student book volume 2

Cultural, Social and Technical

VISIONS

MATHEMATICS

Secondary Cycle Two, Year Two

ANSWER KEY

Visions 4 to *6*

LES ÉDITIONS CEC
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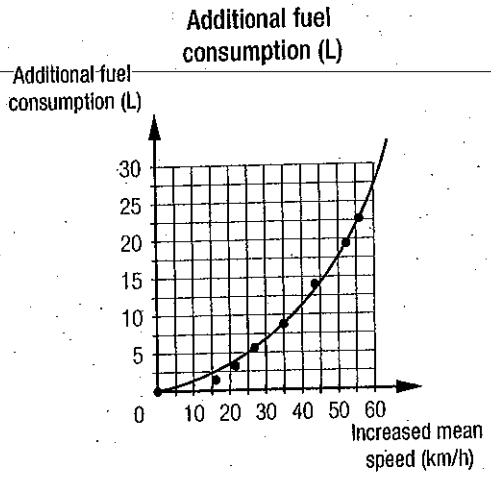
LES 7 Gas savings

The following is an example of a procedure:

1. Represent the situation with a scatter plot and draw a curve that best fits the scatter plot.



2. Determine the rule associated with this curve.

   The scatter plot shows a trend that can be represented by a second-degree polynomial function whose rule is in the form $y = ax^2$. Using the point (50, 18) on this curve, the following equation can be formed:
   $18 = a(50)^2$
   $18 = 2500a$
   $a = 0.0072$

   The rule is therefore $y = 0.0072x^2$.

3. Determine the additional consumption of fuel for the trip between Montréal and Québec City based on speed.

Additional fuel consumption for the trip between Montréal and Québec City

Mean speed (km/h)	Mean speed exceeding 60 km/h (km/h)	Additional fuel consumption (L)
90	30	6.48
100	40	11.52
110	50	18
120	60	25.92

4. Calculate the fuel savings based on the decrease in speed.

Fuel savings

Decrease in speed (km/h)	Fuel savings (L)	Savings based on fuel price of $1.44/L ($)
From 100 to 90	11.52 − 6.48 = 5.04	7.26
From 110 to 90	18 − 6.48 = 11.52	16.59
From 120 to 90	25.92 − 6.48 = 19.44	27.99

LES 8 Buying on credit

The following is an example of a procedure that analyzes the financing options:

1. Compare the different financing options.
   Standard credit card purchase (18% annual interest rate)
   Initial balance: $2,369.25
   First minimum payment: $71.08

	Time (months)	Minimum payment ($)	Balance ($)	Balance and interest ($)
Progression of paying off debt	0	0	2,369.25	2,369.25
	1	71.08	2,298.17	2,332.65
	2	69.98	2,262.67	2,296.61
	3	68.90	2,227.71	2,261.12
	...	...	...	...
	...	...	...	...
After 126 months, 3% of the balance represents less than $10.	126	10.15	328.17	333.10
	127	10.00	323.10	327.94
	128	10.00	317.94	322.71
	129	10.00	312.71	317.40
	...	...	...	...
	...	...	...	...
The debt is fully paid after 172 months (14 years and 4 months).	169	10.00	24.59	24.96
	170	10.00	14.96	15.18
	171	10.00	5.18	5.26
	172	5.26	0	0

Total amount paid: $4,408.96

Credit card cash advances (18% annual interest rate)
Initial balance: $2,369.25
First minimum payment: $72.14

Progression of paying off debt

After 128 months, 3% of the balance represents less than $10.

The debt is fully paid after 173 months (14 years and 5 months).

Time (months)	Minimum payment ($)	Balance ($)	Balance and interest ($)
0	0	2,369.25	2,404.79
1	72.14	2,332.65	2,367.63
2	71.03	2,296.61	2,331.05
3	69.93	2,261.12	2,295.04
...	...	...	...
...	...	...	...
127	10.14	327.95	332.87
128	10.00	322.87	327.71
129	10.00	317.71	322.48
130	10.00	312.48	317.17
...	...	...	...
...	...	...	...
170	10.00	24.16	24.53
171	10.00	14.53	14.74
172	10.00	4.74	4.81
173	4.81	0	0

Total amount paid: $4,477.95

Commercial credit card purchase
(28% annual interest rate)
Initial balance: $2,250.79
First minimum payment: $67.52

Progression of paying off debt

After 260 months, 3% of the balance represents less than $10.

The debt is fully paid after 321 months (26 years and 9 months).

Time (months)	Minimum payment ($)	Balance ($)	Balance and interest ($)
0	0	2,250.79	2,250.79
1	67.52	2,183.26	2,234.21
2	67.03	2,167.18	2,217.75
3	66.53	2,151.22	2,201.41
...	...	...	...
...	...	...	...
259	10.02	324.06	331.63
260	10.00	321.63	329.13
261	10.00	319.13	326.58
262	10.00	316.58	323.96
...	...	...	...
...	...	...	...
318	10.00	21.05	21.54
319	10.00	11.54	11.81
320	10.00	1.81	1.85
321	1.85	0	0

Total amount paid: $8,427.45

Bank-approved line of credit (6% annual interest rate)
Initial balance: $2,369.25
First minimum payment: $71.08

Progression of paying off debt

The debt is fully paid after 37 months (3 years and 1 month).

Time (months)	Minimum payment ($)	Balance ($)	Balance and interest ($)
0	0	2,369.25	2,381.10
1	71.08	2,310.02	2,321.57
2	69.98	2,250.49	2,261.74
3	68.90	2,190.66	2,201.61
...	...	...	...
...	...	...	...
34	71.08	179.94	180.84
35	71.08	109.76	110.31
36	71.08	39.23	39.42
37	39.42	0	0

Total amount paid: $2,598.30

Lease-purchase
Initial balance: $2,369.25
First minimum payment: $142.16

Progression of the lease

The merchandise becomes the property of the buyer after 24 months (2 years).

Time (months)	Minimum payment ($)
0	0
1	142.16
2	142.16
3	142.16
4	142.16
...	...
20	142.16
21	142.16
22	142.16
23	142.16
24	142.16

Total amount paid: $3,411.84

2. Choose the best financing option for someone who can afford to pay a maximum of $150 each month to purchase the household appliances.

a) Standard credit card purchase

Time (month)	Payment ($)	Balance ($)	Balance and interest ($)
0	0	2,369.25	2,369.25
1	150.00	2,219.25	2,252.54
2	150.00	2,102.54	2,134.08
3	150.00	1,984.08	2,013.84
4	150.00	1,863.84	1,891.80
5	150.00	1,741.80	1,767.92
6	150.00	1,617.92	1,642.19
7	150.00	1,492.19	1,514.57
8	150.00	1,364.57	1,385.04
9	150.00	1,235.04	1,253.57
10	150.00	1,103.57	1,120.12
11	150.00	970.12	984.67
12	150.00	834.67	847.19
13	150.00	697.19	707.65
14	150.00	557.65	566.02
15	150.00	416.02	422.26
16	150.00	272.26	276.34
17	150.00	126.34	128.24
18	128.24	0	0
Total	**2,678.24**		

b) Credit card cash advance

Time (month)	Payment ($)	Balance ($)	Balance and interest ($)
0	0	2,369.25	2,404.79
1	150.00	2,254.79	2,288.61
2	150.00	2,138.61	2,170.69
3	150.00	2,020.69	2,051.00
4	150.00	1,901.00	1,929.52
5	150.00	1,779.52	1,806.21
6	150.00	1,656.21	1,681.05
7	150.00	1,531.05	1,554.02
8	150.00	1,404.02	1,425.08
9	150.00	1,275.08	1,294.20
10	150.00	1,144.20	1,161.37
11	150.00	1,011.37	1,026.54
12	150.00	876.54	889.68
13	150.00	739.68	750.78
14	150.00	600.78	609.79
15	150.00	459.79	466.69
16	150.00	316.69	321.44
17	150.00	171.44	174.01
18	150.00	24.01	24.37
19	24.37	0	0
Total	**2,724.37**		

c) Commercial credit card purchase

Time (month)	Payment ($)	Balance ($)	Balance and interest ($)
0	0	2,250.79	2,250.79
1	150.00	2,100.79	2,149.81
2	150.00	1,999.81	2,046.47
3	150.00	1,896.47	1,940.72
4	150.00	1,790.72	1,832.51
5	150.00	1,682.51	1,721.76
6	150.00	1,571.76	1,608.44
7	150.00	1,458.44	1,492.47
8	150.00	1,342.47	1,373.79
9	150.00	1,223.79	1,252.35
10	150.00	1,102.35	1,128.07
11	150.00	978.07	1,000.89
12	150.00	850.89	870.74
13	150.00	720.74	737.56
14	150.00	587.56	601.27
15	150.00	451.27	461.80
16	150.00	311.80	319.08
17	150.00	169.08	173.02
18	150.00	23.02	23.56
19	23.56	0	0
Total	**2,723.56**		

d) Bank-approved line of credit

Time (month)	Payment ($)	Balance ($)	Balance and interest ($)
0	0	2,369.25	2,381.10
1	150.00	2,231.10	2,242.25
2	150.00	2,092.25	2,102.71
3	150.00	1,952.71	1,962.48
4	150.00	1,812.48	1,821.54
5	150.00	1,671.54	1,679.90
6	150.00	1,529.90	1,537.55
7	150.00	1,387.55	1,394.48
8	150.00	1,244.48	1,250.71
9	150.00	1,100.71	1,106.21
10	150.00	956.21	960.99
11	150.00	810.99	815.05
12	150.00	665.05	668.37
13	150.00	518.37	520.96
14	150.00	370.96	372.82
15	150.00	222.82	223.93
16	150.00	73.93	74.30
17	74.30	0	0
Total	**2,474.30**		

e) Lease-purchase

24 payments of $142.16.
The merchandise becomes the property of the buyer after 24 months.
The total amount paid is $3,411.84.

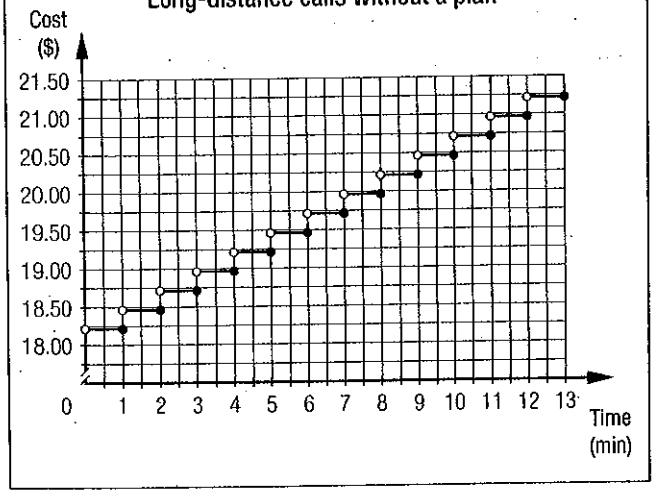
The best financing option is the bank-approved line of credit; with a total amount of $2,474.30 to pay, the buyer only ends up spending $105.05 more than the price of the washer-dryer set. The worst option is the lease-purchase; with a total of $3,411.84 to pay, the buyer ends up spending $1,042.59 more than the price of the washer-dryer set.

LES 9 What plan should I choose?

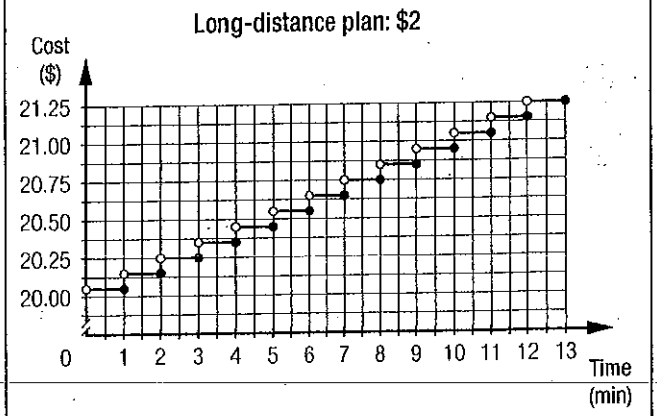
The following is an example of a procedure

1. Represent each long-distance plan on a Cartesian plane.

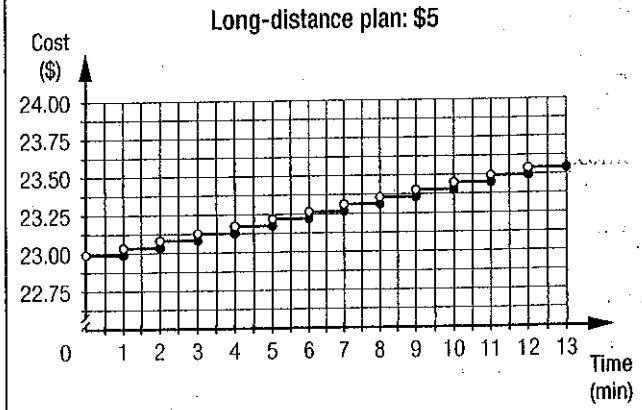
Option: no plan
14.95 + 2.80 + 0.19 = $17.94/month
Long-distance calls at $0.25/min



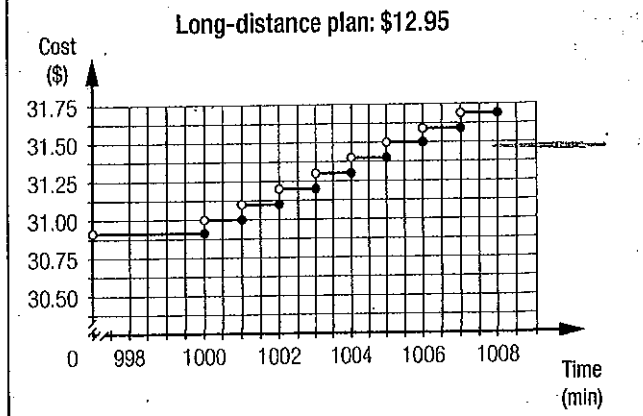
Option: $2-plan
14.95 + 2.80 + 0.19 + 2 = $19.94/month
Long-distance calls at $0.10/min



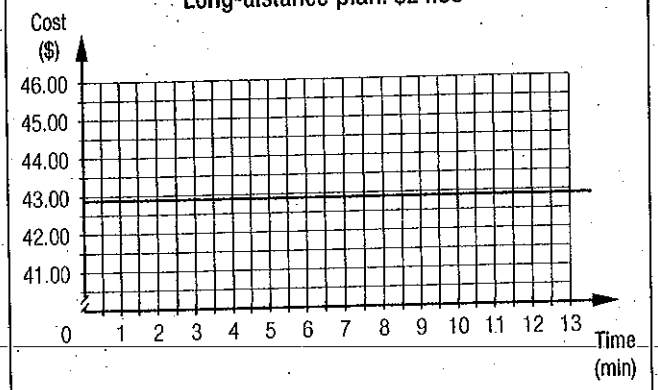
Option: $5-plan
14.95 + 2.80 + 0.19 + 5 = $22.94/month
Long-distance calls at $0.05/min



Option: $12.95-plan
14.95 + 2.80 + 0.19 + 12.95 = $30.89/month
1000 free min
Each minute in excess of the block costs $0.10.

