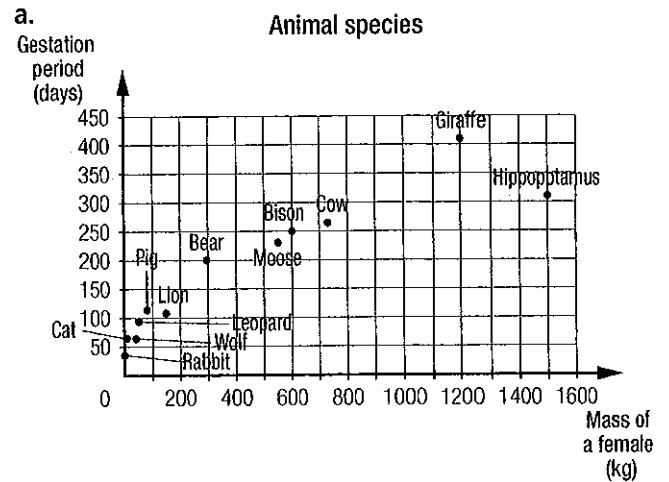
SECTION 2.3

Quantitative interpretation of correlation

Problem

≈ 256.55 kilocalories.

Activity 1



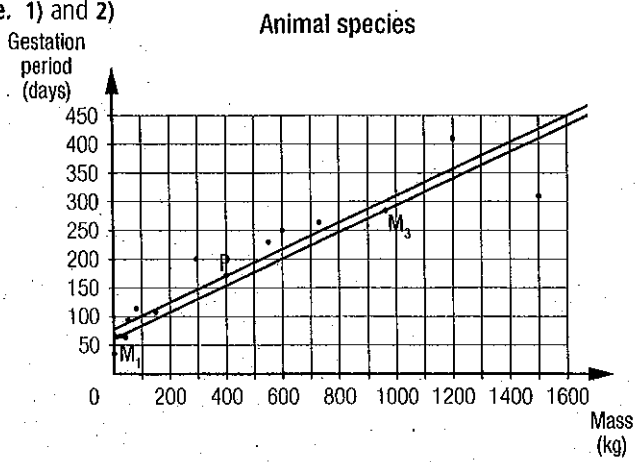
Activity 1 (continued)

b.

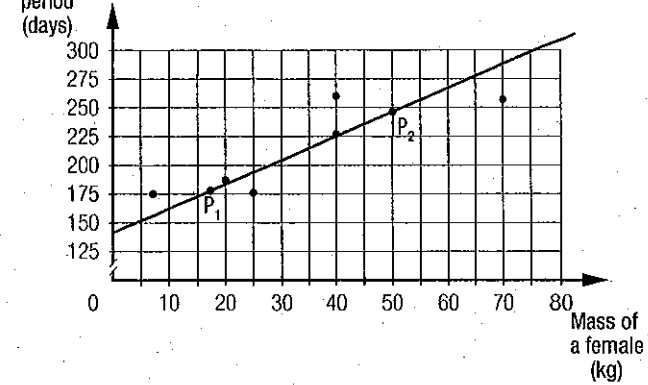
Animal group	Median mass (kg)	Median gestation period (days)	Ordered pair formed by the median mass and the median gestation period
Rabbit, cat, wolf, leopard	24.5	64.5	$M_1(24.5, 64.5)$
Pig, lion, bear, moose	222.5	157	$M_2(222.5, 157)$
Bison, cow, giraffe, hippopotamus	965	287	$M_3(965, 287)$

- c. 1) 404
 d. $\frac{445}{1881} \approx 0.24$
 e. 1) and 2)

2) 169.5



- j. 1) and 2)
 Gestation period (days)



- k. $y \approx 2.1x + 142.9$
 l. 1) ≈ 151.31 days. 2) ≈ 41.44 kg

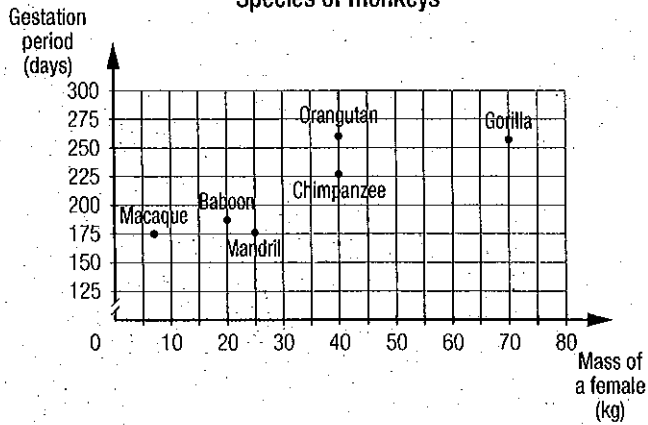
f. $y = \frac{445}{1881}x + \frac{278\ 099}{3762}$ or $y \approx 0.24x + 73.92$

- g. 1) ≈ 334.16 days. 2) ≈ 110.23 kg

Activity 1 (continued)

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h. Species of monkeys



Group of monkeys	Mean mass (kg)	Mean gestation period (days)	Ordered pair formed by the mean mass and the mean gestation period
Baboon, macaque, mandrill	≈ 17.33	≈ 179.33	$P_1(\approx 17.33, \approx 179.33)$
Chimpanzee, gorilla, orangutan	50	248	$P_2(50, 248)$

Activity 2

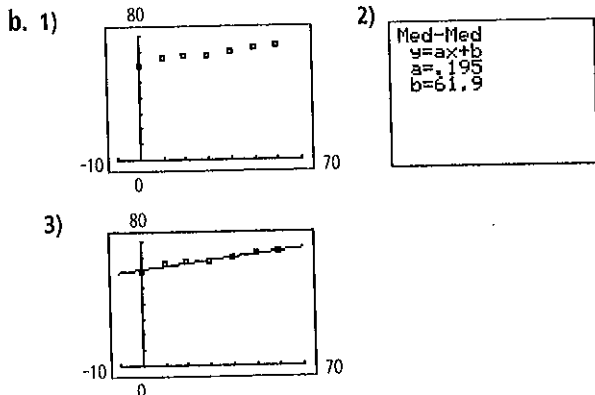
Page 108

- City 1: positive strong correlation.
 City 2: positive moderate correlation.
 City 3: negative moderate correlation.
- In City 3.
- 1) The rectangle associated with the scatter plot representing the data for City 3.
 2) *Several answers possible. Example:* The rectangle is twice as long as it is wide.
- 1) The rectangle associated with the scatter plot representing the data for City 1.
 2) *Several answers possible. Example:* The rectangle is long and narrow.
- 1) The greater the strength of the correlation, the more the statistical study is valid.
 2) The weaker the ratio of $\frac{\text{Measure of the short side}}{\text{Measure of the long side}}$ the stronger the correlation.
- In the case of City 3. It is for the rectangle associated with City 3 that the ratio $\frac{\text{Measure of the short side}}{\text{Measure of the long side}}$ is the greatest, which means that this is the graph in which the line is the least representative of all the points.

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- The value of a represents the slope of the line. The value of b represents the ordinate intercept of the line.



Practice 2.3

1. A 1, B 1, C 3, D 2

2. a) 1) The rectangles all have the same dimensions, 46 mm by 8 mm.
 2) Several answers possible. Example: the slope of the lines in graphs 1 and 2 is negative, while the slope of the line in graph 3 is positive.
 3) The strength of the correlation is the same in all three cases. The correlation coefficient is ≈ -0.83 in graphs 1 and 2, while it is ≈ 0.83 in graph 3.
- b) Graph 1: approximately -0.83 .
 Graph 2: approximately -0.83 .
 Graph 3: approximately 0.83 .

3. C

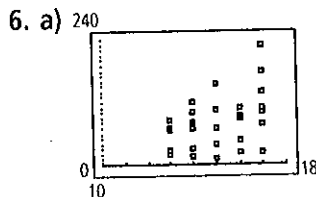
Practice 2.3 (continued)

4. A 3, B 4, C 2, D 1

5.

	Graph a)	Graph b)
1)	≈ -0.77	≈ 0.5
2)	$y \approx -x + 11$	$y \approx 0.97x + 1.4$

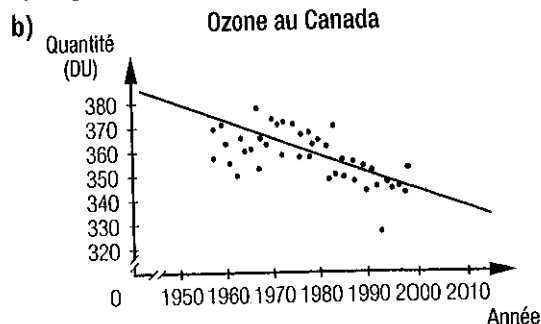
	Graph c)	Graph d)
1)	≈ 0.91	≈ 0.06
2)	$y \approx x + 0.3$	$y \approx 0.15x + 1.7$



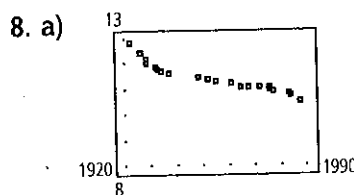
- b) 1) ≈ 0.48
 2) Weak positive correlation.
- c) $y \approx 17.49x - 181.18$
- d) 1) $\approx \$221.17$
 2) No, since the linear correlation is weak.

Practice 2.3 (continued)

7. a) Negative moderate correlation.



- c) $y \approx -0.67x + 1678$
 d) ≈ 338 DU



- b) ≈ -0.95
 c) Linear, negative, and very strong correlation.
 d) $y \approx -0.025x + 60.41$
 e) 1) In 1916. 2) In 2017.

Practice 2.3 (continued)

9. 7, 8, 5, 3, 6, 1, 2, 4

Practice 2.3 (continued)

10. a) Graph 1: ≈ 0.96 b) Graph 2: ≈ 0.85
 c) Graph 3: ≈ -0.44 d) Graph 4: ≈ -0.76
11. a) Table of values 1: $y \approx 0.83x + 0.63$
 Table of values 2: $y \approx -0.59x + 62.16$
 b) Table of values 3: $y \approx 0.7x - 0.34$
 Table of values 4: $y \approx -1.5x + 9.14$

Practice 2.3 (continued)

12. a) ≈ 755.91 min.
 b) ≈ 29.83 kilocalories.
 c) No. According to the equation of the regression line, it would take around 3878 min. Since there are only 1440 min. in a day, it is therefore impossible.
13. a) In graph 2.
 b) 1) Negative strong correlation.
 2) Negative moderate correlation.

Interpretation
of linear
correlation

Problem

Page 119

If the data value that does not appear to follow the trend indicated by the rest of the distribution is excluded, the length of the outstretched arms of a person who is 165 cm tall should be 167.5 cm.

Activity 1

Page 120

- 1) Weak correlation.
2) Strong or very strong correlation.
- On Screen 5, the coefficient is approximately twice that found on Screen 2.
- ≈ 15 objects.
- No. When a data value significantly deviates from the trend exhibited by the other data, the prediction will be more accurate if this data value is not used.

Activity 2

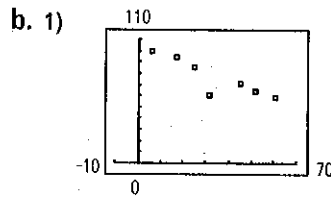
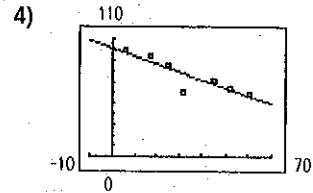
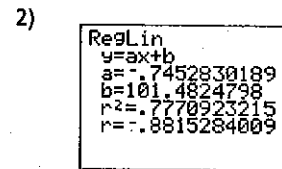
Page 121

- 1) Yes. The points are strongly aligned.
2) Yes. Since the correlation between the variables is strong, we can predict that this student's mark will be $\approx 94.51\%$.
3) Yes. Since the correlation between the variables is strong, we can predict that the snow accumulation will be ≈ 47.33 cm.
- 1) No. The points on the graph are strongly aligned, but there is no relation between a student's mark and the total accumulation of snow. A student can have good marks even when there is no snow.
2) No. There can be a high accumulation of snow without the student improving his or her marks.
- Even though the graph shows a relation between the two variables, it is possible that this may be dependent upon other variables. For example, the greater the snow accumulation, the more a student wants to study, and the more he or she wants to study, the better his or her marks will be.

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- The value of **a** represents the slope of the regression line. The value of **b** represents the ordinate intercept of the regression line. The value of **r** represents the linear correlation coefficient.

3) ≈ -0.88 

Practice 2.4

Page 125

- Ⓐ and Ⓒ.
- 1) ≈ 0.92
2) $y \approx 2.69x - 2.63$
 - 1) ≈ -0.96
2) $y \approx -1.73x + 8.93$
 - 1) ≈ -0.51
2) $y \approx -0.63x + 5$
 - 1) ≈ 0.76
2) $y \approx 1.39x + 0.18$
- Yes. Since the correlation is strong (the correlation coefficient is ≈ 0.85), we can affirm that the older a woman is, the longer her ears are.

Practice 2.4 (continued)

Page 126

- 1) ≈ 0.4
2) ≈ 1
 - $y \approx 0.44x + 37.05$
 - $\approx 74.45\%$
 - 68.8%
- 1) ≈ 0.98
2) ≈ 0.99
 - 1) $y \approx 3.96x + 63.05$
2) $y \approx 5.28x + 41.51$
 - ≈ 16.78 cm
 - ≈ 16.3 cm

Practice 2.4 (continued)

Page 127

- ≈ 91.24 days
b) ≈ 187.87 days.
 - ≈ 139.56 days
d) 100%
- ≈ 1
b) $y \approx 5.62x - 4.54$
 - ≈ 135.9 cm
d) ≈ 41.02 years.

Practice 2.4 (continued)

Page 128

- Several answers possible. Example:
 - the length of a person's hair and his or her height.
 - the effectiveness of an experimental drug.

9. a) 15%
 b) 1) $\approx 38.71\%$ 2) $\approx 22.78\%$
 c) No trees.
 d) A minimum of 346 trees.

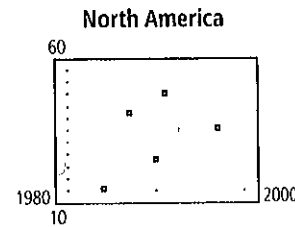
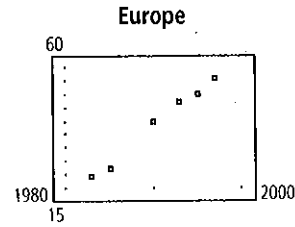
Practice 2.4 (continued)

Page 129

10. a) False. Based on the equation of the regression line associated with this situation, the number of healthy years for an 80-year-old man is ≈ 71.17 years, whereas the number of healthy years for an 80-year-old woman is ≈ 70.17 years.
 b) True. Based on the equation of the regression line associated with this situation, the number of healthy years for women is greater than it is for men.
 c) True. Based on the equation of the regression line associated with this situation, a man whose life expectancy is 55 years should live a healthy life for around 47 years.
 d) False. Based on the equation of the regression line associated with this situation, a woman who will live a healthy life for 59 years has a life expectancy of around 68 years.
 e) False. The linear correlation coefficient associated with this situation is positive and close to 1.

- 4) In Europe. In 1940, the percentage of 12-year-old children in Europe with at least one cavity on a permanent (adult) tooth was $\approx 11.64\%$, as opposed to $\approx 9.46\%$ in North America.

b) 1)



- 2) In North America.
 3) No. As of 1985, Europe had the greatest percentage of 12-year-old children with no cavities.

SPECIAL FEATURES 2

Practice 2.4 (continued)

Page 130

11. a) \$120,400 b) ≈ 57.61 h
 c) 61.18 h d) $\approx \$92,164.82$
 e) 117.7 h

Chronicle of the past

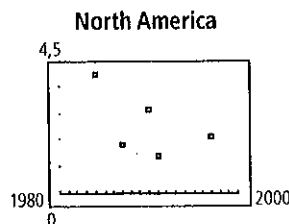
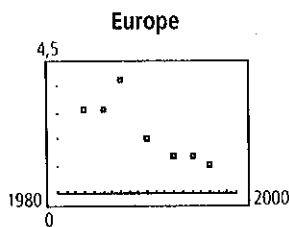
Page 133

1. a) 12 marbles. b) 1.75 marbles.
 2. a) 172.2 cm b) ≈ 158.83 cm
 3. ≈ 0.67
 4. 66

Practice 2.4 (continued)

Page 131

12. a) 1)



- 2) $y \approx -0.19x + 372.76$
 $y \approx -0.14x + 278.36$
 3) North America.

In the workplace

Page 135

1. a) $y \approx 2.62x + 24.21$ b) 181 persons.
 2. a) *Several answers possible. Example: Linear, negative and moderate correlation.*
 b) No. Other factors may come into play, such as the use of an insecticide that would kill an increasing number of bees as years go by, at the same time as the increase in popularity of cell phones.

Overview

Page 136

1. A, B, C, D, 1, 2
 2. 1, 3, 2

3. a) **A**
 b) **A** ≈ 0.91 **B** ≈ -0.7
 C 0 **D** ≈ -0.76
4. **A**

Overview (continued)

Page 137

5. The height of the birches.
6. Group B.
7. **Number of absences in a group of students in the course of a semester**

0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	5	6	7
1	2	3	5	6													
2	0	5	5	6	7	8											
3	0	4	7	7	8												
4	0	1															

8. **A**

Overview (continued)

Page 138

9. a) 75 b) 2.006 m c) Hulk Hogan.
10. a) 12 h b) 20 h
 c) ≈ 20.89 h d) 25 h
11. No. In the contingency table, most of the frequencies are not aligned with one of the diagonals.
12. 63

Overview (continued)

Page 139

13. ≈ 16
14. In the "Deux Rives" Marathon.
15. In 2018.
16. a) 4:53, 4:55, 5:01 b) 4:34, 4:37, 4:45, 4:51
17. a) 70 b) 100 c) 39 d) 2

Overview (continued)

Page 140

18. a) **②** and **③**. b) **①** and **③**. c) **②**
 d) **②** e) **②**
19. Yes. The linear correlation coefficient associated with the situation is ≈ 0.98 .