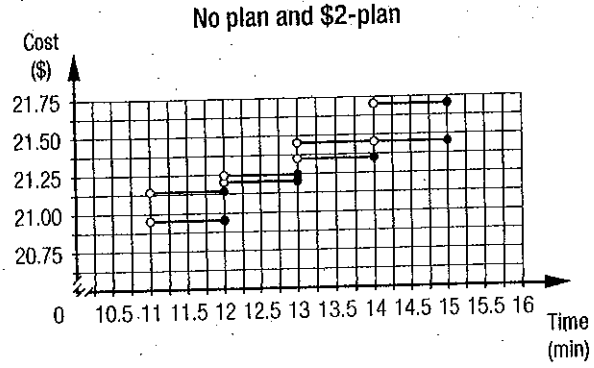


Option: $24.95-plan
14.95 + 2.80 + 0.19 + 24.95 = $42.89/month
All long-distance calls are free.



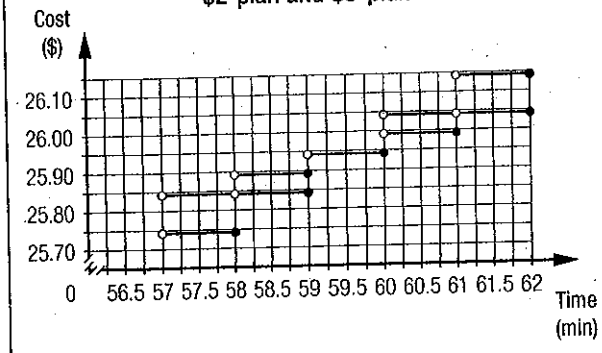
2. On the same Cartesian plane, represent several plans in order to compare them two at a time.

No plan —— $2-plan ——



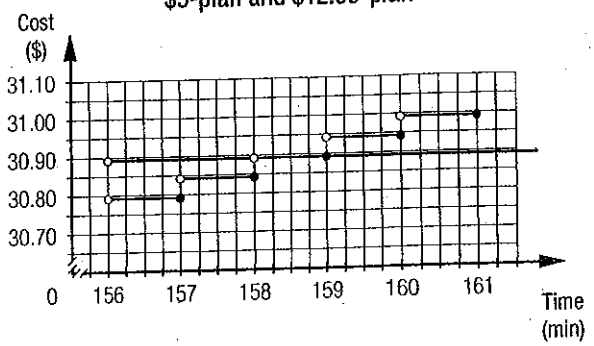
If the total duration of the long-distance calls does not exceed 13 min, it is more beneficial not to go with a plan.

$2-plan —— $5-plan ——



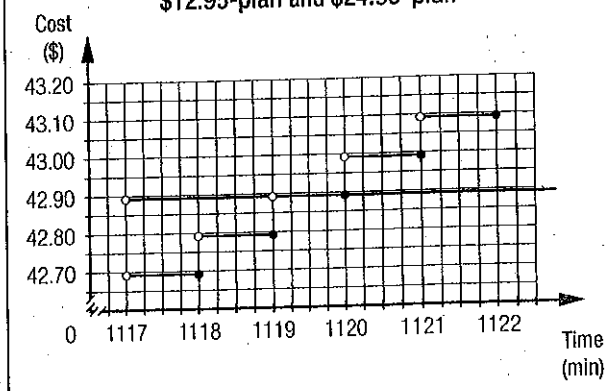
If the total duration of the long-distance calls is more than 13 min, but does not exceed 59 min, the $2-plan offers the best value. If the total duration of the long-distance calls is more than 59 min, but does not exceed 60 min, the $2-plan is equivalent to the $5-plan.

$5-plan —— $12.95-plan ——



If the total duration of the long-distance calls is more than 60 min, but does not exceed 158 min, the $5-plan offers the best value. If the total duration of the long-distance calls is more than 158 min, but does not exceed 159 min, the $5-plan is equivalent to the $12.95-plan.

$12.95-plan —— $24.95-plan ——



If the total duration of the long-distance calls is more than 159 min, but does not exceed 1119 min, the $12.95-plan offers the best value. If the total duration of the long-distance calls is more than 1119 min, but does not exceed 1120 min, the $12.95-plan is equivalent to $24.95-plan. If the total duration is more than 1120 minutes, the $24.95-plan is more beneficial.

3. Compare the plans in relation to the duration of the long-distance calls over the span of a month.

Comparing the different plans
(The shaded cells represent the best value.)

Duration of long-distance calls (min)	Cost ($)				
	No plan	$2-plan	$5-plan	$12.95-plan	$24.95-plan
5	19.19	20.44	23.19	30.89	42.89
12	20.94	21.14	23.54	30.89	42.89
13	21.19	21.24	23.59	30.89	42.89
14	21.44	21.34	23.64	30.89	42.89
20	22.94	21.94	23.94	30.89	42.89
58	32.44	25.74	25.84	30.89	42.89
59	32.69	25.84	25.89	30.89	42.89
60	32.94	25.94	25.94	30.89	42.89
61	33.19	26.04	25.99	30.89	42.89
120	47.94	31.94	28.94	30.89	42.89
157	57.19	35.64	30.79	30.89	42.89
158	57.44	35.74	30.84	30.89	42.89
159	57.69	35.84	30.89	30.89	42.89
160	57.94	35.94	30.94	30.89	42.89
161	58.19	36.04	30.99	30.89	42.89
500	142.94	69.94	47.94	30.89	42.89
1118	297.44	131.74	78.84	42.69	42.89
1119	297.69	131.84	78.89	42.79	42.89
1120	297.94	131.94	78.94	42.89	42.89
1121	298.19	132.04	78.99	42.99	42.89
2000	517.94	219.94	122.94	130.89	42.89

Conclusion

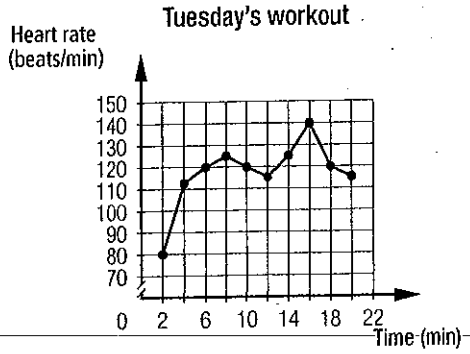
- It is beneficial for someone not to go with a plan if the long-distance calls do not last longer than 13 min/month.
- The $2-plan is beneficial for someone who makes more than 13 min/month of long-distance calls without exceeding 60 min/month.
- The $5-plan is beneficial for someone who makes more than 60 min/month of long-distance calls without exceeding 159 min/month.
- The $12.95-plan is beneficial for someone who makes more than 159 min/month of long-distance calls without exceeding 1120 min/month.
- The $24.95-plan is only beneficial for someone who makes 1120 min/month or more of long-distance calls. For a month of 30 days, this time represents more than 37 min/day of long-distance calls.

REVISION 4

Prior learning 1

Page 4

a.



b.

Workout	Maximum heart rate	Minimum heart rate	Periods when heart rate increased
Thursday	136	64	[0, 20], [28, 32] and [36, 40]

Periods when heart rate decreased
[20, 28], [32, 36] and [40, 48]

c. 132 beats/min.

Prior learning 2

Page 5

a. -$25,000. It is the sum that the self-employed worker had to invest to start his company.

b. Five months. It is the time required for the self-employed worker to pay off the initial amount invested.

c. The function is negative over the interval [0, 5]. This interval represents the period in which the worker's company was in debt.

The function is positive over the interval [5, 12]. This interval represents the period in which the worker made profit.

d. It is decreasing over the interval [0, 2], because the slope of the corresponding line segment is negative. It is increasing over the interval [2, 12], because the slope of the corresponding line segment is positive.

e. Domain: [0, 12]; range: [-50 000, 100 000].

f. No, because for the inverse of this function, each value of the independent variable is not associated to, at most, a value of the dependent variable.

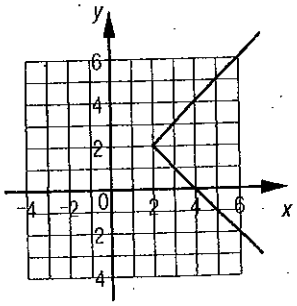
g. Domain: [-50 000, 100 000]; range: [0, 12].

Knowledge in action

Page 9

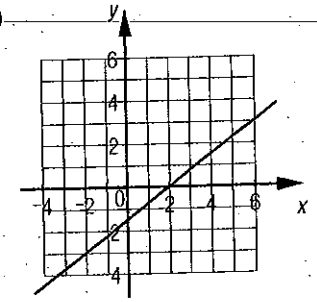
1. A, B, C

2. a) 1)



2) The inverse is not a function.

b) 1)



2) The inverse is a function.

Knowledge in action (*cont'd*)

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3. a) 1) Domain: $[-4, 4]$, range: $[-2, 5]$.
   2) The initial value is 1.
   3) Minimum: -2, maximum: 5.
   4) The function is increasing for $x \in [-4, 2]$; it is decreasing for $x \in [2, 4]$.
   5) Zeros: $\{-1, 3\}$.
   6) The function is positive for $x \in [-1, 3]$, it is negative for $x \in [-4, -1] \cup [3, 4]$.

   b) 1) Domain: $\mathbb{R}$, range: $\mathbb{R}$.
   2) The initial value is 3.
   3) This function has no extrema.
   4) This function is decreasing for $x \in \mathbb{R}$.
   5) Zero: $\{3\}$.
   6) The function is positive for $]-\infty, 3]$, it is negative for $[3, +\infty[$.

   c) 1) Domain: $[-3, 4]$, range: $[-1, 2]$.
   2) The initial value is 0.
   3) Minimum: -1, maximum: 2.
   4) The function is increasing for $x \in [-3, -2] \cup [1, 4]$, it is decreasing for $x \in [-2, 1]$.
   5) Zeros: $\{0, 2\}$.
   6) The function is positive for $x \in [-3, 0] \cup [2, 4]$, it is negative for $x \in [0, 2]$.

   d) 1) Domain: $[-2, 2]$, range: $[1, 3]$.
   2) The initial value is approximately 2.5.
   3) Minimum: 1, maximum: 3.
   4) The function is increasing for $x \in [-2, 2]$.
   5) This function does not have a zero.
   6) The function is positive for $x \in [-2, 2]$. This function is never negative.

4. a) **Cost of a taxi ride**

Distance covered (km)	Cost ($)
0	3.15
5	9.65
10	16.15
15	22.65
20	29.15
25	35.65
30	42.15

   b) $25.25

   c) 23 km

Knowledge in action (*cont'd*)

Page 11

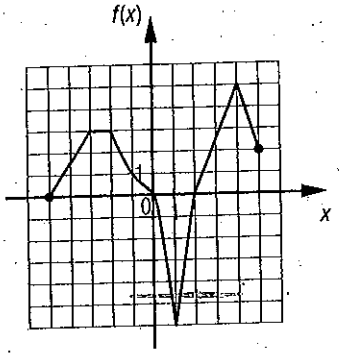
5. a) 1) 1800 people.
   2) 500 people.

   b) 1 h, 3 h, 10 h and 14 h after opening.

   c) $[1, 3]$ and $[10, 14]$.

   d) 1) It is increasing.
   2) It is decreasing over $[2, 5] \cup [6, 9]$ and increasing over $[5, 6]$.

6. *Several answers possible. Example:*



Knowledge in action (*cont'd*)

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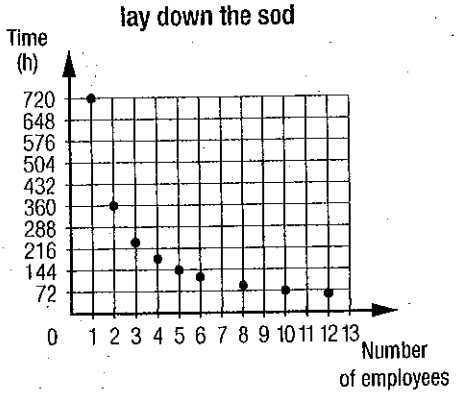
7. a) **Time required to lay down the sod**

Number of employees	Time (h)
1	720
2	360
3	240
4	180
5	144
6	120
8	90
10	72
12	60

b) **Time required to lay down the sod**

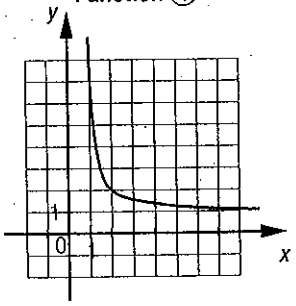


c) The function is decreasing.

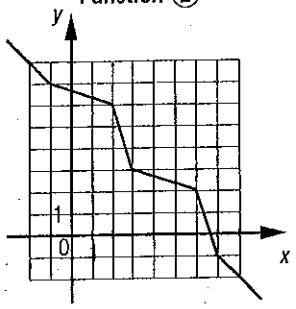
d) It does not represent a proportional situation because the graphical representation of the situation is not a line passing through the origin of the Cartesian plane.

8. a) The two graphical representations are symmetrical in relation to the angle bisector of the first and third quadrant.

b) **Inverse of Function ①**



Inverse of Function ②



You can notice that in each case, the graphical representation of the function is identical to the graphical representation of its inverse.

c) The curve of the inverse of a function whose graphical representation is symmetrical in relation to the angle bisector of the first and third quadrant is equivalent to the curve of the function itself.

SECTION 4.1 Real functions

Page 13

Problem

The evacuation process will be initiated 20 h after the start of the flood.

Page 14

Activity 1

a. 1) $1,200
   2) At 1 week, 3 weeks, 5 weeks, etc., every 2 weeks.

b. 1) $600
   2) The number of weeks between two pays, 2 weeks.

c. *Several answers possible. Example:* Employee ① spends his entire pay in a span of 2 weeks. This is why his assets decrease regularly between each pay and reaches $0. Employee ② has no expenses. This is why his assets do not change between two pays. Employee ③ spends less money than she earns, which allows her to save regularly. Employee ④ spends more than she earns, which means that her debts increase regularly.

d. Employee ①: $1,200, Employee ②: $9,600, Employee ③: ≈ $2,040, Employee ④ : ≈ $336.

Page 15

Activity 2

a. Curve ③.

b. 1) Curve ①.
2) Curve ②.
3) Curve ③.

c. An exponential function.

d. Approximately 7.42 micrograms of dioxins for each kilogram of breast milk.