Overview (continued)

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20. a) 25
 b) Yes, the correlation coefficient is ≈ 0.96 .
 c) 1) 95 2) 2 cousins. 3) 3 cousins.
 d) ≈ 175 cm

Overview (continued)

Page 142

21. a) No. The curve definitely appears to represent the relationship between the families' annual income and the percentage of income used to cover basic needs.
 b) Yes. The scatter plot shows a decreasing trend.
 c) 1) A family with an annual income of around \$18,000, below the low income cut-off, that uses around 45% of its income to cover basic needs.
 2) A family with an annual income of around \$25,000, that uses around 95% of its income to cover basic needs.
 d) 10 families.
22. Based on the correlation coefficient associated with the situation, which is ≈ -0.84 , the affirmation is true.

Overview (continued)

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23. a) In Denmark. b) In Denmark.
 c) ≈ 169.33 cm d) ≈ 184.79 cm
 e) Around 1875. f) $y \approx 0.1x - 27.54$
 g) $y \approx 0.11x - 30.88$ h) Around 2198.

Bank of problems

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24. *Several answers possible.* Example: Mathematically speaking, the incandescent light bulb is more attractive than the fluorescent light bulb since the fluorescent light bulb is 3.26 more expensive than the incandescent light bulb, but only lasts 2.67 times longer. However, one could also choose the fluorescent light bulb in order to preserve the environment, since it is recyclable.
25. Towards the age of 11.

Bank of problems (continued)

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26. ≈ 290 kJ
27. Yes, William will take part in the field trip.

LES 5

Soil fertilization

The following is an approach to verify whether the producer is able to fertilize all of his fields for less than \$18,000.

1. Calculate the area of each field.

- Draw an altitude from vertex E or Field ④ to obtain the triangle DEM.
- Draw a segment joining points B and F so as to obtain the triangle BDF.
- Prove the triangle DEM and triangle BDF are similar.

STATEMENT	JUSTIFICATION
$\angle EDM \cong \angle BDF$	These are all exterior angles of an equilateral triangle.
$\frac{m \overline{DF}}{m \overline{DM}} = 2$	In an equilateral triangle, altitude acts both as a median and a perpendicular bisector.
$\frac{m \overline{DB}}{m \overline{DE}} = 2$	By hypothesis.
$\triangle DEM \sim \triangle BDF$	Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar. (SAS)

- Since $\triangle DEM \sim \triangle BDF$, angle BFD is a right angle. Using the Pythagorean theorem :

$$m \overline{BF} = \sqrt{600^2 - 300^2}$$

$$m \overline{BF} \approx 519.62 \text{ m}$$
- Since segment BF forms a right angle with an extension of segment AD, it represents the altitude of the triangle ABD from point B. The area of the Field ① is:

$$A_1 = \frac{b \times h}{2} = \frac{m \overline{AD} \times m \overline{BF}}{2}$$

$$A_1 \approx \frac{300 \times 519.62}{2}$$

$$A_1 \approx 77\,943 \text{ m}^2$$

- Prove that the triangles formed by the Fields ①, ② and ③ are congruent.

STATEMENT	JUSTIFICATION
$\angle ADB \cong \angle BEC \cong \angle CFA$	These are all the exterior angles of an equilateral triangle.
$\overline{AF} \cong \overline{BD} \cong \overline{CE}$	By hypothesis.
$\overline{AD} \cong \overline{BE} \cong \overline{CF}$	By hypothesis.
$\triangle ABD \cong \triangle BCE \cong \triangle ACF$	Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar. (SAS)

Fields ①, ② and ③ therefore, have the same area of approximately 77 943 m².

- Using the Pythagorean theorem, we obtain:

$$m \overline{EM} \approx \sqrt{300^2 - 150^2}$$

$$m \overline{EM} \approx 259.81 \text{ m}$$

- Calculate the area of the Field ④.

$$A_4 = \frac{b \times h}{2} = \frac{m \overline{DF} \times m \overline{EM}}{2}$$

$$A_4 \approx \frac{300 \times 259.81}{2}$$

$$A_4 \approx 38\,971.5 \text{ m}^2$$

- The total area is equal to the sum of the areas of Fields ① to ④, which is :

$$3 \times 77\,943 + 38\,971.5 \approx 272\,800.5 \text{ m}^2$$

2. Calculate the total quantity of fertilizer required for each field as well as the costs.

Field	Type of fertilizer	Cost (\$/ton)	Cost (\$/kg)	Quantity (kg/m ²)	Area of field (m ²)	Total quantity (kg)	Total quantity (tons)	Total cost (\$)
①	A	450	0.45	0.2	77 943	15 588.6	15.59	7,014.81
②	B	300	0.30	0.1	77 943	7794.3	7.79	2,338.27
③	C	240	0.24	0.25	77 943	19 485.75	19.49	4,676.54
④	A	450	0.45	0.2	38 971.5	7794.3	7.79	3,507.40
							Total	17,537.02

Conclusion

The grain farmer can fertilize all of his fields for less than \$18,000.

LES 6

Roof trusses

The following is an approach to determine the quantity of wood required to construct the shed's roof trusses at the lowest possible cost.

1. Determine the length of each segment of the plan.

Statement	Justification
$m \overline{BG} = 3.75 \text{ cm}$	By Pythagorean theorem, $(m \overline{BG})^2 = (m \overline{BH})^2 + (m \overline{GH})^2$.
$m \overline{AH} = 4 \text{ cm}$	In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse. $m \overline{BH}^2 = m \overline{AH} \times m \overline{GH}$
$m \overline{AB} = 5 \text{ cm}$	By Pythagorean theorem, $(m \overline{AB})^2 = (m \overline{AH})^2 + m \overline{BH}^2$.
$m \overline{BC} = \frac{45}{16}$ or approximately 2.81 cm	In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse. $(m \overline{BG})^2 = m \overline{AB} \times m \overline{BC}$.
$m \overline{CG} = \frac{75}{16}$ or approximately 4.69 cm	By Pythagorean theorem, $(m \overline{CG})^2 = (m \overline{BC})^2 + (m \overline{BG})^2$.
$m \overline{GF} = \frac{225}{64}$ or approximately 3.52 cm	In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse. $(m \overline{CG})^2 = m \overline{AG} \times m \overline{GF}$
$m \overline{CF} = \frac{375}{64}$ or approximately 5.86 cm	By Pythagorean theorem, $(m \overline{CF})^2 = (m \overline{CG})^2 + (m \overline{FG})^2$.
$m \overline{CD} = \frac{1125}{256}$ or approximately 4.39 cm	In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse. $(m \overline{CF})^2 = m \overline{AC} \times m \overline{CD}$
$m \overline{DF} = \frac{1875}{256}$ or approximately 7.32 cm	By Pythagorean theorem, $(m \overline{DF})^2 = (m \overline{CD})^2 + (m \overline{CF})^2$.
$m \overline{FE} = \frac{5625}{1024}$ or approximately 5.49 cm	In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse. $(m \overline{DF})^2 = m \overline{AF} \times m \overline{FE}$
$m \overline{DE} = \frac{9375}{1024}$ or approximately 9.16 cm	By Pythagorean theorem, $(m \overline{DE})^2 = (m \overline{DF})^2 + (m \overline{FE})^2$.

2. Determine the length of each piece of wood required to construct a roof truss.

Piece	Segment shown on the plan	Length on the plan (cm)	Actual length (m)
1	\overline{AD}	$\frac{3125}{256}$ or approximately 12.21	$\frac{3125}{1024}$ or approximately 3.05
2	\overline{DE}	$\frac{9375}{1024}$ or approximately 9.16	$\frac{9375}{4096}$ or approximately 2.29
3	\overline{AE}	$\frac{15\,625}{1024}$ or approximately 15.26	$\frac{15\,625}{4096}$ or approximately 3.81
4	\overline{BH}	3	$\frac{3}{4}$ or 0.75
5	\overline{BG}	$\frac{15}{4}$ or 3.75	$\frac{15}{16}$ or approximately 0.94
6	\overline{CG}	$\frac{75}{16}$ or approximately 4.69	$\frac{75}{64}$ or approximately 1.17
7	\overline{CF}	$\frac{375}{64}$ or approximately 5.86	$\frac{375}{256}$ or approximately 1.46
8	\overline{DF}	$\frac{1875}{256}$ or approximately 7.32	$\frac{1875}{1024}$ or approximately 1.83

3. Determine the amount of pieces of each type of wood required to construct 10 roof trusses.

Pieces of Type A (2.4 m)

Amount	Used to make:	Final cut
35	10 segments of 1.83 m 10 segments of 2.29 m 10 segments of 1.46 m 10 segments of 0.94 m 10 segments of 1.17 m	5 pieces of 0.06 m 10 pieces of 0.57 m 10 pieces of 0.11 m

Pieces of Type B (3.65 m)

Amount	Used to make:	Final cut
10	10 segments of 3.05 m	10 pieces of 0.60 m

Pieces of Type C (4.85 m)

Amount	Used to make:	Final cut
10	10 segments of 3.81 m 10 segments of 0.75 m	10 pieces of 0.29 m

Note: We preferred using the pieces of Type A since it is the least expensive (\$0.83/m), even if it requires more final cuts than the pieces of Type B (\$1.09/m) or C (\$1.13/m).

4. Calculate the cost to construct the shed's roof trusses.

Type of piece	Price (\$)	Amount	Total (\$)
A	1.99	35	69.65
B	3.99	10	39.90
C	5.49	10	54.90
Total			164.45

REVISION 3

Prior learning 1

- Isosceles triangle.
- 69°
- $\frac{27\sqrt{10}}{2}$ or approximately 42.69 cm²
 - $\frac{3\sqrt{10}}{2}$ or approximately 4.74 cm²
 - $6\sqrt{10}$ or approximately 18.97 cm²
 - $6\sqrt{10}$ or approximately 18.97 cm²
 - $\frac{27\sqrt{10}}{49}$ or approximately 1.74 cm²

Knowledge in action

- Isosceles and acute triangles
 - Isosceles.
 - Acute triangle
 - Scalene.
 - Equiangular triangle and acute triangle
 - Equilateral.
 - Equilateral and acute triangle
 - Isosceles.
 - Obtuse triangle
 - Scalene.
 - Right triangle
 - Scalene.
- 5.2 cm²
 - 4 cm²
 - 5.04 cm²
 - 3.52 cm²
 - $\frac{36\pi}{25}$ or approximately 4.52 cm²
 - 3.3 cm²

4. a) 13.5 cm^2 b) 24 cm^2
 c) Approximately 1.37 cm^2 d) 8 cm^2
5. a) 1) $\frac{3\sqrt{2}}{2}$ or approximately 2.12 cm
 2) The diagonals of a square are perpendicular and intersect at their midpoints. Pythagorean theorem.
- b) 1) 120°
 2) The consecutive angles of a parallelogram are supplementary.
- c) 1) $\sqrt{5}$ or approximately 2.24 cm .
 2) The diagonals of a rhombus intersect at their midpoints and form a right angle. Pythagorean theorem.
- d) 1) $\frac{\sqrt{41}}{2}$ or approximately 3.20 cm
 2) Pythagorean theorem
6. At $5\sqrt{65}$ or approximately 40.31 m .
7. $36\sqrt{3}$ or approximately 62.35 cm^2
8. a) $8\sqrt{37}$ or approximately 48.66 mm^2 b) $\approx 64.12 \text{ cm}$

9. Approximately 275 cm
10. a) The parallelogram has two pairs of parallel sides, whereas a trapezoid only has one pair. Moreover, the opposite sides of the parallelogram are congruent, whereas this is not the case for the sides of a trapezoid.
 b) The isosceles trapezoid has two congruent diagonals.
 c) The four sides of a rhombus are congruent.
 d) All four angles of a square are right angles and the diagonals are congruent.
11. Approximately 36.14 m
12. a) Approximately 14.43 cm^2 b) Approximately 19.29 cm^2
 c) Approximately 17.56 cm^2

13. a)

STATEMENT	JUSTIFICATION
$m \angle BCD + m \angle CDB + m \angle DBC = 180^\circ$	The sum of the measure of the interior angles of the triangle is 180° .
$m \angle BCD = 38^\circ + 48^\circ = 180^\circ$	$m \angle CDB = 38^\circ$ and $m \angle DBC = 48^\circ$
$m \angle BCD = 94^\circ$	

- b) $m \angle GHD = m \angle CDB = 38^\circ$, because angles GHD and CDB are corresponding angles.
 c) $m \angle ABG = m \angle CBD = 48^\circ$, because angles ABG and CBD are vertically opposite angles.

d)

STATEMENT	JUSTIFICATION
$m \angle BGF = m \angle CBA$	These angles are corresponding.
$m \angle CBA = 180^\circ - m \angle CBD$	Angles CBA and CBD are supplementary.
$m \angle CBA = 180^\circ - 48^\circ = 132^\circ$	$m \angle CBD = 48^\circ$
$m \angle BGF = m \angle CBA = 132^\circ$	

14. a) Angle 1 :

- 1) 40° 2) The angles opposite a parallelogram are congruent.

Angle 2 :

- 1) 140° 2) The consecutive angles of a parallelogram are supplementary.

Angle 3 :

- 1) 40° 2) Angle 3 and Angle A are corresponding. In addition, Angles 2 and 3 are supplementary.

Angle 4 :

- 1) 40° 2) Angle A and Angle 4 are corresponding.

Angle 5 :

- 1) 50° 2) The acute angles in a right triangle are complementary.

Angle 6 :

- 1) 40° 2) Angle 4 and Angle 6 are corresponding.

b) Angle 1 :

- 1) 96° 2) Angle 1 and the 84° angle are supplementary.

Angle 2 :

- 1) 84° 2) Angle 2 is corresponding to an angle that is vertically opposite the 84° angle.

Angle 3 :

- 1) 65° 2) The sum of the measures of the interior angles of a triangle is 180° .

Angle 4 :

- 1) 65° 2) Angles 3 and 4 are vertically opposite.

Angle 5 :

- 1) 25° 2) The acute angles of a right triangle are complementary. Angle 4 and the acute angle of the right triangle are corresponding.

Angle 6 :

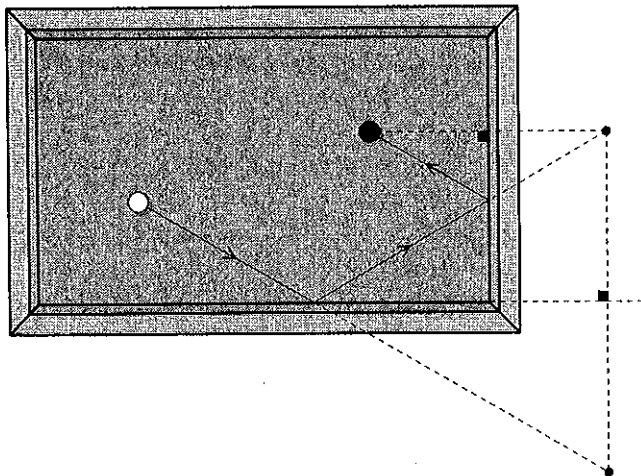
- 1) 115° 2) This angle is supplementary to a 65° angle.

15. 374 arrows.

Problem

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Several answers possible. Example:



Activity 1

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- a. A rotation.
 b. 1) Side EF. 2) Side FG. 3) Side EG.
 c. 1) 4 cm 2) 3 cm 3) 5 cm
 d. 1) The altitudes are congruent.
 2) The perimeters are equivalent.
 3) The areas are equivalent.

Activity 2

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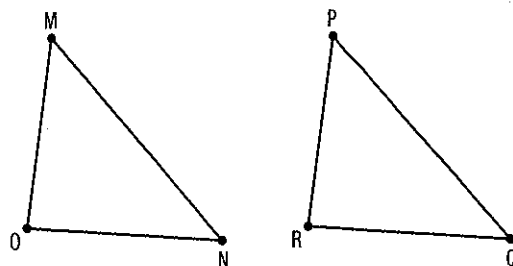
- a. 40° , 128° and 12° .
 b. 1) 100 m 2) 40° , 128° and 12° .
 c. Both triangles are congruent.
 d. Only one triangle.

Technomath

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- a. 1) These triangles have two pairs of congruent sides.
 2) All of their angles are congruent.
 3) These triangles have two pairs of congruent sides and one pair of congruent angles.
 b. No.
 c. 1) No. 2) No.

- d. 1) Several answers possible. Example:



- 2) These triangles are congruent only if the two pairs of congruent sides are situated on both sides of the pair of congruent angles.

Knowledge in action 3.1

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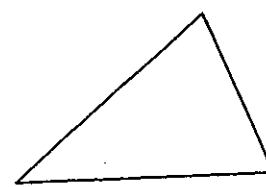
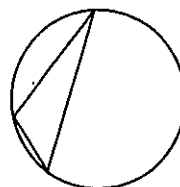
1. a) SSS. b) ASA. c) ASA. d) SAS.
 2. a) ASA. b) ASA. c) ASA. d) SAS.
 e) SAS. f) SAS. g) SSS. h) SSS.

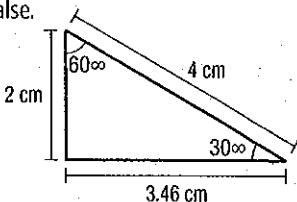
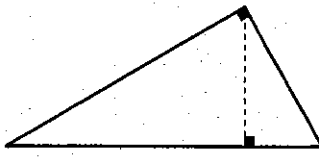
Knowledge in action 3.1 (cont'd)

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3.	Hypothesis	Conclusion
a)	All sides of a triangle are congruent.	This triangle is equilateral.
b)	The diagonals of a quadrilateral are congruent and intersect at their midpoints.	This quadrilateral is a square.
c)	Two angles have a vertex and a side in common.	These angles are adjacent.
d)	A square.	The angles are right angles.
e)	A parallelogram.	The diagonals intersect at their midpoints.