Page 16

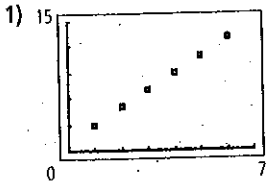
Technomath

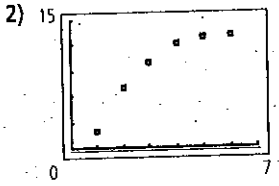
a. Activating the **Off** option in Screen **2** or Screen **4** would prevent the display of the scatter plot in question.

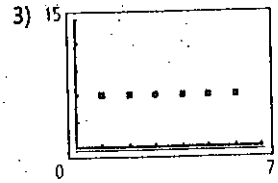
b. 1st difference: Screen **2** corresponds to **Plot1**, whereas Screen **4** corresponds to **Plot2**.
2nd difference: The type of graph selected in Screen **2** is a scatter plot, whereas the type of graph selected in Screen **4** is a broken line.
3rd difference: In Screen **2**, the selected lists are **L1** and **L2**, whereas in Screen **4**, the selected lists are **L1** and **L3**.
4th difference: In Screen **2**, the mark of the selected points is a square, whereas in Screen **4**, the mark of the selected points is a cross.

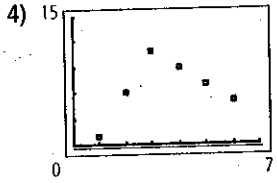
c. 1) Screen **5**. 2) Screen **3**.

d. *Several answers possible. Examples:*

1) 

2) 

3) 

4) 

Page 19

Practice 4.1

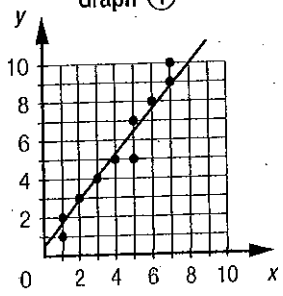
1. a) Exponential function.
b) First-degree polynomial function.
c) Periodic function.
d) Periodic function.
e) Piecewise function.
f) Step function.

Page 20

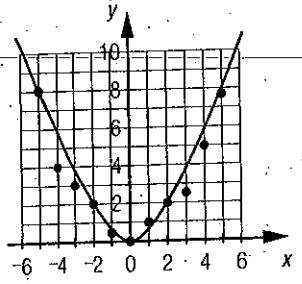
Practice 4.1 (*cont'd*)

2. A 1, B 5, C 3, D 2, E 4

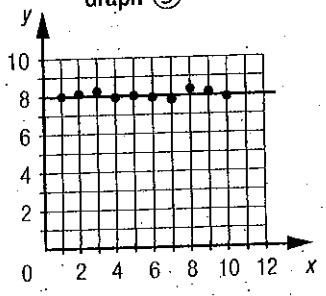
3. a) First-degree polynomial function.
b) Graph ①



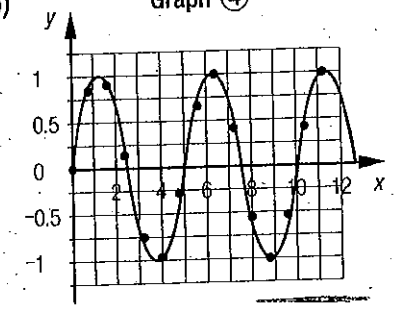
a) Second-degree polynomial function.
b) Graph ②



a) Zero-degree polynomial function.
b) Graph ③



a) Periodic function.
b) Graph ④



Practice 4.1 (*cont'd*)

4.

Table of values ①

a) A zero-degree polynomial function. b) $f(x) = 6$

c) $f(12.5) = 6$

Table of values ②

a) An inverse variation function. b) $f(x) = \frac{120}{x}$

c) $f(12.5) = 9.6$

Table of values ③

a) A first-degree polynomial function. b) $f(x) = 3x$

c) $f(12.5) = 37.5$

Table of values ④

a) A first-degree polynomial function. b) $f(x) = -0.5x + 8$

c) $f(12.5) = 1.75$

5.

Function ①

a) A second-degree polynomial function.

b) 1) Domain: $\mathbb{R}$, range: $]-\infty, 4]$.
2) The function is increasing over the interval $]-\infty, 3]$ and decreasing over the interval $[3, +\infty[$.
3) The function is negative over the interval $]-\infty, 0] \cup [6, +\infty[$ and positive over the interval $[0, 6]$.
4) The maximum of the function is 4.
5) 0
6) $\{0, 6\}$

Function ②

a) A piecewise function.

b) 1) Domain: $[-4, 7]$, range: $[-4, 4]$.
2) The function is increasing over the intervals $[-4, 1]$ and $[3, 6]$, the function is constant over the interval $[-3, 0]$ and the function is decreasing over the intervals $[1, 3]$ and $[6, 7]$.
3) The function is negative over the interval $[2, 5]$ and positive over the interval $[-4, 2] \cup [5, 7]$.
4) The maximum of the function is 4 and its minimum is -4.
5) 1
6) $\{-4, 2, 5\}$

Function ③

a) An exponential function.

b) 1) Domain: $\mathbb{R}$, range: $]0, +\infty[$.
2) The function is increasing over $\mathbb{R}$.
3) The function is positive over $\mathbb{R}$.
4) The function does not have a maximum nor a minimum.
5) $\{1\}$
6) The function does not have a zero.

Function ④

a) A step function.

b) 1) Domain: $]0, 12]$, range: $\{1, 2, 3, 4, 5, 6\}$.
2) The function is increasing over the interval $]2, 6]$ and decreasing over the intervals $]0, 3]$ and $]3, 12]$.
3) The function is positive over the interval $]0, 12]$.
4) The maximum of the function is 6 and its minimum is 1.
5) $\{\}$
6) $\{\}$

Practice 4.1 (*cont'd*)

6. a) A piecewise function.

b) 1 °C

c) -3 °C

d) 1) At 4:00 a.m.
2) At 10:00 a.m.

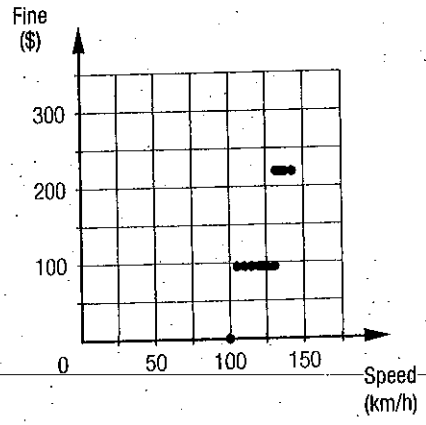
e) 1) At 1:00 a.m. and 7:00 a.m.
2) From 1:00 a.m. to 7:00 a.m.
3) From 12:00 a.m. to 1:00 a.m. and 7:00 a.m. to 12:00 p.m.

7. a)

Speeding fines

Speed (km/h)	Fine ($)	Speed (km/h)	Fine ($)
100	0	128	95
105	95	130	95
110	95	132	220
115	95	134	220
120	95	136	220
122	95	138	220
124	95	140	220
126	95		

b) Fines for speeding



c) A step function.

Practice 4.1 (*cont'd*) — Page 23

8. a) 45 patrons.
   b) For the 3 first hours as of 4:00 p.m.
   c) 40 patrons.
   d) At 5:00 p.m. and 9:00 p.m.

9. a) 1) A first-degree polynomial function.
      2) A zero-degree polynomial function.
      3) A first-degree polynomial function.
      4) A piecewise function.
   b) A periodic function.

SECTION 4.2 Second-degree polynomial function

Problem — Page 24

No. You would need a CD whose recordable surface has a "width" greater than 35 mm.

Activity 1 — Page 25

a. 1) No. Variations of the independent variable do not bring about zero variations of the dependent variable.
   2) No. Constant variations of the independent variable do not bring about constant and non-zero variables from the dependent variable.
   3) No. The product of the values of each ordered pair is not constant.

b. The numbers are 195.84, 587.52, 979.2, 1370.88, 1762.56, 2154.24, 2545.92, 2937.6, 3329.28, 3720.96, 4112.64.

c. A second-degree polynomial function.

d. From left to right:
   $\times 4, \times 9, \times 16, \times 25, \times 36, \times 49, \times 64, \times 81, \times 100, \times 121$.

e. The numbers in Column **B** are all the squares of the numbers in Column **A**.

f. 28 200.96 m

Activity 2 — Page 26

a. A second-degree polynomial function.

b. 1) From left to right, the value of the coefficient of x^2 gets progressively further from 0.
   2) From left to right, the curves progressively become vertically stretched.

c. The further the coefficient of x^2 is from 0, the more the curve is vertically stretched.

d. 1) From left to right, the value of the coefficient of x^2 gets progressively further from 0.
   2) From left to right, the curves progressively become vertically stretched.

e. The further the value of the coefficient of x^2 is from 0, the more the curve is vertically stretched.

f. 1) The sign of the coefficient of x^2 is different.
   2) One corresponds to the reflection of the other about the x-axis.

g. A change in the sign of the coefficient of x^2 causes a reflection of the curve about the horizontal axis, the x-axis.

Technomath — Page 27

a. The curves pass through the origin of the Cartesian plane and they are symmetrical in relation to the y-axis.

b. The coefficients of x^2 are -2.5, 0.6, 4.

c. In the case of a function whose rule is written in the form $y = ax^2$, the coefficient of x^2 has the same value as the y-coordinate of the point whose x-coordinate is 1.

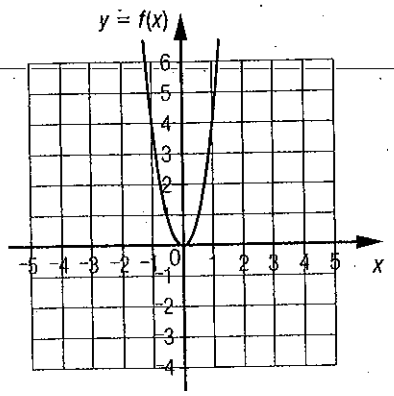
d. 1) The curve is open upward.
   2) The curve is open downward.
   3) The opening of the curve enlarges or the curve compresses vertically.
   4) The opening of the curve diminishes or the curve stretches vertically.

Practice 4.2

1. a) 1) $f(x) = 4x^2$

x	y
-6	144
-4	64
-2	16
0	0
2	16
4	64
6	144

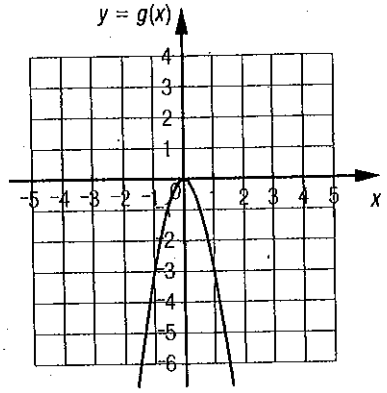
2)



b) 1) $g(x) = -3x^2$

x	y
-6	-108
-4	-48
-2	-12
0	0
2	-12
4	-48
6	-108

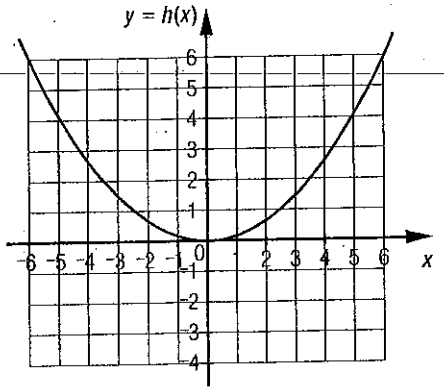
2)



c) 1) $h(x) = \frac{1}{6}x^2$

x	y
-6	6
-4	$\frac{8}{3}$
-2	$\frac{2}{3}$
0	0
2	$\frac{2}{3}$
4	$\frac{8}{3}$
6	6

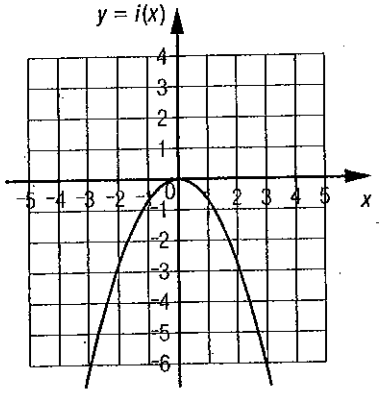
2)



d) 1) $i(x) = -0.7x^2$

x	y
-6	-25.2
-4	-11.2
-2	-2.8
0	0
2	-2.8
4	-11.2
6	-25.2

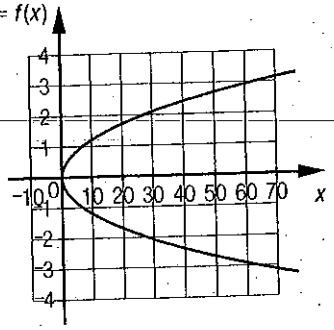
2)



2. a)

x	y
63	-3
28	-2
7	-1
0	0
7	1
28	2
63	3

b) $y = f(x)$



c) No, because in the case of the inverse, each value of the independent variable is not associated to, at most, a value of the dependent variable.

3. a) g
b) h
c) f and g.
d) h and i.

Practice 4.2 (*cont'd*) — Page 31

4. a)

Rule	Coordinates of the vertex	Maximum	Minimum
$f(x) = 16x^2$	(0, 0)	–	0
$g(x) = -5x^2$	(0, 0)	0	–
$h(x) = 9x^2$	(0, 0)	–	0
$i(x) = -3x^2$	(0, 0)	0	–

b) The vertex is always located at coordinates (0, 0). If a is positive, the function does not have a maximum and its minimum is 0. If a is negative, the function does not have a minimum and its maximum is 0.

5. A 4, B 3, C 2, D 1

6. a) 1) $A = c^2$
2) $A = 0.5c^2$
b) 1) 40.96 cm^2
2) 57.245 cm^2
c) 1) 18 cm
2) 9.4 cm

Practice 4.2 (*cont'd*) — Page 32

7.