4. a) False. The diagonals of an isosceles trapezoid are also congruent.
 b) False. If the two other angles measure 70° , the triangle is not an obtuse triangle.
 c) False. Two acute angles of a parallelogram or rhombus are not supplementary.
 d) True.
 e) True.
 f) False. g) False.



	1) Hypothesis	1) Conclusion	2) Example or counterexample
a)	We double the length of the legs of a right triangle.	The length of the hypotenuse is doubled.	True. If $a^2 + b^2 = c^2$ then $(2a)^2 + (2b)^2 = (2c)^2$ $(2a)^2 + (2b)^2 = (2c)^2$ because $\Leftrightarrow 4a^2 + 4b^2 = 4c^2$ $\Leftrightarrow 4(a^2 + b^2) = 4c^2$ $\Leftrightarrow a^2 + b^2 = c^2$.
b)	The height of a right triangle is double its base.	One of the acute angles measures twice the size of the other acute angle.	False. 
c)	Given a right triangle.	The altitude separates the triangle into 2 congruent triangles.	False. 

6. (B) are (D) are congruent by ASA and (E) and (I) are congruent by SSS.

Knowledge in action 3.1 (cont'd)

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7. By measuring the three sides and making sure they are congruent with the sides of the other triangles.
8. a) 1 triangle. b) 1 triangle.
c) An infinite number of triangles.
9. a) No. b) Yes, by SAS. c) Yes, by SSS.
10. a) 1) The opposite sides of a parallelogram are congruent.
2) If a transversal intersects two parallel lines, the alternate interior angles are congruent.
3) SSS, SAS or ASA.
b) A rotation.
11. (C) and (D).

13. 1) $\overline{MP} \parallel \overline{QN}$

2) $MO \cong ON$

3) These angles are vertically opposite, therefore they are congruent.

4) By hypothesis.

5) If a transversal intersects two parallel lines, the alternate interior angles are congruent.

6) By ASA.

7) Since $\overline{OP} \cong \overline{OQ}$.

14. 1) $\angle ABC \cong \angle ACB$

2) $\overline{DA} \cong \overline{AE}$

3) Since $\angle ABC \cong \angle ACB$ by hypothesis, $\triangle ABC$ is an isosceles triangle.

4) In an isosceles triangle, the sides opposite the congruent angles are congruent.

5) These angles are vertically opposite, therefore they are congruent.

6) By hypothesis.

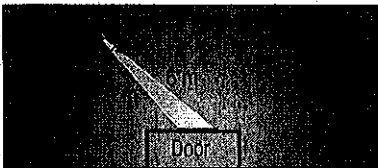
7) By SAS.

8) The corresponding sides of two congruent triangles are congruent.

Knowledge in action 3.1 (cont'd)

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12.



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Knowledge in action 3.1 (cont'd)

15.

Hypothesis:	Point C is the midpoint of segments AE and BD.
Conclusion:	$\overline{AB} \parallel \overline{DE}$

STATEMENT	JUSTIFICATION
$\overline{AC} \cong \overline{CE}$	Point C is the midpoint of segment AE, by hypothesis.
$\angle ACB \cong \angle DCE$	These angles are vertically opposite, therefore they are congruent.
$\overline{BC} \cong \overline{CD}$	Point C is the midpoint of segment BD, by hypothesis.
$\triangle ABC \cong \triangle CDE$	By SAS.
$\angle BAC \cong \angle CED$	In congruent triangles, corresponding angles are congruent.
$\overline{AB} \parallel \overline{DE}$	If alternate interior angles are congruent, the segments intersected by a transversal are parallel.

16.

Hypothesis:	<ul style="list-style-type: none"> • ABDE is a rectangle. • Point C is the midpoint of segment BD.
Conclusion:	$\triangle ABC \cong \triangle EDC$

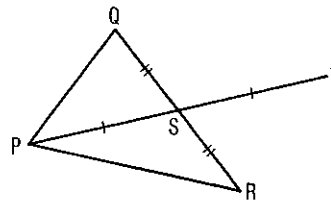
STATEMENT	JUSTIFICATION
$\overline{AB} \cong \overline{DE}$	The opposite sides of a rectangle are congruent.
$m\angle ABC = m\angle CDE = 90^\circ$	The angles in a rectangle are right angles.
$\overline{BC} \cong \overline{CD}$	Point C is the midpoint of segment BD, by hypothesis.
$\triangle ABC \cong \triangle EDC$	By SAS.

17.

Hypothesis:	<ul style="list-style-type: none"> • Segment DB is the bisector of angle ADC. • $\overline{AD} \cong \overline{CD}$
Conclusion:	$\overline{AB} \cong \overline{BC}$

STATEMENT	JUSTIFICATION
$\overline{AD} \cong \overline{CD}$	By hypothesis.
$\angle ADB \cong \angle BDC$	Since, by hypothesis, segment DB is the bisector of angle ADC, it divides angle ADC in two congruent angles.
$\overline{BD} \cong \overline{BD}$	A segment is always congruent to itself (reflexivity).
$\triangle ABD \cong \triangle CBD$	By SAS.
$\overline{AB} \cong \overline{BC}$	In congruent triangles, the corresponding sides are congruent.

18. a)



b)

Hypothesis:	<ul style="list-style-type: none"> • PQR is a triangle. • $\overline{PS} \cong \overline{ST}$ • $\overline{QS} \cong \overline{SR}$
Conclusion:	$\overline{PR} \cong \overline{QT}$

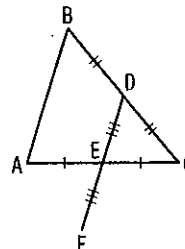
STATEMENT	JUSTIFICATION
$\overline{QS} \cong \overline{SR}$	By hypothesis.
$\angle QST \cong \angle PSR$	These angles are opposite their vertex, therefore they are congruent.
$\overline{PS} \cong \overline{ST}$	By hypothesis.
$\triangle QST \cong \triangle PSR$	By SAS.
$\overline{PR} \cong \overline{QT}$	In congruent triangles, the corresponding sides are congruent.

c)

Hypothesis:	<ul style="list-style-type: none"> • PQR is a triangle. • $\overline{PS} \cong \overline{ST}$ • $\overline{QS} \cong \overline{SR}$
Conclusion:	$\overline{PQ} \cong \overline{RT}$

STATEMENT	JUSTIFICATION
$\overline{QS} \cong \overline{SR}$	By hypothesis.
$\angle QSP \cong \angle RST$	These angles are opposite their vertex, therefore they are congruent.
$\overline{PS} \cong \overline{ST}$	By hypothesis.
$\triangle QSP \cong \triangle RST$	By SAS.
$\overline{PQ} \cong \overline{RT}$	In congruent triangles, the corresponding sides are congruent.

19. a)



b)

Hypothesis:	<ul style="list-style-type: none"> • ABC is a triangle. • E is the midpoint of segment AC. • D is the midpoint of segment BC. • $\overline{ED} \cong \overline{EF}$
Conclusion:	$\overline{AF} \cong \overline{CD}$

STATEMENT	JUSTIFICATION
$\overline{ED} \cong \overline{EF}$	By hypothesis.
$\angle DEC \cong \angle AEF$	These angles are vertically opposite, therefore they are congruent.
$\overline{AE} \cong \overline{EC}$	Since, by hypothesis, E is the midpoint of segment AC.
$\triangle AEF \cong \triangle DEC$	By SAS.
$\overline{AF} \cong \overline{CD}$	In congruent triangles, the corresponding sides are congruent.

20.

Triangle EFG	Triangle LMN
<p>If the volume of the pyramid is equal to $\frac{20\,000}{3}$ cm³ and if the height (segment EG) is equal to 10 cm, then:</p> $\text{Volume} = \frac{\text{Area of base} \times \text{height}}{3} = \frac{20\,000}{3}$ <p>Area of base \times height = 20 000 Area of base \times 10 = 20 000 Area of base = 2000 $m \overline{DC} = \sqrt{2000}$ $m \overline{DC} = 20\sqrt{5}$ $m \overline{GF} = \frac{20\sqrt{5}}{2}$ $m \overline{GF} = 10\sqrt{5}$ $m \overline{EF}^2 = m \overline{EG}^2 + m \overline{GF}^2$ $m \overline{EF}^2 = 10^2 + (10\sqrt{5})^2$ $m \overline{EF}^2 = 600$ $m \overline{EF} = 10\sqrt{6}$</p> <p>The lengths of the sides of triangle EFG are 10 cm, $10\sqrt{5}$ cm and $10\sqrt{6}$ cm.</p>	<p>If $m \angle NML = 30^\circ$ and $m \angle LNM = 90^\circ$, then triangle LMN is a $30^\circ - 60^\circ - 90^\circ$. Since segment LM measures 20 cm, segment LN measures 10 cm because in a $30^\circ - 60^\circ - 90^\circ$ triangle, the length of the side opposite the 30° angle is equal to half the length of the hypotenuse.</p> $m \overline{LM}^2 = m \overline{LN}^2 + m \overline{NM}^2$ $20^2 = 10^2 + m \overline{NM}^2$ <p>Therefore, $400 = 100 + m \overline{NM}^2$ $300 = m \overline{NM}^2$ $10\sqrt{3} = m \overline{NM}$.</p> <p>The lengths of the sides of triangle LMN are $10\sqrt{3}$ cm, 10 cm and 20 cm.</p>

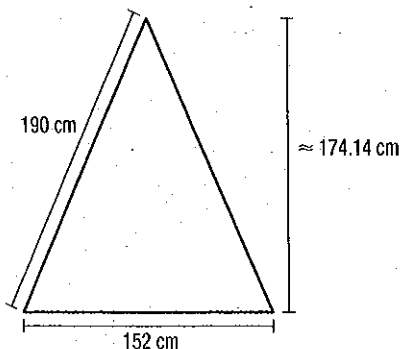
Triangles EFG and LMN are not congruent.

SECTION 3.2

Similar airplanes

Problem

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Activity 1

Page 168

- Both segments are parallel.
- They are corresponding angles formed by a transversal intersecting two parallel segments.
- It is the same angle, it is therefore congruent to itself (reflexivity).
- $\frac{12}{7}$ or $\frac{7}{12}$
- Approximately 14.57 m.

Activity 2

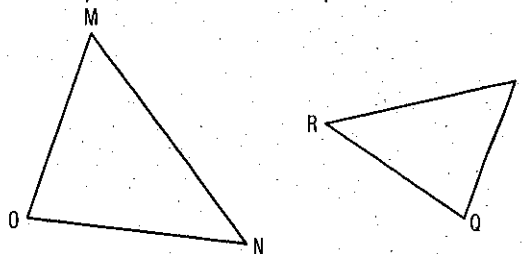
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- Line l_3 was moved to a lower position.
 - The angle formed by the transversals was increased in size.
 - Lines l_1 , l_2 and l_3 were inclined.
- Screen 1 1) $\frac{15}{14}$ 2) $\frac{15}{14}$
 Screen 2 1) $\frac{15}{14}$ 2) $\frac{15}{14}$
 Screen 3 1) $\frac{10}{7}$ 2) $\frac{10}{7}$
 Screen 4 1) $\frac{3}{2}$ 2) $\frac{3}{2}$
 - Several possible answers. Example: When three parallel lines are intersected by two transversals, the segments determined by the transversals are proportional.

Technomath

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- 1) 3 pairs of angles. 2) None.
- 1) pair of angles.
- 1) $\frac{10}{5} = \frac{12}{6} = \frac{6}{3} = 2$ 2) $\frac{7.5}{5} = \frac{4.5}{3} = 1.5$
 3) $\frac{6}{3} = \frac{12}{6} = 2$
- Yes, triangles ABC and DEF are similar.
- 1) No. 2) No.
- Several possible answers. Example:



- Triangles MNO and triangles PQR are similar, only if the pairs of proportional sides are situated on both sides of the pair of congruent angles.

Knowledge in action 3.2

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- a) SAS. b) AA. c) SSS.
 d) AA. e) SAS. f) SSS.

2. a) 30° b) 51° c) 35°
 d) 49° e) 145° f) 41°

Knowledge in action 3.2 (cont'd)

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3. a) $x = 3$ cm and $y = 2.8$ cm
 b) $x = 2.8$ cm and $y \approx 0.71$ cm
 c) $x = 1$ cm and $y = 3.51$ cm
 d) $x = 1.35$ cm and $y = 4.9$ cm
 e) $x = 5.76$ cm and $y \approx 1.78$ cm
 f) $x \approx 1.77$ cm and $y \approx 2.96$ cm
4. No, it can be a $40^\circ - 70^\circ - 70^\circ$ triangle or a $40^\circ - 40^\circ - 100^\circ$ triangle.
5. Angle AED is congruent with angle BEC, because they are vertically opposite.
 Angle EDA is congruent with angle EBC, because they are alternate interior angles, formed by a transversal intersecting two parallel lines.
 Angle DAE is congruent with angle ECB, because they are alternate interior angles, formed by a transversal intersecting two parallel lines.
6. These triangles are similar by SAS.
7. These triangles are similar by AA.
8. These triangles are similar by SAS.

Knowledge in action 3.2 (cont'd)

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9. ≈ 15.95 km
10. a) $\frac{14}{15}$ cm b) 2.5 cm c) 3.1 cm
 d) $\frac{16}{3}$ cm e) 3.5 cm
11. 14.875 cm
- 12.

Hypothesis:	1) $\overline{AN} \parallel \overline{BC}$
Conclusion:	$\frac{m \overline{NQ}}{m \overline{MQ}} = \frac{m \overline{AQ}}{m \overline{CQ}}$

STATEMENT	JUSTIFICATION
$\angle AQN \cong \angle CQM$	Common angle.
$\angle NAQ \cong \angle MCQ$	2) If a transversal intersects two parallel lines, then the corresponding angles are congruent.
$\triangle ANQ \sim \triangle CMQ$	3) By AA.
$\frac{m \overline{NQ}}{m \overline{MQ}} = \frac{m \overline{AQ}}{m \overline{CQ}}$	4) The ratio between the lengths of the corresponding sides of two similar triangles is constant.

Hypothesis:	Segments AE, CD and FB are the three altitudes of isosceles triangle ABC.
Conclusion:	$\frac{m \overline{AE}}{m \overline{CD}} = \frac{m \overline{BE}}{m \overline{BD}}$

STATEMENT	JUSTIFICATION
$\angle AEB \cong \angle CDB$	The altitude of a triangle forms a right angle with its base.
$\angle ABE \cong \angle DBC$	An angle is always congruent to itself (reflexivity).
$\triangle ABE \sim \triangle CDB$	By AA.
$\frac{m \overline{AE}}{m \overline{CD}} = \frac{m \overline{BE}}{m \overline{BD}}$	The ratio of the lengths of the corresponding sides of the similar triangles is constant.

Knowledge in action 3.2 (cont'd)

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14.

Hypothesis:	<ul style="list-style-type: none"> • ABCD is a trapezoid. • Diagonals BD and AC intersect at point E.
Conclusion:	$\frac{m \overline{BE}}{m \overline{ED}} = \frac{m \overline{AE}}{m \overline{CE}}$

STATEMENT	JUSTIFICATION
$\angle ABE \cong \angle EDC$	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
$\angle BEA \cong \angle CED$	These angles are vertically opposite, therefore they are congruent.
$\triangle ABE \sim \triangle CDE$	By AA.
$\frac{m \overline{BE}}{m \overline{ED}} = \frac{m \overline{AE}}{m \overline{CE}}$	The ratio of the lengths of the corresponding sides of the similar triangles is constant.

15. Lot B is the better bargain.
16. a) 1) $\angle CAB \cong \angle EDB$, because these angles are corresponding since they are formed by a transversal intersecting two parallel lines. $\angle ABC \cong \angle DBE$, because an angle is always congruent to itself (reflexivity). Therefore $\triangle ABC \sim \triangle DBE$ by AA.
 2) $\angle CAB \cong \angle CFE$, because these angles are corresponding since they are formed by a transversal intersecting two parallel lines. $\angle ACB \cong \angle FCE$, because an angle is always congruent to itself (reflexivity). Therefore $\triangle ABC \sim \triangle FEC$ by AA.
 3) Since $\triangle ABC \sim \triangle DBE$ and $\triangle ABC \sim \triangle FEC$, $\triangle DBE \sim \triangle FEC$ by transitivity.
- b) 1) 2.14 cm 2) Approximately 5.72 cm.
 3) Approximately 7.03 cm. 4) Approximately 7.03 cm.
17. a) 137 m
 b) There would not have been any visible shadow on the ground.

18. Approximately 195.13 cm.
 19. Approximately 1535.91 m.
 20. 13.6 m

21. a) 1) 4.08 cm 2) 6.12 cm
 3) 3.54 cm 4) ≈ 7.07 cm

b)

Pieces	Angles (°)			Sides (m)		
1)	16.4	30	133.6	1.8	2.61	1.02
2)	73.6	48.4	58	2.04	2.31	1.8
3)	60	30	90	1.53	≈ 0.89	≈ 1.77
4)	48.4	41.6	90	1.73	≈ 2.31	1.53
5)	60	60	60	1.02	1.02	1.02
6)	120	30	30	1.02	1.02	≈ 1.77

22. $x = 4.8$ m, $y = 7.2$ m and $z = 12$ m

23. $c = 4$ m

SECTION 3.3

Metric relations

Problem

Approximately 4.77 m.