Rule	Domain	Range	Zero	Increasing interval	Decreasing interval
$f(x) = x^2$	$\mathbb{R}$	$[0, +\infty[$	0	$[0, +\infty[$	$]-\infty, 0]$
$g(x) = 6x^2$	$\mathbb{R}$	$[0, +\infty[$	0	$[0, +\infty[$	$]-\infty, 0]$
$h(x) = -2x^2$	$\mathbb{R}$	$]-\infty, 0]$	0	$]-\infty, 0]$	$[0, +\infty[$
$i(x) = -1.5x^2$	$\mathbb{R}$	$]-\infty, 0]$	0	$]-\infty, 0]$	$[0, +\infty[$

8. a) The speed is associated to a first-degree polynomial function and the distance is associated to a second-degree polynomial function.
b) 1) 5 m/s
2) 62.5 m
c) 1) 57.5 s
2) $\approx$ 112.25 s

Practice 4.2 (*cont'd*) — Page 33

9. a) **Steel balls**

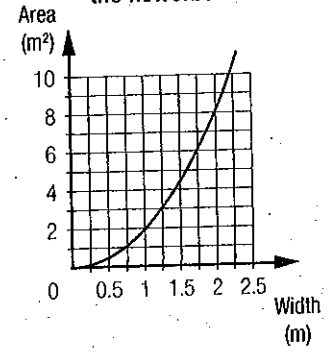
Radius (mm)	Surface area (mm^2)
1	$4\pi \approx 12.57$
2	$16\pi \approx 50.27$
3	$36\pi \approx 113.1$
4	$64\pi \approx 201.06$
5	$100\pi \approx 314.16$

b) $\approx$ 1261.67 mm^2

10. a) At 112.36 m. (56.18) b) $\approx$ 3.16 s (4.47 s)
c) Yes, because he is approximately 315.06 m from his point of ejection. No (157.5)

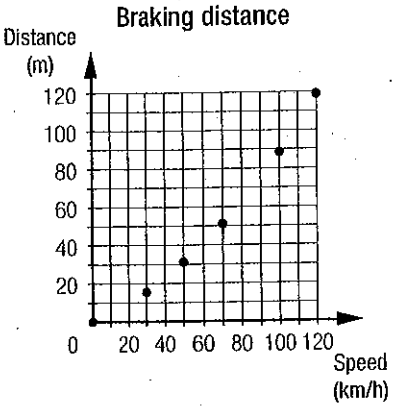
Practice 4.2 (*cont'd*) — Page 34

11. a) $A = 2x^2$
b)

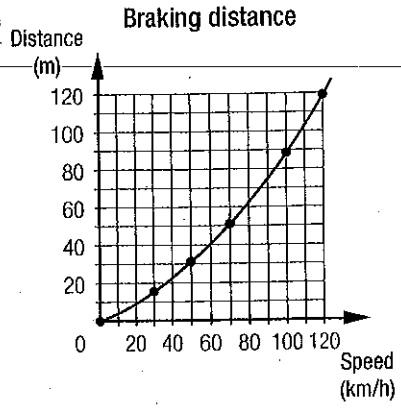


c) 1) The curve would be vertically stretched.
2) The curve would be vertically compressed.

12. a)



b)



c) $y \approx 0.01x^2$

d) 1) $\approx$ 92.72 m
2) $\approx$ 224.36 m
3) $\approx$ 293.04 m

SECTION 4.3 Exponential functions

Problem

Page 35

This situation is a paradox because its issue depends on the way it is approached. In fact, if it is approached by taking the steps into consideration, it would seem as though Achilles would have never overtaken the turtle since in order to do this, he would have had to cover an infinite number of steps. However, if this situation is considered in terms of time, a well-defined answer can be formed.

Activity 1

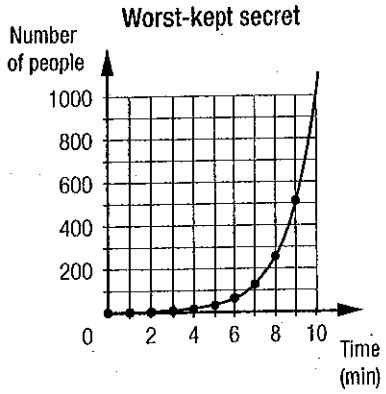
Page 36

a.

Worst-kept secret

Time (min)	Calculation	Number of people aware of the secret
0	1×2^0	1
1	$1 \times 2 = 1 \times 2^1$	2
2	$1 \times 2 \times 2 = 1 \times 2^2$	4
3	$1 \times 2 \times 2 \times 2 = 1 \times 2^3$	8
4	$1 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^4$	16
5	$1 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^5$	32
6	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^6$	64
7	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^7$	128
8	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^8$	256
9	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^9$	512
...	...	...
t	1×2^t	

b.



c. An exponential function, because the graphical representation is a curve whose growth becomes increasingly larger.

d. 1) 5 min 2) 14 min 3) 19 min

e. 1) 1 048 576 people.
2) 33 554 432 people.
3) 8 589 934 592 people.

Activity 2

a. The number of bacteria decreases by half at each application.

b. **Antibiotic ointment**

Number of applications	Calculation	Number of bacteria ($\times 10^6$)
0	1024×0.5^0	1024
1	$1024 \times 0.5 = 1024 \times 0.5^1$	512
2	$1024 \times 0.5 \times 0.5 = 1024 \times 0.5^2$	256
3	$1024 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^3$	128
4	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^4$	64
5	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^5$	32
6	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^6$	16
7	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^7$	8
...	...	...
n	1024×0.5^n	

c. 1024 million bacteria.

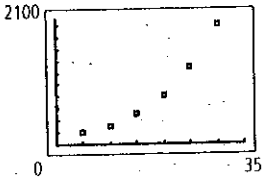
d. By 0.5.

e. The distance between the curve and the x-axis constantly decreases, but it will never be zero.

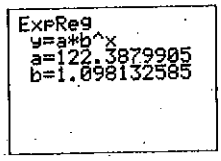
Technomath

a. The value of a represents the initial value of the curve that is used as a model of this situation. The value of b represents the base.

b. 1)



2)



$y = 122.39(1.1)^x$

3)

Practice 4.3

1. a) 81 b) 16 807 c) 4 d) 5
e) 2 f) 2 g) 5 h) 3

2.

	Rule of the function	Domain	Range	Base	Initial value	Variation
a)	$f(x) = 3\left(\frac{1}{5}\right)^x$	$\mathbb{R}$	$]0, +\infty[$	$\frac{1}{5}$	3	Decreasing
b)	$g(x) = 2.5^x$	$\mathbb{R}$	$]0, +\infty[$	2.5	1	Increasing
c)	$h(x) = 0.7^x$	$\mathbb{R}$	$]0, +\infty[$	0.7	1	Decreasing
d)	$i(x) = 2(\pi)^x$	$\mathbb{R}$	$]0, +\infty[$	π	2	Increasing
e)	$j(x) = -2\left(\frac{2}{3}\right)^x$	$\mathbb{R}$	$]-\infty, 0[$	$\frac{2}{3}$	-2	Increasing
f)	$k(x) = -7(2.6)^x$	$\mathbb{R}$	$]-\infty, 0[$	2.6	-7	Decreasing

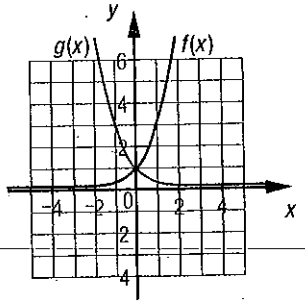
3. a) 1) 486, 1458, 4374, ... 2) $y = 2(3)^x$

b) 1) $4, \frac{4}{9}, \frac{4}{81}, \ldots$ 2) $y = 26\,244\left(\frac{1}{9}\right)^x$

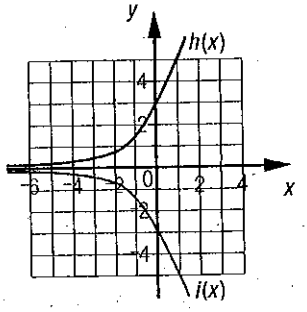
c) 1) 2.5, 25, 250, ... 2) $y = 0.000\,25(10)^x$

d) 1) $12, 3, \frac{3}{4}, \ldots$ 2) $y = 3072\left(\frac{1}{4}\right)^x$

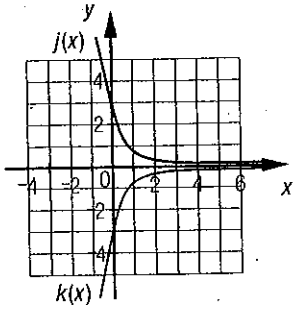
4. a) 1)



2)



3)



b) 1) A reflection about the *y*-axis.
2) A reflection about the *x*-axis.
3) A reflection about the *x*-axis.

Practice 4.3 (*cont'd*)

5. Approximately 1197 frogs will be present.

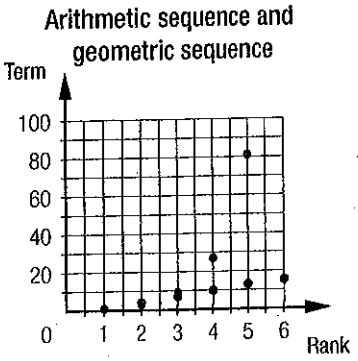
6. C and B.

7. $234.74

8. Screen **1**: $\approx -4.44 \times 10^{30}$
Screen **2**: $\approx 579\,725\,821.7$
Screen **3**: $\approx 5.19 \times 10^{50}$

Practice 4.3 (*cont'd*)

9. a)

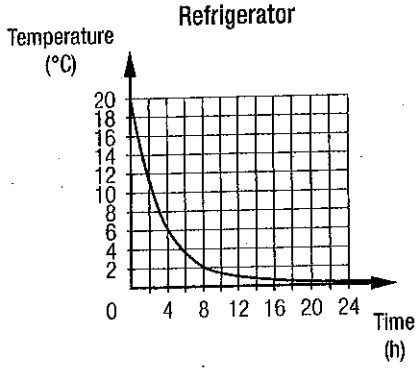


b) Arithmetic sequence: first-degree polynomial function.
Geometric sequence: exponential function.

c) Arithmetic sequence: $y = 3x - 2$.
Geometric sequence: $y = \frac{1}{3}(3)^x$ or $y = 3^{x-1}$.

10. a) 3.5 **b)** 65, 325
c) 36, 2916 **d)** 0.625, 160

11. a)



b) 1) $y = 0$
2) The asymptote represents the minimum temperature of the refrigerator.

c)]0, 20]

d) 20°C

12. a) $y = 4(6)^x$ **b)** $y = 3(1.5)^x$
c) $y = -1(3)^x$ **d)** $y = 2(0.5)^x$

Practice 4.3 (*cont'd*)

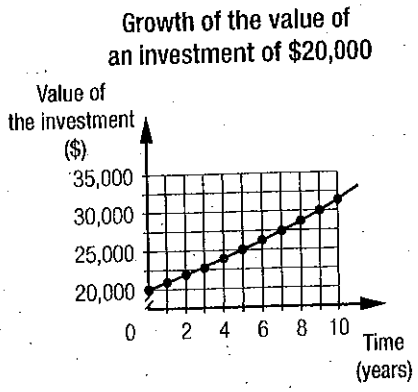
13. a) 29 640 446 people.

b) 1 600 584 people.

c) 38 555 629 people.

d) Because nothing guarantees that in the next few years the growth of the population of Canada would develop at the same rate as it did in the previous years.

14. a)

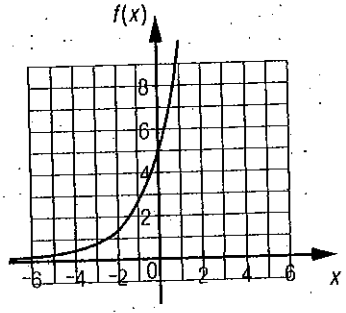


b) $31,366.24

15. a)

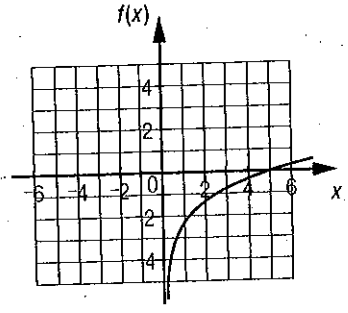
x	$f(x)$
−3	$\frac{5}{8}$
−2	$\frac{5}{4}$
−1	$\frac{5}{2}$
0	5
1	10
2	20
3	40

b)



c) Domain: $\mathbb{R}$; range: $]0, +\infty[$.

d)



e) Domain: $]0, +\infty[$; range: $\mathbb{R}$.

f) Yes. It consists of a relation where each value of the independent variable is associated, at most, to a value of the dependent variable.

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Practice 4.3 *(cont'd)*

16. $7,691.15

17. a) 1) 1.5% 2) $\approx$ 14.03%

b) 1) 98.5% 2) $\approx$ 85.97%

18. a) 6 376 210 235 people.

b) $P = 6\,376\,210\,235(1.0122)^n$ where n represents the number of years since 2004 and P represents the world population.

c) $P = 6\,376\,210\,235(1.0122)^{46}$
$= 11\,138\,172\,630$ personnes.

19. a) $N = 3000(2)^h$ where n represents the number of bacteria and h represents time in hours.

b) 1) After 6 h. 2) After 10 h.

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Practice 4.3 *(cont'd)*

20. a) $26,500 b) 106%

c) $26,522.50 d) 106.09%

e)

Plan A			
Time (mths)	Time (years)	Calculation	Value of investment ($)
0	0	$25\,000(1.06)^0$	25,000
12	1	$25\,000(1.06)^1$	26,500
24	2	$25\,000(1.06)^2$	28,090
36	3	$25\,000(1.06)^3$	29,775.40
48	4	$25\,000(1.06)^4$	31,561.92
...	...	...	...
	x	$25\,000(1.06)^x$	

Plan B			
Time (mths)	Time (years)	Calculation	Value of investment ($)
0	0	$25\,000(1.03)^0$	25 000
6	0.5	$25\,000(1.03)^1$	25,750
12	1	$25\,000(1.03)^2$	26,522.50
18	1.5	$25\,000(1.03)^3$	27,318.18
24	2	$25\,000(1.03)^4$	28,137.72
30	2.5	$25\,000(1.03)^5$	28,981.85
36	3	$25\,000(1.03)^6$	29,851.31
42	3.5	$25\,000(1.03)^7$	30,746.85
48	4	$25\,000(1.03)^8$	31,669.25
...	...	...	...
	x	$25\,000(1.03)^{2x}$	

f) Plan B is the most profitable because in this plan, the second portion of 3% interest is calculated on an amount where 3% had already been added.

SECTION 4.4

Step, periodic and piecewise functions

Problem

Page 48

The best time to go see the falls would be in the afternoon on January 19 at 2:15 p.m.

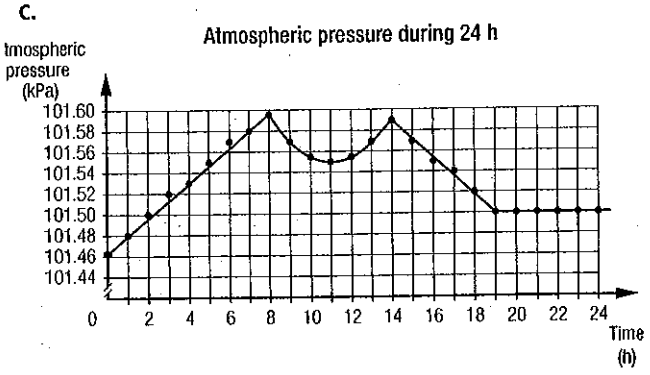
Activity 1

Page 49

a. 1) A first-degree polynomial function.
2) A second-degree polynomial function.
3) A first-degree polynomial function.
4) A zero-degree polynomial function.

b. A piecewise function.

c.

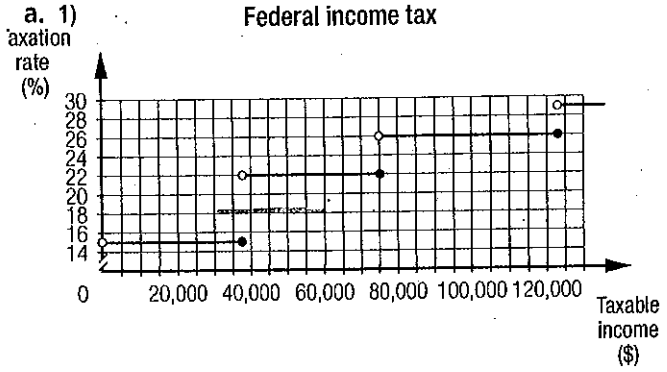


d. 1) ≈ 101.51 kPa
2) ≈ 101.59 kPa
3) ≈ 101.58 kPa
4) ≈ 101.515 kPa

Activity 2

Page 50

a. 1)



2) A step function.
3) Domain: [0, +∞[(if you were to assume that it is possible to have an infinitely large taxable income), range: {15, 22, 26, 29}.

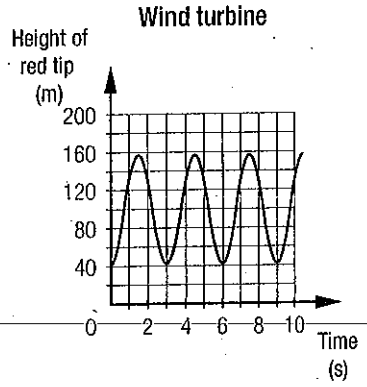
b. 1) $5,250
2) $14,017.23
3) $32,889.27

c. 1) Greater than $75,769 without exceeding $123,184.
2) $54,327.05

Activity 3

Page 51

a.



b. A periodic function.

c. Domain: [0, 10]; range: [42.5, 157.5].

d. Every 3 s.

e. 1) Its range would become [62.5, 137.5] and the graph would be vertically compressed.
2) The period of the function would be 4 s (the red tip would be at its initial position every 4 s) and the graph would be horizontally stretched.

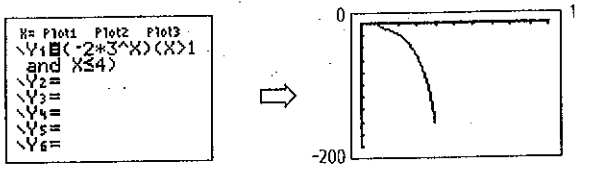
Technomath

Page 52

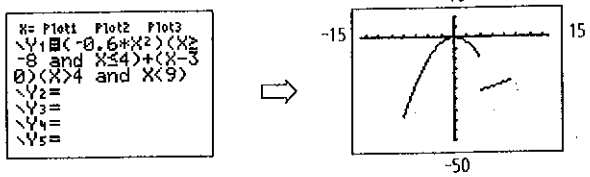
a. 1) Domain: [-3, 4[
2) Range: [0, 8[

b. 1) Domain: [-8, 3]
2) Domain: [3, 6]
3) Domain: [-8, 6]
4) Range: [8, 74]

c. 1)



2)



Practice 4.4

Page 54

1. a) A piecewise function.
b) A piecewise function.
c) A periodic function.
d) A step function.

2. a) A step function.
b) A piecewise function.
c) A periodic function.

Practice 4.4 (*cont'd*)

Page 55

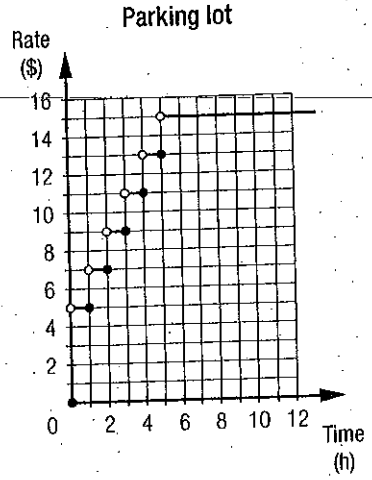
3. a) 1) A piecewise function.
2) 2 3) 8 4) 4

b) 1) A periodic function.
2) 1 3) 4 4) {4, 10, 16, 22}

c) 1) A step function.
2) 0 3) 10 4) [4, 6[

d) 1) A piecewise function.
2) 12 3) 8 4) {4, 8}

4. a)



b) $15

c) No, because the range of the function is {5, 7, 9, 11, 13, 15}.

5. a) 6

b) [-1, 2]

c) 1) 1 2) 2 3) 1

Practice 4.4 (*cont'd*)

Page 56

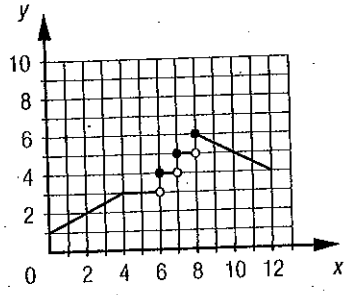
6. a) The moment when the program retrieves a file.

b) 4 files.

c) Domain: [0, 35]; range: {0, 1, 2, 3, 4}.

d) [0, 20[. It consists of the time interval during which the program has not yet found a single file.

7. a)



b) A piecewise function.

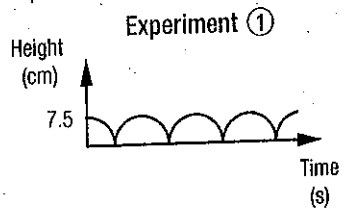
c) $4

d) In the interval [7, 8[.

8. Experiment ①

a) A periodic function.

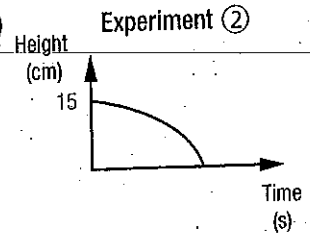
b)



Experiment ②

a) A piecewise function.

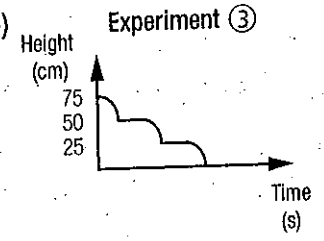
b)



Experiment ③

a) A piecewise function.

b)

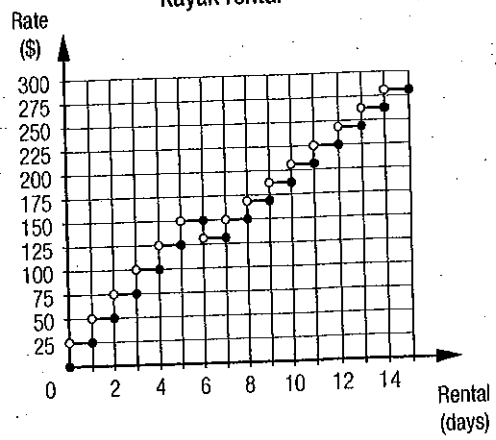


Practice 4.4 (*cont'd*)

Page 57

9. No, since the nature of the periodic function is that more than one value of the independent variable can be associated to the value of the dependent variable.

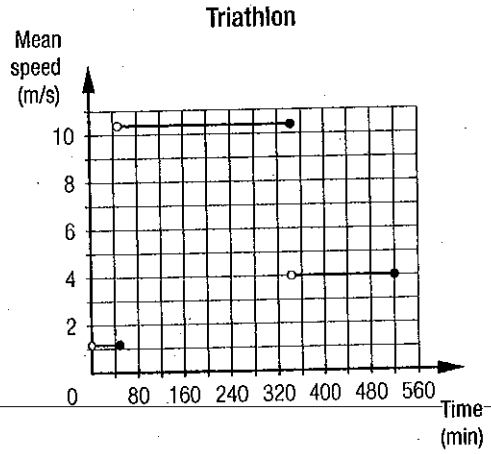
10. a)



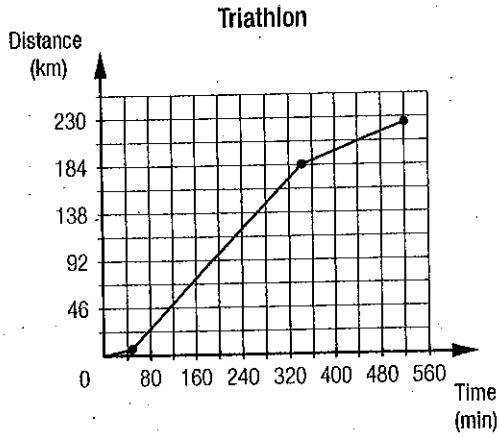
b) It is cheaper to rent the kayak for 7 days. The 6-day rental at $25/day comes to $150, whereas the 7-day rental at $18.75/day comes to $131.25.

c) The individual can use the kayak for a maximum period of 8 days.

11. a) 1)



2)



b) Cycling.

c) 8 h 37 min 3 s.

Practice 4.4 (*cont'd*)

Page 58

12. a) 1) At approximately 2.17 L/s.

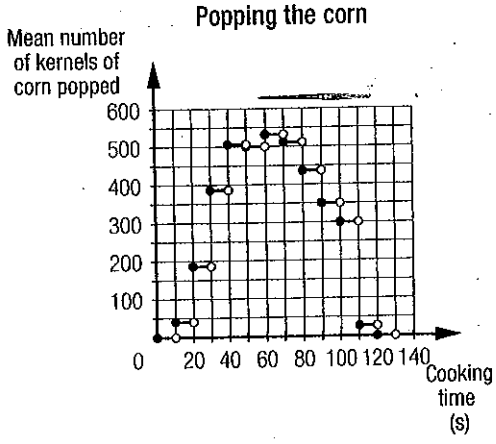
2) At approximately 0.48 L/s.

b) 1) 6.5 L

2) ≈ 4.33 L

c) After approximately 2.31 s and after approximately 22.62 s.

13. a)



b) 1) Approximately 8705 kernels have popped.

2) Approximately 22 560 kernels have popped.

3) Approximately 37 780 kernels have popped.

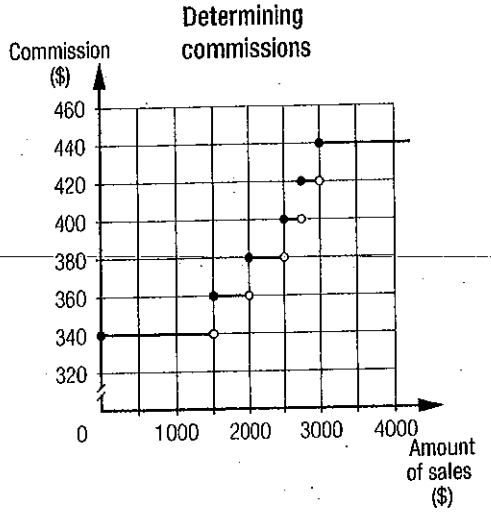
c) ≈ 48 s

Practice 4.4 (*cont'd*)

Page 59

14. Approximately 2 h 40 min or 160 min.

15. a)



b) 1) $340 2) $380
3) $420 4) $440

c) $2,500.01

d) No. For this to happen, the salesman must obtain a commission of $10, which is impossible based on the table provided of calculations of commission.

SPECIAL FEATURES 4

Chronicle of the past

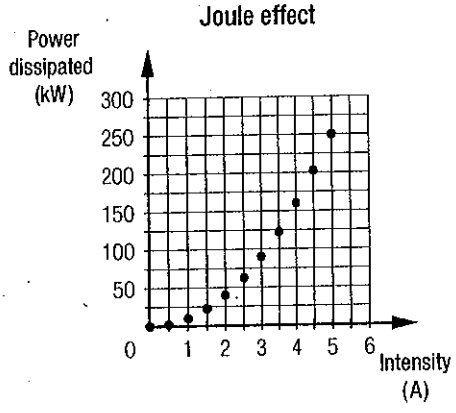
Page 61

1. a) A first-degree polynomial function.

b) ≈ 8 A

c) Its slope would be less steep.

2. a)



b) A second-degree polynomial function.

c) The curve would be vertically compressed.

In the workplace

Page 63

1. a) 4.5 bars. b) 7 bars.
2. The duration of the ascent in scuba diving is 84 min.
3. 5.5 bars.

Overview

Page 64

1. a) To a periodic function.
   b) To an exponential function.
   c) To a second-degree polynomial function.
   d) To a first-degree polynomial function.
2. a) To a second-degree polynomial function.
   b) To an exponential function.
   c) To a piecewise function.
   d) To a periodic function.

Overview (*cont'd*)

Page 65

2. e) To a first-degree polynomial function.
   f) To a step function.
3. **Graph ①**
   a) 1) Domain: $\mathbb{R}$; range: [-3, 3].
      2) {..., -8, -4, 0, 4, 8, ...}
      3) Maximum: 3; minimum: -3.
      4) 0
   b) A periodic function.
   c) Its inverse is not a function because more than one value of the dependent variable can be associated to a value of the independent variable.

   Graph ②
   a) 1) Domain: $\mathbb{R}$; range: [0, +∞[.
      2) 0
      3) There is no maximum, minimum: 0.
      4) 0
   b) A second-degree polynomial function.
   c) Its inverse is not a function because, more than one value of the dependent variable can be associated to a value of the independent variable.

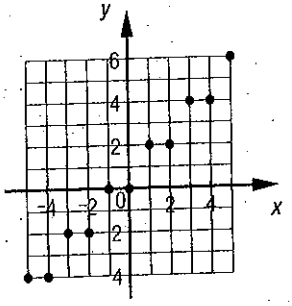
   Graph ③
   a) 1) Domain: $\mathbb{R}$; range:]0, +∞[.
      2) There is no zero.
      3) There is no maximum or minimum.
      4) 1
   b) An exponential function.
   c) Its inverse is a function because, not more than one value of the dependent variable can be associated to a value of the independent variable.

   Graph ④
   a) 1) Domain: $\mathbb{R}$; range: [-3, +∞[.
      2) {0.5, 3.5}
      3) There is no maximum, minimum: -3.
      4) 1
   b) A piecewise function.
   c) Its inverse is not a function because, more than one value of the dependent variable can be associated to a value of the independent variable.
4. To an exponential function because, each new term is obtained by multiplying the previous one by 2.