Activity 1

- a. 1) Since $\angle ADC \cong \angle BAC$ (two right angles) and $\angle DCA \cong \angle ACB$ (it is the same angle), these triangles are similar by AA.
 2) Since $\angle ADB \cong \angle BAC$ (two right angles) and $\angle DBA \cong \angle ABC$, these triangles are similar by AA.
 3) Since $\triangle \text{blue} \sim \triangle \text{pink}$ and $\triangle \text{pink} \sim \triangle \text{green}$, then $\triangle \text{blue} \sim \triangle \text{green}$ by transitivity.

b. 1) $\frac{m \overline{CD}}{m \overline{AC}} = \frac{m \overline{AC}}{m \overline{BC}} = \frac{m \overline{AD}}{m \overline{AB}}$
 2) $\frac{m \overline{AC}}{m \overline{AD}} = \frac{m \overline{BC}}{m \overline{AB}} = \frac{m \overline{AB}}{m \overline{DB}}$
 3) $\frac{m \overline{AC}}{m \overline{AB}} = \frac{m \overline{AD}}{m \overline{BD}} = \frac{m \overline{CD}}{m \overline{AD}}$

c. 1) 4 cm 2) $\frac{16}{3}$ cm 3) $\frac{20}{3}$ cm

Technomath

- a. 1) By SSS, because the ratio of the lengths of the corresponding sides are constant.

$$\frac{m \overline{BC}}{m \overline{AB}} = \frac{m \overline{CH}}{m \overline{BH}} = \frac{m \overline{BH}}{m \overline{AH}}$$
 2) By AA, $m \angle BAC = m \angle BAH$ (identity) and $m \angle ABC = m \angle BHA = 90^\circ$ by hypothesis.
 3) By AA, $m \angle BCA = m \angle BCH$ (identity) and $m \angle ABC = m \angle BHC = 90^\circ$ by hypothesis.
 b. 1) Screen 3: 32.955 cm². Screen 4: 35.6 cm².
 Screen 5: 83.916 cm². Screen 6: 22.5 cm².
 2) Screen 3: 32.955 cm². Screen 4: 35.6 cm².
 Screen 5: 83.916 cm². Screen 6: 22.5 cm².
 c. Several possible answers. Example: In a right triangle, the product of the length of the hypotenuse by the corresponding altitude equals the product of the length of the sides of the right angle.
 d. 1) Yes, but only if it is a right angle isosceles triangle.
 2) No.

Knowledge in action 3.3

1. a) 1) 40 cm 2) $(m \overline{AB})^2 = (m \overline{AC})^2 + (m \overline{CB})^2$
 b) 1) 40.5 cm 2) $(m \overline{CD})^2 = m \overline{AD} \times m \overline{BD}$
 c) 1) approximately 1.98 cm 2) $(m \overline{AB})^2 = m \overline{AD} \times m \overline{AC}$
 d) 1) 12 cm 2) $(m \overline{BC})^2 = m \overline{CD} \times m \overline{AC}$
 2. a) 24 cm
 b) $\frac{120}{13}$ or approximately 9.23 cm
 c) $\frac{50}{13}$ or approximately 3.85 cm
 3.

Lengths of segments

a)	a	b	c	m	n	h
b)	9	12	15	5.4	9.6	7.2
c)	$4\sqrt{5}$ 8.9	$8\sqrt{5}$ 17.8	20	4	16	8
	10	7.5	12.5	8	4.5	6

4. $\frac{3136}{1077}$ or approximately 2.91 cm.

Knowledge in action 3.3 (cont'd)

5. a) 1) Since the sum of the measures of the interior angles of a triangle is 180° ,
 $m \angle A + m \angle B + m \angle C = 180^\circ$
 $60^\circ + m \angle B + 30^\circ = 180^\circ$
 $m \angle B = 90^\circ$.

- 2) Since in a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse.
- b) $40\sqrt{3} + 1.6$ or approximately 70.88 m.
6. $\frac{5\sqrt{2}}{2}$ or approximately 3.54 m
7. $\frac{29}{3}$ or approximately 9.67 cm
8. a)

Calculation of length	Geometric statement
$(m\overline{AD})^2 + (m\overline{DC})^2 = (m\overline{AC})^2$ $(m\overline{AD})^2 + 5^2 = 8^2$ $(m\overline{AD})^2 + 25 = 64$ $(m\overline{AD})^2 = 39$ $(m\overline{AD}) = \sqrt{39}$	Pythagorean theorem.
$(m\overline{AC})^2 = m\overline{AD} \times m\overline{AB}$ $8^2 = \sqrt{39} \times m\overline{AB}$ $\frac{64\sqrt{39}}{39} = m\overline{AB}$	In a right triangle, the length of each side of the right angle is the proportional mean between the length of its projection over the hypotenuse and the length of the entire hypotenuse.
$m\overline{AD} + m\overline{BD} = m\overline{AB}$ $\sqrt{39} + m\overline{BD} = \frac{64\sqrt{39}}{39}$ $m\overline{BD} = \frac{64\sqrt{39}}{39} - \sqrt{39}$ $m\overline{BD} = \frac{25\sqrt{39}}{39} \approx 4.0$	The length of a segment is the sum of the lengths of segments that compose it.

b)

Calculation of length	Geometric statement
$(m\overline{BD})^2 + (m\overline{CD})^2 = (m\overline{BC})^2$ $\left(\frac{25\sqrt{39}}{39}\right)^2 + 5^2 = (m\overline{BC})^2$ $\frac{625}{39} + 25 = (m\overline{BC})^2$ $\frac{1600}{39} = (m\overline{BC})^2$ $\frac{40\sqrt{39}}{39} = (m\overline{BC})$ ≈ 6.4	Pythagorean theorem.

9. Approximately 323.69 m.

Knowledge in action 3.3 (cont'd)

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10. a) 5 m
 b) Approximately 8.33 m. c) 5 m
 d) Approximately 2.08 m. e) Approximately 5.42 m.

11. Approximately 11.08 m.
 12. Approximately 74.88 m.

Knowledge in action 3.3 (cont'd)

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13. 27.6 m
 14. $\approx \$5,273.23$
 15. Approximately 2.67 m.
 16. Approximately 327.02 cm².
 17. Approximately 3079.49 cm².

SPECIAL FEATURES

3

Chronicle of the past

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1. ≈ 4.71 cm

2. a)

Hypothesis:	<ul style="list-style-type: none"> • Lines AB and DC are parallel. • Lines AD and BC intersect at point O.
Conclusion:	$\triangle ABO \sim \triangle DCO$

STATEMENT	JUSTIFICATION
$\overline{AB} \parallel \overline{CD}$	By hypothesis.
$\angle BAO \cong \angle CDO$	When a transversal intersects two parallel lines, the corresponding angles are congruent.
$\angle ABO \cong \angle DCO$	When a transversal intersects two parallel lines, the corresponding angles are congruent.
$\triangle ABO \sim \triangle DCO$	By AA.

b) Yes. Since the pairs of sides AO and DO and AB and DC are two pairs of corresponding sides of similar triangles, the ratio of their lengths is the same.

3.

Hypothesis:	<ul style="list-style-type: none"> • $m\overline{AB} = 2$ m • $m\overline{AC} = 3.2$ m • $m\overline{AD} = 1.25$ m
Conclusion:	$\triangle ABC \sim \triangle ADB$

STATEMENT	JUSTIFICATION
$\frac{m\overline{AC}}{m\overline{AB}} = \frac{m\overline{AB}}{m\overline{AD}}$	Since $\frac{m\overline{AC}}{m\overline{AB}} = \frac{3.2}{2} = 1.6$ and $\frac{m\overline{AB}}{m\overline{AD}} = \frac{2}{1.25} = 1.6$.
$\angle BAD \cong \angle BAC$	An angle is congruent to itself (identity).
$\triangle ABC \sim \triangle ADB$	By SAS

1. a)

Hypothesis:	$\overline{BF} \parallel \overline{CE}$
Conclusion:	$\triangle BDF \sim \triangle CDE$

STATEMENT	JUSTIFICATION
$\angle FBD \cong \angle ECD$	Since segments BF and CE are parallel (by hypothesis), corresponding angles FBD and ECD are congruent.
$\angle BDF \cong \angle CDE$	An angle is congruent to itself (identity).
$\triangle BDF \sim \triangle CDE$	By AA.

b) $\frac{170}{3}$ or approximately 56.67 cm

2. a)

Hypothesis:	<ul style="list-style-type: none"> • BCEF is a rectangle. • $m \overline{AF} = m \overline{DE}$
Conclusion:	$\triangle ABF \cong \triangle CDE$

STATEMENT	JUSTIFICATION
$\overline{BF} \cong \overline{CE}$	The opposite sides of a rectangle are congruent.
$m \angle BFA = m \angle CED = 90^\circ$	The exterior angles of a rectangle are right angles.
$m \overline{AF} = m \overline{DE}$	By hypothesis.
$\triangle ABF \cong \triangle CDE$	By SAS.