Overview (*cont'd*)

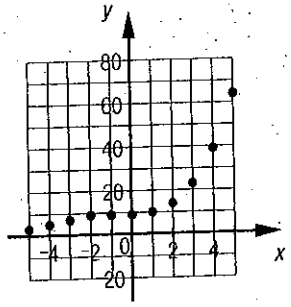
Page 66

5. a) $y = -\frac{7}{11}x + \frac{27}{11}$ b) $y = \frac{1}{16}x^2$
   c) $y = 2\left(\frac{1}{2}\right)^x$ d) $y = \frac{1}{x}$
6. a) 1)



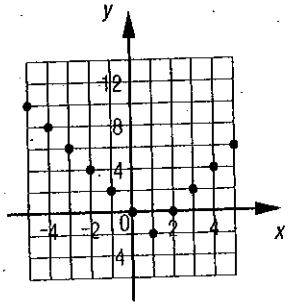
   2) A step function.

   b) 1)



   2) An exponential function.

   c) 1)



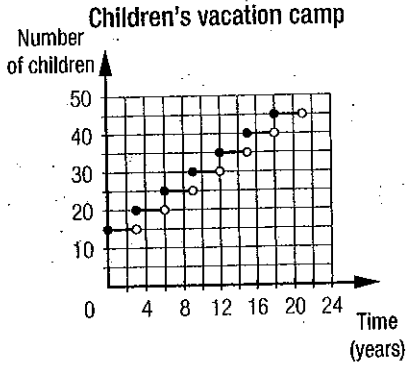
   2) A piecewise function.

Overview (*cont'd*)

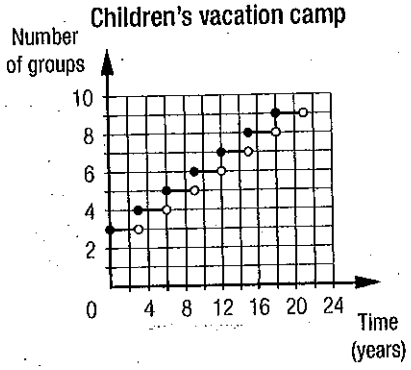
7. a) Children's vacation camp

Time since the camp was founded (years)	Number of children	Number of groups	Number of leaders	Number of children for each leader
0	15	3	3	5
1	15	3	3	5
2	15	3	3	5
3	20	4	4	5
4	20	4	4	5
5	20	4	4	5
6	25	5	5	5
7	25	5	5	5
8	25	5	5	5
9	30	6	6	5
10	30	6	6	5

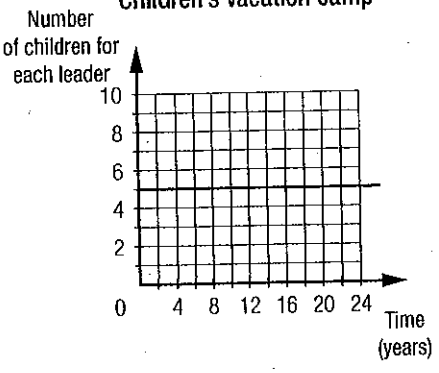
b) 1) Children's vacation camp



2) Children's vacation camp



3) Children's vacation camp



c) 1) 45 children. **2)** 9 groups.

d) A zero-degree polynomial function.

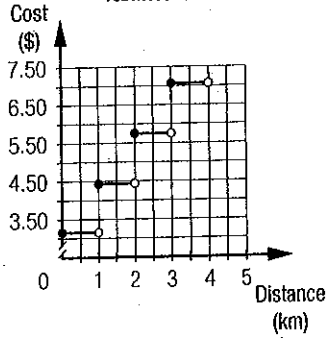
Overview (*cont'd*)

8. a) $f(x) = -2x^2$ **b)** $f(x) = -2(3)^x$

9. a) 10

b) 1) 1 **2)** 1 **3)** 2

10. a) Taximeter



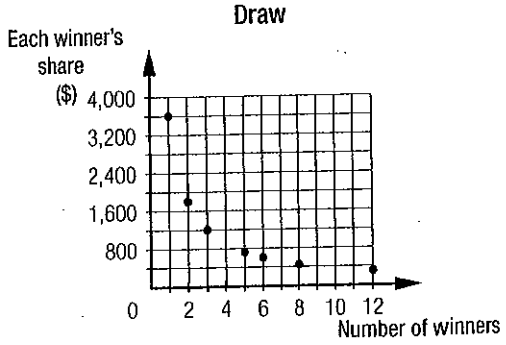
b) $14.85

c) 6 km and more, but less than 7 km.

Overview (*cont'd*)

11. a) $3,600

b) Draw



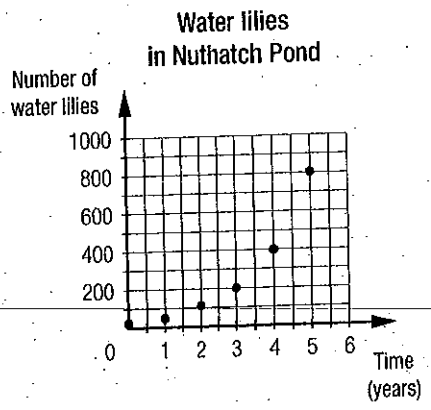
c) To an inverse variation function.

d) $240

12. a) 1) 10 min 2) 40 min

b) 55 min c) $\approx$ 10.92 km

13. a)



b) To an exponential function.

c) After a little more than 7 years.

Overview (*cont'd*)

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14. a)

x	0	0.1	0.25	0.4	0.49	0.5	0.6	0.75	0.9
y	0	0	0	0	0	1	1	1	1

0.99	1	1.1	1.25	1.4	1.49	1.5	1.6	1.75	2
1	1	1	1	1	1	2	2	2	2

b) It is rounded off to the nearest unit.

15. *Several answers possible.* Any initial value held between 3.5 mg/mL and 8 mg/mL would be acceptable.

Overview (*cont'd*)

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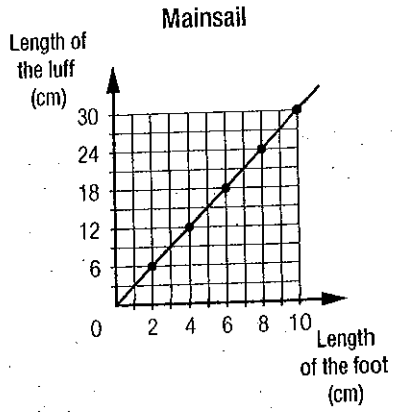
16. a)

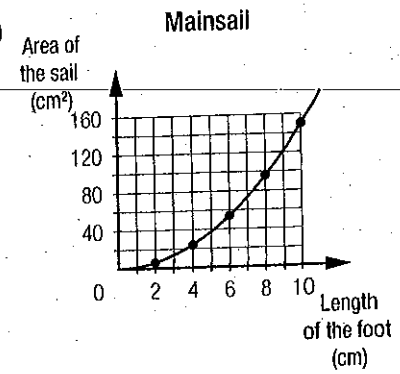
Mainsail

Length of the foot (cm)	Length of the luff (cm)	Area of the sail (cm^2)
2	6	6
4	12	24
6	18	54
8	24	96
10	30	150

b) 1)



2)



c) 1) $y = 3x$ 2) $y = 1.5x^2$

d) 15 cm

17. a) A second-degree polynomial function.

b) 9 m

c) At 6 s.

Bank of problems

Page 72

18. Canada's population will be approximately 48 075 000 people.

19. The step function is the most appropriate, because it shows that the value of the record remains constant as long as he or she is not beaten, which complies with reality. The model of the first-degree polynomial function is to be rejected, because it assumes that at a certain time, the record would end up being zero, which is impossible.

Bank of problems (*cont'd*)

Page 73

20. The medicine does not meet the doctor's requests. The effects go away approximately 73 min after it is administered, which is not long enough.

21. A Malthusian catastrophe should occur in 24 years.

VISION 5 | From right triangles to trigonometric relations

LES 10 | *Measuring time*

The following is a procedure used to draw hour lines on the dial plate of a sundial that can be used at a latitude of 52°.

The base AO of the style is fixed at 20 cm.

1. Calculate the dimensions of the style. The angle of inclination of the style must be 52° since it is being used at a latitude of 52°.

$\sin 52° = \frac{m\,\overline{AQ}}{m\,\overline{AO}} \Rightarrow m\,\overline{AQ} = m\,\overline{AO} \times \sin 52° = 20 \times \sin 52° \approx 15.76$ cm

2. Determine the position of points P on the dial plate.

Since the triangles, with vertices point A, point Q and a point on the line, are right triangles, certain trigonometric ratios can be established.

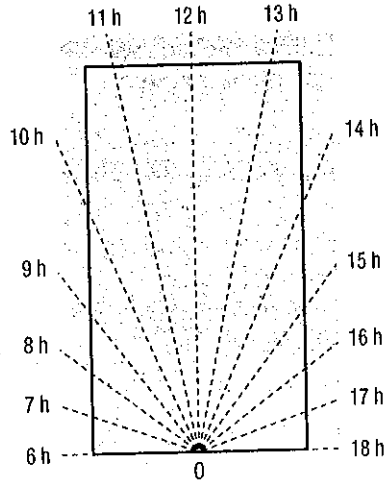
Angle	Measure of angle (°)	Ratio	Distance of point P in relation to point A
AQP_1	90		
AQP_2	75	$\tan 75° = \frac{m\,\overline{AP_2}}{m\,\overline{AQ}}$	$m\,\overline{AP_2} = m\,\overline{AQ} \times \tan 75° \approx 58.82$ cm
AQP_3	60	$\tan 60° = \frac{m\,\overline{AP_3}}{m\,\overline{AQ}}$	$m\,\overline{AP_3} = m\,\overline{AQ} \times \tan 60° \approx 27.3$ cm
AQP_4	45	$\tan 45° = \frac{m\,\overline{AP_4}}{m\,\overline{AQ}}$	$m\,\overline{AP_4} = m\,\overline{AQ} \times \tan 45° \approx 15.76$ cm
AQP_5	30	$\tan 30° = \frac{m\,\overline{AP_5}}{m\,\overline{AQ}}$	$m\,\overline{AP_5} = m\,\overline{AQ} \times \tan 30° \approx 9.1$ cm
AQP_6	15	$\tan 15° = \frac{m\,\overline{AP_6}}{m\,\overline{AQ}}$	$m\,\overline{AP_6} = m\,\overline{AQ} \times \tan 15° \approx 4.22$ cm
AQP_7	0	$\tan 0° = \frac{m\,\overline{AP_7}}{m\,\overline{AQ}}$	0
AQP_8	15	$\tan 15° = \frac{m\,\overline{AP_8}}{m\,\overline{AQ}}$	$m\,\overline{AP_8} = m\,\overline{AQ} \times \tan 15° \approx 4.22$ cm
AQP_9	30	$\tan 30° = \frac{m\,\overline{AP_9}}{m\,\overline{AQ}}$	$m\,\overline{AP_9} = m\,\overline{AQ} \times \tan 30° \approx 9.1$ cm
AQP_{10}	45	$\tan 45° = \frac{m\,\overline{AP_{10}}}{m\,\overline{AQ}}$	$m\,\overline{AP_{10}} = m\,\overline{AQ} \times \tan 45° \approx 15.76$ cm
AQP_{11}	60	$\tan 60° = \frac{m\,\overline{AP_{11}}}{m\,\overline{AQ}}$	$m\,\overline{AP_{11}} = m\,\overline{AQ} \times \tan 60° \approx 27.3$ cm
AQP_{12}	75	$\tan 75° = \frac{m\,\overline{AP_{12}}}{m\,\overline{AQ}}$	$m\,\overline{AP_{12}} = m\,\overline{AQ} \times \tan 75° \approx 58.82$ cm
AQP_{13}	90		

3. Calculate the angles that help draw the hour lines.

Since the triangles, with vertices point O, point A and one of the previously determined point P's, are right triangles, it is possible to establish certain trigonometric ratios.

Triangle	Time	Tangent of angle O	Measure of angle O (°)
AOP_1	6 h		90
AOP_2	7 h	$\frac{m\overline{AP_2}}{m\overline{AO}} \approx \frac{58.82}{20}$	$m\angle O \approx \arctan\frac{58.82}{20} \approx 71.22$
AOP_3	8 h	$\frac{m\overline{AP_3}}{m\overline{AO}} \approx \frac{27.3}{20}$	$m\angle O \approx \arctan\frac{27.3}{20} \approx 53.77$
AOP_4	9 h	$\frac{m\overline{AP_4}}{m\overline{AO}} \approx \frac{15.76}{20}$	$m\angle O \approx \arctan\frac{15.76}{20} \approx 38.24$
AOP_5	10 h	$\frac{m\overline{AP_5}}{m\overline{AO}} \approx \frac{9.1}{20}$	$m\angle O \approx \arctan\frac{9.1}{20} \approx 24.46$
AOP_6	11 h	$\frac{m\overline{AP_6}}{m\overline{AO}} \approx \frac{4.22}{20}$	$m\angle O \approx \arctan\frac{4.22}{20} \approx 11.92$
AOP_7	12 h	$\frac{m\overline{AP_7}}{m\overline{AO}} = \frac{0}{20}$	$m\angle O = \arctan 0 = 0$
AOP_8	13 h	$\frac{m\overline{AP_8}}{m\overline{AO}} \approx \frac{4.22}{20}$	$m\angle O \approx \arctan\frac{4.22}{20} \approx 11.92$
AOP_9	14 h	$\frac{m\overline{AP_9}}{m\overline{AO}} \approx \frac{9.1}{20}$	$m\angle O \approx \arctan\frac{9.1}{20} \approx 24.46$
AOP_{10}	15 h	$\frac{m\overline{AP_{10}}}{m\overline{AO}} \approx \frac{15.76}{20}$	$m\angle O \approx \arctan\frac{15.76}{20} \approx 38.24$
AOP_{11}	16 h	$\frac{m\overline{AP_{11}}}{m\overline{AO}} \approx \frac{27.3}{20}$	$m\angle O \approx \arctan\frac{27.3}{20} \approx 53.77$
AOP_{12}	17 h	$\frac{m\overline{AP_{12}}}{m\overline{AO}} \approx \frac{58.82}{20}$	$m\angle O \approx \arctan\frac{58.82}{20} \approx 71.22$
AOP_{13}	18 h		90

4. Represent the hour lines.

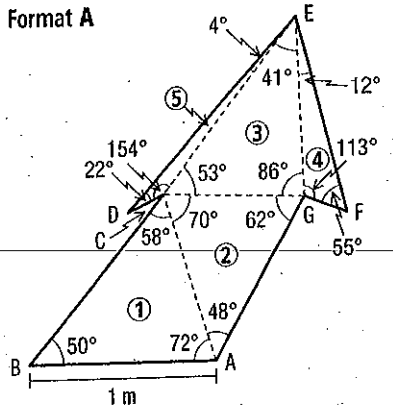


LES 11 Measuring pressure

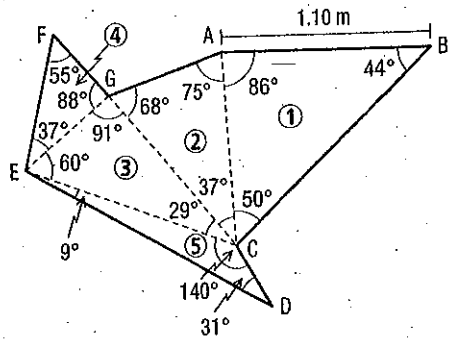
The following is a procedure that calculates the pressure exerted by the sculpture based on the format of the chosen bases:

1. Cut out the base of each format into triangles and measure the angle formed.

Format A



Format B



2. Calculate the measure of each side of each triangle.

It is possible to calculate the length of each side by using the sine law. The following is an example of the calculation for the side BC of Format **A**:

$$\frac{1}{\sin 58°} = \frac{m\overline{BC}}{\sin 72°} \Rightarrow m\overline{BC} = \frac{1 \times \sin 72°}{\sin 58°} \approx 1.12 \text{ m}$$

Format A		Format B	
Side	Length (m)	Side	Length (m)
$\overline{AB}$	1	$\overline{AB}$	1.10
$\overline{BC}$	≈ 1.12	$\overline{BC}$	≈ 1.43
$\overline{AC}$	≈ 0.90	$\overline{AC}$	≈ 1
$\overline{AG}$	≈ 0.96	$\overline{AG}$	≈ 0.65
$\overline{CG}$	≈ 0.76	$\overline{CG}$	≈ 1.04
$\overline{CD}$	≈ 0.22	$\overline{CD}$	≈ 0.36
$\overline{CE}$	≈ 1.16	$\overline{CE}$	≈ 1.20
$\overline{DE}$	≈ 1.35	$\overline{DE}$	≈ 1.50
$\overline{EG}$	≈ 0.93	$\overline{EG}$	≈ 0.58
$\overline{EF}$	≈ 1.04	$\overline{EF}$	≈ 0.71
$\overline{FG}$	≈ 0.23	$\overline{FG}$	≈ 0.43

3. Calculate the areas of the bases of both formats.

Since the measurements of all the angles and sides of the triangles are known, it is possible to calculate the area of each triangle by using Hero's formula or the trigonometric formula. The following are examples of calculations for Triangles ① and ② of Format **A**. The other areas have been calculated based on the same principles.

$A_1 = \frac{1 \times 1.12 \times \sin 50°}{2} \approx 0.43 \text{ m}^2$

$A_2 = \sqrt{1.31(1.31 - 0.9)(1.31 - 0.96)(1.31 - 0.76)}$
$\approx 0.32 \text{ m}^2$

Format A		Format B	
Triangle	Area (m^2)	Triangle	Area (m^2)
①	≈ 0.43	①	≈ 0.55
②	≈ 0.32	②	≈ 0.31
③	≈ 0.35	③	≈ 0.30
④	≈ 0.10	④	≈ 0.12
⑤	≈ 0.05	⑤	≈ 0.14
Area of the base $\approx 1.258 \text{ m}^2$		Area of the base $\approx 1.426 \text{ m}^2$	

4. Calculate the volume and mass of each base.

The bases are in the shape of a right prism. Their volume is therefore given by the formula $V = A_{base} \times h$. Moreover, since the density of each material is given in g/cm^3, it is best to convert the obtained volumes into cm^3. As a result, $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$.

	Format A
Volume (m^3)	$1.258 \times 0.25 \approx 0.3145$
Volume (cm^3)	314 500
Mass of the base (g)	$314\,500 \times 2.75 = 864\,875$
Mass of the base (kg)	$864\,875 \div 1000 = 864.875$
Total mass of the sculpture (kg)	$1500 + 864.875 = 2364.875$
Exerted pressure on the surface (Pa)	$\frac{9.8 \times 2364.875}{1.258} \approx 18\,422.71$

	Format B
Volume (m^3)	$1.426 \times 0.20 \approx 0.2852$
Volume (cm^3)	285 200
Mass of the base (g)	$285\,200 \times 2.52 = 718\,704$
Mass of the base (kg)	$718\,704 \div 1000 = 718.704$
Total mass of the sculpture (kg)	$1500 + 718.704 = 2218.704$
Exerted pressure on the surface (Pa)	$\frac{9.8 \times 2218.704}{1.426} \approx 15\,247.76$

5. Conclusion

Format **B** is the safest because, it allows the exerted pressure from the sculpture and the base to be less than 16 000 Pa.

REVISION 5

Prior learning 1 — Page 76

a. $\frac{12.5}{4.26} \approx 2.93$

b. 1) $\frac{m\,\overline{DE}}{m\,\overline{JK}} = \frac{m\,\overline{DF}}{m\,\overline{JL}}$ 2) $\frac{m\,\overline{CD}}{m\,\overline{IJ}} = \frac{m\,\overline{AF}}{m\,\overline{GL}}$

c. 1) 25°, because the sum of the interior angles of a triangle is 180°.
2) 115°, because it consists of an angle that is supplementary to angle 4.
3) 90°, quadrilateral HIJL is a square.
4) 25°, because angle 6 is supplementary to the sum of angles 2 and 5.

d. 1) ≈ 13.67 m
2) $\approx 1{:}146.71$
3) $\approx 24.80 \text{ m}^2$

Knowledge in action — Page 79

1. ❶ D, ❷ A, ❸ B, ❹ C

2. a) $x = \frac{70}{3}$ or $23.\overline{3}$. b) $y = 4.5$
c) $z = 1.04$ d) $a = 7$ or -7.

3. a) 1) $\frac{65}{34}$ 2) $\frac{34}{9}$ 3) 22

b) Model **3)**, because the ratios of the dimensions are not identical to the dimensions of the original.

Knowledge in action (*cont'd*) — Page 80

4. a) 54° b) 79.55° c) 45°
d) 23.7° e) 0°

5. 7 km

6. a) The sum of the measures of angles A and D is always equal to 180°. It is the same for the sum of the measures of angles B and C.

b) 1) $m\angle ABD = 90°$, because it is an angle formed by a base and height of a trapezoid.
$m\angle CBD + m\angle BCD = 90°$, because they are both acute angles of right triangle BCD.
Conclusion:
$m\angle ABD +$
$m\angle CBD + m\angle BCD = m\angle ABC + m\angle BCD$
$= 90° + 90° = 180°$

2) $m\angle CAE = 90°$, because it is an angle formed by a base and height of a parallelogram.
$m\angle EAF + m\angle AFE = 90°$, because they are both acute angles of the right triangle AEF.
Conclusion:
$m\angle CAE +$
$m\angle EAF + m\angle AFE = m\angle CAF + m\angle AFE$
$= 90° + 90° = 180°$

7. a) Length of the sides of Triangle ②: 5 u, 3.75 u and 6.25 u.

Length of the sides of Triangle ③: 6.25 u, ≈ 4.69 u and ≈ 7.81 u.

Length of the sides of Triangle ④: ≈ 7.81 u, ≈ 5.86 u and ≈ 9.77 u.

Length of the sides of Triangle ⑤: ≈ 9.77 u, ≈ 7.32 u and ≈ 12.21 u.

b) ≈ 0.41 **c)** ≈ 88.67 u²

SECTION 5.1 Trigonometric ratios

Problem — Page 81

The students can provide explanations similar to the ones provided below.

In regards to each ramp, the following must be completed:

1. The employee must measure the three sides of the triangle that form the base of the ramp.
2. The employee must then make sure that these three measurements validate the Pythagorean theorem. If this is the case, it means the base has the shape of a right triangle and the employee can go to Step **3.** If this is not the case, the ramp does not meet the stated conditions.
3. The employee must calculate the ratio $\frac{\text{length of the side opposite to the 12° angle}}{\text{length of the hypotenuse}}$ and verify that this ratio is equal to $\frac{1.6}{7.7}$, which is the ratio obtained with the measures of the provided model. If this is the case, it means the base has the shape of a right triangle with an angle measuring 12°.

Activity 1 — Page 82

a. *Several answers possible. Example:* Triangles that have two congruent corresponding angles are similar.

b. ≈ 9.33 m, 10.89 m and 12.44 m.

c. 1) ≈ 0.766, ≈ 0.766 and ≈ 0.766.

2) The ratios are almost equal in each case.

3) SIN 50 ≈ 0.766

4) The sine of an angle can calculate the value of the ratio $\frac{\text{length of the side opposite to an acute angle}}{\text{length of the hypotenuse}}$ when this angle is associated to a right triangle.

d. 1) ≈ 0.643, ≈ 0.643 and ≈ 0.643.

2) The ratios are almost equal in each case.

3) COS 50 ≈ 0.643

4) The cosine of an angle can calculate the value of the ratio $\frac{\text{length of the side adjacent to an acute angle}}{\text{length of the hypotenuse}}$ when this angle is associated with a right triangle.

e. 1) ≈ 1.192, ≈ 1.191 and ≈ 1.191.

2) The ratios are almost equal in each case.

3) TAN 50 ≈ 1.192

4) The tangent of an angle can calculate the value of the ratio $\frac{\text{length of the side opposite to an acute angle}}{\text{length of the side adjacent to this acute angle}}$ when this angle is associated to a right triangle.

Technomath — Page 83

a. 1) Respectively ≈ 0.75, ≈ 0.42, ≈ 0.42 and ≈ 0.34.

2) Yes.

b. 1) Respectively ≈ 0.66, ≈ 0.91, ≈ 0.91 and ≈ 0.94.

2) No, the contrary. The ratio seems to decrease.

c. 1) It approaches 1. 2) It approaches 1.

Practice 5.1 — Page 85

1.

m ∠ R	sin R	cos R	tan R
15°	**≈ 0.26**	**≈ 0.97**	**≈ 0.27**
30°	**0.5**	**≈ 0.87**	**≈ 0.58**
45°	**≈ 0.71**	**≈ 0.71**	**1**
60°	**≈ 0.87**	**0.5**	**≈ 1.73**
75°	**≈ 0.97**	**≈ 0.26**	**≈ 3.73**

2. a) sin G ≈ 0.81 cos G ≈ 0.58 tan G ≈ 1.4

b) sin G ≈ 0.93 cos G ≈ 0.37 tan G = 2.5

c) sin G ≈ 0.81 cos G ≈ 0.58 tan G = 1.4

d) sin G = $\frac{6}{7}$ ≈ 0.86 cos G ≈ 0.52 tan G ≈ 1.66

e) sin G = 0.5 cos G ≈ 0.87 tan G ≈ 0.58

f) sin G ≈ 0.85 cos G = $\frac{25}{48}$ ≈ 0.52 tan G ≈ 1.64

Practice 5.1 (*cont'd*) — Page 86

3. a) m ∠ C = 34° m $\overline{AB}$ ≈ 2.29 cm m $\overline{BC}$ ≈ 3.40 cm

b) m ∠ D = 57° m $\overline{DE}$ ≈ 5.37 cm m $\overline{DF}$ ≈ 2.92 cm

c) m ∠ H = 30° m $\overline{GI}$ = 2.5 cm m $\overline{HI}$ ≈ 4.33 cm

d) m ∠ J = 20° m $\overline{JL}$ ≈ 4.40 cm m $\overline{JK}$ ≈ 4.68 cm

e) m ∠ N = 65° m $\overline{MN}$ ≈ 5.68 cm m $\overline{MO}$ ≈ 5.15 cm

f) m ∠ P = 45° m $\overline{PR}$ = 3.3 cm m $\overline{PQ}$ ≈ 4.67 cm

4. a) At approx. 6.18 m. **b)** ≈ 5.33 m **c)** At approx. 1 m.

5. a) False, because $\frac{\sin S}{\sin T} = \frac{\frac{s}{r}}{\frac{t}{r}} = \frac{s}{t}$; $\frac{\cos S}{\cos T} = \frac{\frac{t}{r}}{\frac{s}{r}} = \frac{t}{s}$ and $\frac{s}{t} \neq \frac{t}{s}$.

b) True, because $\tan S = \frac{s}{t}$; $\frac{\cos T}{\cos S} = \frac{\frac{s}{r}}{\frac{t}{r}} = \frac{s}{t}$ and $\frac{s}{t} = \frac{s}{t}$.

c) False, because $\frac{s}{r} + \frac{t}{r} = \frac{s+t}{r}$; $\tan S = \frac{s}{t}$ and $\frac{s+t}{r} \neq \frac{s}{t}$.

d) True, because $\sin R = \frac{r}{r} = 1$.

e) True, because the hypotenuse is the longest side in any right triangle.

Practice 5.1 *(cont'd)* — Page 87

6. ≈ 9.19 cm

7. a) $>$ **b)** $=$ **c)** $<$ **d)** $=$

8. Tree ①: ≈ 68.63 m
Tree ②: ≈ 69.17 m
Tree ③: ≈ 81.35 m

Practice 5.1 *(cont'd)* — Page 88

9. a)

	sine	cosine	$\frac{\text{sine}}{\text{cosine}}$	tangent
Angle D	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{3}{4}$
Angle F	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$	$\frac{4}{3}$

b) 1. The tangent of an angle is equivalent to the ratio $\frac{\text{sine}}{\text{cosine}}$.
2. The sine of an angle is equivalent to the cosine of its complement.
3. The cosine of an angle is equivalent to the sine of its own complement.

c) 1. $\tan A = \frac{a}{c}$, $\sin A = \frac{a}{b}$, $\cos A = \frac{c}{b}$

$$\frac{\sin A}{\cos A} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{c} = \tan A$$

2. $\sin A = \frac{a}{b}$ and $\cos C = \frac{a}{b}$.
3. $\cos A = \frac{c}{b}$ and $\sin C = \frac{c}{b}$.

10. No, because the boat travelled a distance of approximately 74.64 m.

11. a) Approximately 10.95 s. **b)** Approximately 13.1 s.
c) Approximately 1.67 m/s.

Practice 5.1 *(cont'd)* — Page 89

12. ≈ 2 m

13. $\approx 5.14 \times 10^{13}$ km

14. a) $\approx 37\,658.18$ km **b)** ≈ 8199.57 km

Practice 5.1 *(cont'd)* — Page 90

15. ≈ 313.96 m

16. The speed of this plane is approximately 396.74 m/s (Mach 1.3) or 1428.25 km/h.

SECTION 5.2 Finding missing measurements

Problem — Page 91

The angle of emission CBD must measure approximately 113.17°.

Activity 1 — Page 92

a. 1) ≈ 0.0436 **2)** 2.5° **3)** Yes.

b. 1) The measure of an angle based on the value of the sine associated to this angle.
2) 60°

c. 1) ≈ 0.9994 **2)** 2° **3)** Yes.

Activity 1 *(cont'd)* — Page 93

d. 1) The measure of an angle based on the value of the cosine associated to this angle.
2) 75°

e. 1) ≈ 0.06116 **2)** 3.5° **3)** No.

f. 1) The measure of an angle based on the value of the tangent associated to this angle.
2) 0°

g. 1) 30° for each angle A.
2) In all right triangles, if the length of the side opposite to an angle is equal to half the length of the hypotenuse, this angle is equal to 30°. Conversely, in all right triangles with a 30° angle, the length of the side opposite to this angle is equal to half the length of the hypotenuse.