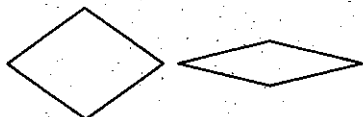
b) $\frac{5\sqrt{5}}{4}$ or approximately 2.80 m

Overview

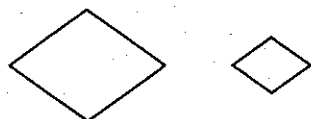
- SSS.
 - SAS.
 - ASA.
 - SSS.
- Congruent triangles by SAS.
 - Similar triangles by SSS.
 - Neither one.
 - Neither one.
 - Similar triangles by SSS.
 - Similar triangles by AA.

Overview (cont'd)

3. a) False.



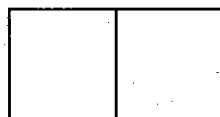
- True.
- True.
- False.



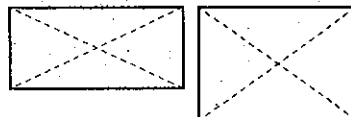
4. a) No.



b) Yes.



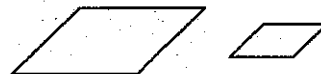
c) No.



d) No.



e) No.



- $x = 3.6$ cm
 - $x = 2.88$ cm
 - $x = 1.2$ cm
 - $x = \frac{\sqrt{19}}{5}$ or approximately 0.87 cm

Overview (cont'd)

6. a)

Hypothesis:	ABCD is a rectangle.
Conclusion:	$m \overline{AC} = m \overline{BD}$

STATEMENT	JUSTIFICATION
$m \overline{AB} = m \overline{CD}$	The opposite sides of a rectangle are congruent.
$m \angle ABC = m \angle BCD = 90^\circ$	The angles of a rectangle are right angles.
$m \overline{BC} = m \overline{BC}$	A segment is always congruent to itself (reflexivity).
$\triangle ABC \cong \triangle BCD$	By SAS.
$m \overline{AC} = m \overline{BD}$	Two congruent triangles have corresponding congruent sides.

b)

Hypothesis:	<ul style="list-style-type: none"> • ABCD is a quadrilateral. • $\overline{AB} \parallel \overline{CD}$ • $m\overline{AB} = m\overline{CD}$
Conclusion :	$\overline{AD} \parallel \overline{BC}$

STATEMENT	JUSTIFICATION
$m\overline{AB} = m\overline{CD}$	By hypothesis.
$\overline{AB} \parallel \overline{CD}$	By hypothesis.
$m\angle ABD = m\angle BDC$	When two parallel lines are intersected by a transversal, the alternate internal angles are congruent.
$m\overline{BD} = m\overline{BD}$	A segment is congruent to itself (reflexivity).
$\triangle ABD \cong \triangle DBC$	By SAS.
$m\overline{AD} = m\overline{BC}$	Two congruent triangles have the corresponding congruent sides.
$m\angle ADB = m\angle DBC$	Two congruent triangles have the corresponding congruent sides.
$\overline{AD} \parallel \overline{BC}$	When two alternate internal angles are congruent, the lines intersected by the transversal are parallel.

c)

Hypothesis:	<ul style="list-style-type: none"> • ABCD is a parallelogram. • Diagonals BD and AC intersect at point E.
Conclusion:	$m\overline{BE} = m\overline{ED}$ and $m\overline{AE} = m\overline{EC}$

STATEMENT	JUSTIFICATION
$\overline{AB} \parallel \overline{CD}$	By hypothesis.
$m\angle ABD = m\angle BDC$	When two parallel lines are intersected by a transversal, the alternate interior angles are congruent.
$m\overline{AB} = m\overline{CD}$	The opposite sides of a parallelogram are congruent.
$\overline{AD} \parallel \overline{BC}$	By hypothesis.
$m\angle BAC = m\angle ACD$	When two parallel lines are intersected by a transversal, the alternate internal angles are congruent.
$\triangle ABE \cong \triangle CDE$	By ASA.
$m\overline{BE} = m\overline{ED}$	Two congruent triangles have corresponding congruent sides.
$m\overline{AE} = m\overline{EC}$	Two congruent triangles have corresponding congruent sides.

7. By measuring diagonals BD and AC. If these diagonals are congruent, it is because the floor is rectangular.

8. a) Triangles ABE, ABD, ADE, ACD, CDE, BCD, BCE and ABC.

b) Pairs of triangles ABC and ABD, ACD and BCD, and BCE and ADE.

9. a) 1) $\frac{12}{17}$ or approximately 0.71 cm 2) 1.62 cm

3) $\frac{71}{45}$ or approximately 1.58 cm 4) 2.44 cm

b) No. Since lines EF and GH are not parallel, it is impossible to apply Thales' theorem.

Overview (cont'd)

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10. $\frac{\sqrt{6}}{2}$ or approximately 1.22 km

Room	Volume (m ³)
Bathroom	13.5
Living room	216
Bedroom	121.5
Dining room	168.75
Shed	$\frac{243}{32}$ or approximately 7.59

12. $m\overline{AB} = 5$ cm, $m\overline{BE} = 3$ cm, $m\overline{CD} = 4.2$ cm

13. $2000\sqrt{6}$ or approximately 4898.98 m²

Overview (cont'd)

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14. Approximately 9.07 m.

15. a) 1) Several answers possible. Example: Triangles ABD, BCD, ADF and BDF are all similar to triangle ABC.

2) Triangle ECG is congruent therefore it is similar to triangle EBG.

3) None of the triangles are similar to triangle BED.

b) 1) Triangle EBG is congruent with triangle ECG.

2) None of the triangles are congruent with triangle FDB.

16. a) SSS, ASA or SAS.

b) AA.

c) Pythagorean theorem:

$$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2$$

d) The measurement of side AB is the proportional mean between the measurement of segment AC and the measurement of segment AE:

$$(m\overline{AB})^2 = (m\overline{AC}) \times (m\overline{AE})$$

e) Pythagorean theorem:

$$(m\overline{AB})^2 = (m\overline{AE})^2 + (m\overline{BE})^2$$

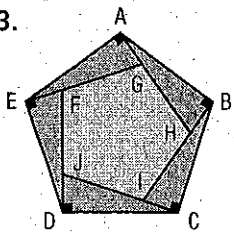
17. Triangle **A**, **C**, **D** and **F** are congruent.

Triangle **B** and **E** are congruent.

Bank of problems

18. $m \overline{OK} = \frac{3\sqrt{6}}{2}$ or approximately 3.67 cm
19. $a = 6$ cm. Pythagorean theorem.
 $b = 4.8$ cm. In a right triangle, the product of the measurements of the hypotenuse and the corresponding altitude equals the product of the measurements of the sides of the right angle.
 $c = 3.84$ cm. In similar triangles, the measures of the corresponding sides are proportional.
20. Approximately 12.09 cm².
21. Approximately 18.32 cm.
22. Point D must be placed 9 cm from point A on segment AC.

Bank of problems (cont'd)



STATEMENT	JUSTIFICATION
The sum of the interior angles of a regular pentagon is 540°.	In a regular pentagon, the sum of the measures of the interior angles is given by the formula $(n - 2) \times 180^\circ$.
$m \angle BAE = m \angle ABC = m \angle BCD$ $= m \angle CDE = m \angle AED$ $= \frac{540^\circ}{5} = 108^\circ$	In a regular pentagon, all the interior angles are congruent.
$m \angle EAG = m \angle ABH = m \angle BCI = m \angle CDJ = m \angle DEF = 90^\circ$	By hypothesis.
$m \angle BAH = m \angle BAE - m \angle EAG = 108^\circ - 90^\circ = 18^\circ$	Angles EAG and BAH are adjacent.
$m \angle CBI = m \angle ABC - m \angle ABH = 108^\circ - 90^\circ = 18^\circ$	Angles ABH and CBI are adjacent.
$m \angle DCJ = m \angle BCD - m \angle BCI = 108^\circ - 90^\circ = 18^\circ$	Angles BCI and DCJ are adjacent.
$m \angle EDF = m \angle CDE - m \angle CDJ = 108^\circ - 90^\circ = 18^\circ$	Angles JDC and EDF are adjacent.
$m \angle AEG = m \angle AED - m \angle DEF = 108^\circ - 90^\circ = 18^\circ$	Angles FED and AEG are adjacent.
$m \overline{AB} = m \overline{BC} = m \overline{CD} = m \overline{DE} = m \overline{AE}$	ABCDE is a regular pentagon.
$\triangle ABH \cong \triangle BCI \cong \triangle CDJ \cong \triangle DEF \cong \triangle AEG$	By ASA.

24. $\frac{3\sqrt{1099}}{5}$ or approximately 19.89 km

Hypothesis:	ABCDE is a regular pentagon.
Conclusion:	Diagonals AC and EB are congruent.

STATEMENT	JUSTIFICATION
$\overline{AE} \cong \overline{AB}$	A regular polygon is equilateral.
$\angle EAB \cong \angle ABC$	A regular polygon is equiangular.
$\overline{AB} \cong \overline{BC}$	A regular polygon is equilateral.
$\triangle EAB \cong \triangle ABC$	By SAS.
$\overline{AC} \cong \overline{EB}$	The corresponding sides of two congruent triangles are congruent.

26. Approximately 14.06 m².