Technomath — Page 94

a. They are all 30°-60°-90° triangles. In each triangle, the smallest side measures half the length of the hypotenuse.

b. 1)

Trigonometric ratio	Screen 3	Screen 4
$\frac{\text{length of side opposite to 30° angle}}{\text{length of hypotenuse}}$	0.5	0.5
$\frac{\text{length of side adjacent to 30° angle}}{\text{length of hypotenuse}}$	≈ 0.87	≈ 0.87
$\frac{\text{length of side opposite to 60° angle}}{\text{length of hypotenuse}}$	≈ 0.87	≈ 0.87
$\frac{\text{length of side adjacent to 60° angle}}{\text{length of hypotenuse}}$	0.5	0.5

Trigonometric ratio	Screen 5	Screen 6
$\frac{\text{length of side opposite to 30° angle}}{\text{length of hypotenuse}}$	0.5	0.5
$\frac{\text{length of side adjacent to 30° angle}}{\text{length of hypotenuse}}$	≈ 0.87	≈ 0.87
$\frac{\text{length of side opposite to 60° angle}}{\text{length of hypotenuse}}$	≈ 0.87	≈ 0.87
$\frac{\text{length of side adjacent to 60° angle}}{\text{length of hypotenuse}}$	0.5	0.5

2) The length of the side opposite to the 30° angle is equal to half the length of the hypotenuse.

c. 1) *Several answers possible.*
2) There is only one measure of an angle whose sine equals 0.5 — 30°.

d. *Several answers possible.*

Practice 5.2 — Page 96

1. a) ≈ 9.79° **b)** 30° **c)** ≈ 60.46°
d) ≈ 24.49° **e)** ≈ 44.77° **f)** ≈ 74.93°
g) ≈ 34.99° **h)** 45°

2. b), d) and f).

3. a) ≈ 19.88° **b)** ≈ 73.74° **c)** ≈ 8.11°
d) ≈ 83.11° **e)** ≈ 21.8° **f)** ≈ 88.00°

4.

	Length of side adjacent to angle A (cm)	Length of side opposite to angle A (cm)	Length of hypotenuse (cm)
Triangle ①	12	≈ 32.88	35
Triangle ②	≈ 51.96	30	60
Triangle ③	11.4	15.5	≈ 19.24
Triangle ④	45.76	≈ 26.42	≈ 52.84
Triangle ⑤	≈ 0.20	≈ 0.40	0.45
Triangle ⑥	34.5	46	57.5

	Measure of angle A (°)	Measure of angle B (°)
Triangle ①	≈ 69.95	≈ 20.05
Triangle ②	30	60
Triangle ③	≈ 53.67	≈ 36.33
Triangle ④	30	60
Triangle ⑤	63	27
Triangle ⑥	≈ 53.13	≈ 36.87

5. ≈ 51.34°

Practice 5.2 (*cont'd*) — Page 97

6. a) m ∠A = 48.89° m ∠C = 41.11°
m $\overline{AB}$ = 6.11 cm
b) m ∠E = 60° m ∠F = 30°
m $\overline{DF}$ ≈ 17.32 cm
c) m ∠O = 30° m $\overline{MN}$ = 4.5 cm
m $\overline{MO}$ ≈ 7.79 cm
d) m ∠H = 40.02° m ∠G = 49.98°
m $\overline{HI}$ = 23.82 cm
e) m ∠L = 66° m $\overline{KL}$ ≈ 13.36 mm
m $\overline{JL}$ ≈ 32.84 mm
f) m ∠Q = 76.66° m ∠R = 13.34°
m $\overline{PR}$ = 72.98 cm

7. a) ≈ 36.87° **b)** ≈ 56.31°

8. ≈ 28.69°

Practice 5.2 (*cont'd*) — Page 98

9. a) ≈ 49.99° **b)** ≈ 40.11°
c) ≈ 57.26°, ≈ 57.26°, ≈ 65.48°

10. ≈ 8.72 cm

11. ≈ 82.16 m

Practice 5.2 (*cont'd*) — Page 99

12. ≈ 3649.89 m

13. a) ≈ 3 km **b)** ≈ 20.38°
c) ≈ 33.02°, ≈ 14.57°

Practice 5.2 (*cont'd*) — Page 100

14. a) ≈ 14.97 km
b) Plane **A**: ≈ 18.88°, Plane **B**: 30°.
c) ≈ 1300.97 m

15. a) ≈ 25° **b)** ~~≈ 2.37~~ m

Practice 5.2 (*cont'd*) — Page 101

16. a) ≈ 6.78 m **b)** ≈ 20.42 m
c) ≈ 12.23 m **d)** ≈ 22.92 m

17. a) ≈ 7.21° **b)** ≈ 7.21°
c) ≈ 39 315.76 km

SECTION 5.3

Calculating the area of any triangle

Problem

Page 102

Thomas gains approximately 4.66° by placing himself at point B rather than point A.

Activity 1

Page 103

a. Michelle's observation is accurate, because in triangle ABC, 83° → 2.25 cm, 62° → 2 cm and 35° → 1.3 cm, and in triangle DEF, 107° → 2.35 cm, 41° → 1.6 cm and 37° → 1.3 cm.

b. The equalities are false (the fundamental property of the proportions is not respected).

c. No, because the two examples mentioned refute this conjecture.

Activity 1 (*cont'd*)

Page 104

d. Joseph's observation is accurate, because in triangle ABC, sin 83° ≈ 0.99, sin 62° ≈ 0.88 and sin 35° ≈ 0.57 and in triangle DEF, sin 107° ≈ 0.96, sin 41° ≈ 0.66 and sin 32° ≈ 0.53.

e. The equalities are true (the fundamental property of the proportions is respected).

f. Yes, because the two examples mentioned validate this conjecture.

g. 1) ≈ 0.26 and ≈ 0.26. 2) 0.5 and ≈ 0.17.
3) ≈ 0.71 and ≈ 0.71. 4) ≈ 0.87 and ≈ 0.87.
5) ≈ 1.00 and ≈ 0.94.

h. 1) No, because the previous results show that there are several angles whose sine is less than or equal to the sine of a smaller angle.
2) Yes, because the previous results do not contain any counterexamples that would refute Audrey's observation.

Activity 2

Page 105

a. 1) 4.86 cm² 2) 4.86 cm²

b. 1) ≈ 4.46 cm² 2) ≈ 4.46 cm²

c. 1) The area of this triangle cannot be calculated because there is not enough information to determine the length of one of its heights.
2) ≈ 4.84 cm²

d. Hero's formula can calculate the area of a triangle quickly while it is tedious or impossible to calculate the height unless the lengths of all three sides are known.

Activity 2 (*cont'd*)

Page 106

e. 1. The area of a triangle is equal to half the product of the base of the triangle multiplied by its height.
2. $\overline{BD}$ is the height relative to the base AC and is therefore perpendicular to it.
3. In a right triangle, the sine of an angle is equal to the ratio between the length of the side opposite to this angle and the length of the hypotenuse.
4. In a proportion, the product of the extremes is equal to the product of the means.
5. In an equality, a term can be substituted by an expression that is equivalent to it.

f. ≈ 50.33 cm²

g. 1) $A = \frac{m\,\overline{DE} \times m\,\overline{DF} \times \sin D}{2}$

2) $A = \frac{m\,\overline{DE} \times m\,\overline{EF} \times \sin E}{2}$

3) $A = \frac{m\,\overline{EF} \times m\,\overline{DF} \times \sin F}{2}$

h. You must know the lengths of the two sides as well as the measure of the angle found between these two sides.

Technomath

Page 107

a. 1) 3 u, 6 u, 7 u 2) 8 u
3) 16 u

b. 1) ≈ 17.32 cm² 2) ≈ 4.60 dm²
3) ≈ 34 751.83 m²

c.

```
PROGRAM:HERO
:ClrHome
:Disp "LENGTH OF
 SIDES"
:Prompt A,B,C
:(A+B+C)/2→P
:Disp "AREA OF
TRIANGLE"
```

```
:Disp √(P(P-A)
(P-B)(P-C)
:End
```

Practice 5.3

Page 109

1. a) ≈ 7.93 **b)** ≈ 61.28 **c)** ≈ 68.33
d) ≈ 15.35 **e)** ≈ 25.73 **f)** ≈ 48.70

2. a) ≈ 11.92 cm **b)** ≈ 12.40 cm **c)** ≈ 8.81 cm
d) ≈ 36.68° **e)** ≈ 54.60° **f)** ≈ 131.42°
g) 96° **h)** ≈ 4.34 cm **i)** ≈ 135.83°

3.

	m $\overline{BC}$	m $\angle C$	m $\angle B$	Area of ΔABC
a)	4.8 cm	66°	48°	≈ 8.56 cm^2
b)	10 cm	$\approx 26.01°$	$\approx 87.99°$	≈ 23.99 cm^2
c)	5 cm	$\approx 61.28°$	$\approx 52.72°$	≈ 9.55 cm^2

Practice 5.3 (*cont'd*) — Page 110

4. a) 1) True. 2) False. 3) True.
   b) 1) The sine of an angle and the sine of its supplementary are equivalent.
   2) The cosine of an angle and the cosine of its supplementary are opposite signs.
   3) The tangent of an angle and the tangent of its supplement are opposite signs.
5. a) False. b) False. c) False.
6. a) ≈ 13.34 cm^2 b) ≈ 8.44 cm^2 c) ≈ 5.23 cm^2
7. ≈ 30.26 cm^2

Practice 5.3 (*cont'd*) — Page 111

8. a) ≈ 61.90 m^2 b) ≈ 43.77 m^2
9. a) ≈ 9.93 cm^2 b) ≈ 9.59 cm^2 c) ≈ 6.97 cm^2
10. $104,635.37
11. $\approx 121.28°$
12. At approximately 1258.29 m.

Practice 5.3 (*cont'd*) — Page 112

13. a) No, because she gets a measure that is less than 90° for angle B and greater than 90° for angle C, whereas the drawing shows that it should be the contrary.
    b) In these calculations, this student forgot to take into consideration the fact that angle B is obtuse.
    c) m $\angle B \approx 129.94°$
    m $\angle C \approx 20.06°$
    m $\overline{AB} \approx 2.06$ cm
14. ≈ 37.54 m
15. a) 91.44 m, 91.44 m, ≈ 125.89 m
    b) ≈ 4174.91 m^2

Practice 5.3 (*cont'd*) — Page 113

16. ≈ 45.14 m
17. a) ≈ 11.62 m b) ≈ 8.16 m c) ≈ 6.63 m
    d) ≈ 15.24 m e) ≈ 8.21 m

SPECIAL FEATURES 5

Chronicle of the past — Page 115

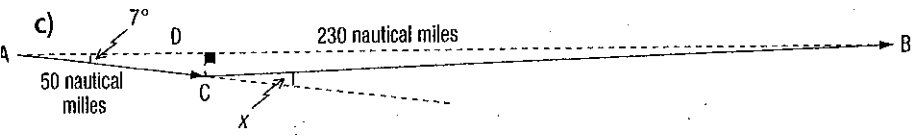
1. a) $\approx 1.8°$ b) $\approx 89.1°$ c) $\approx 400\,001.3$ km
2.

Measure of angle AOB subtended at the centre (°)	Length of chord AB (dm)	Measure of angle AOC subtended at the centre (°)	Sine of angle AOC $\left(\frac{m\,\overline{AC}}{m\,\overline{AO}}\right)$
0	0.00	0	0
10	0.174	5	0.087
20	0.347	10	0.1735
30	0.518	15	0.259
40	0.684	20	0.342
50	0.845	25	0.4225
60	1	30	0.5
70	1.147	35	0.5735
80	1.286	40	0.643
90	1.414	45	0.707
100	1.532	50	0.766
110	1.638	55	0.819
120	1.732	60	0.866
130	1.813	65	0.9065
140	1.879	70	0.9395
150	1.932	75	0.966
160	1.97	80	0.985
170	1.992	85	0.996
180	2	90	1

3. Arab mathematicians were able to construct a sine table by determining, for a given angle, half of the chord that was associated to the double of this angle in Hipparchus' chord tables.

In the workplace — Page 117

1. a) Approximately 248.37 m.
   b) Approximately 415.56 m.
   c) Yes, because his or her boat is located at approximately 175.62 m from the shore.
2. a) $\approx 8.93°$
   b) ≈ 0.88 km



Overview — Page 118

1. a) $x \approx 4.10$ cm and $y \approx 2.09$ cm.
   b) $x \approx 60.89°$ and $y \approx 3.23$ cm.
   c) $x \approx 47.42°$ and $y \approx 42.58°$.
   d) $x \approx 68.46°$ and $y \approx 3.33$ cm.
   e) $x \approx 4.27$ cm and $y \approx 3.09$ cm.
   f) $x \approx 130.62°$ and $y \approx 2.33$ cm.
2. a) ≈ 5.41 cm^2 b) ≈ 10.81 cm^2
   c) ≈ 11.54 cm^2 d) ≈ 3.78 cm^2
   e) ≈ 7.59 cm^2 f) ≈ 5.11 cm^2
3. ≈ 19.32 m

Overview (*cont'd*) — Page 119

4. a) ≈ 8.33 cm^2 b) ≈ 2498.04 mm^2
   c) ≈ 0.43 km^2 d) $\approx 18\,596.76$ cm^2
5. a) $\approx 3.58°$ b) 17.28 m^3
   c) ≈ 28.86 m^2
6. a) ≈ 1784.77 m^2 b) $\approx 11\,898.48$ m^3
7. a) ≈ 218.81 m b) ≈ 17.97 m
   c) ≈ 219.54 m
8. $\approx 10\,524.21$ cm^3

Overview (*cont'd*) — Page 120

9. a) At approximately 12.81 m. b) $\approx 51.34°$
10. At approximately 53.44 km/h.
11. ≈ 8.78 m
12. ≈ 271.89 m

Overview (*cont'd*) — Page 121

13. a) **Aiguille du Midi cable car**

	1st stage	2nd stage
Length (m)	2553	2867
Vertical difference (m)	1279	1470
Base station altitude (m)	1038	2317
Top station altitude (m)	2317	3787

   b) 1) $\approx 30.06°$ 2) $\approx 30.85°$
   c) ≈ 5419.87 m
14. $\approx \$199{,}734.67$
15. a) $\approx 153.62°$
   b) Any length of paddle would work, because the minimum distance between the tip of the paddle and the second kayaker would be approximately 106.07 cm.

Overview (*cont'd*) — Page 122

16. a) ≈ 26.11 m
   b) $90°$, $\approx 23.54°$, $\approx 66.46°$
17. a) At approximately 9.37 m in height.
   b) 1) 600 m 2) $\approx 0.76°$
18. ≈ 17.04 m

Overview (*cont'd*) — Page 123

19. a) ≈ 159.66 m
   b) $\approx 19\,225.22$ m^2
20. a) Approximately 4.85 m in height.
   b) ≈ 2.23 m

Overview (*cont'd*) — Page 124

21. a) Approximately 101.08 cm.
   b) $\approx 1.90°$
   c) ≈ 4.81 cm
22. ≈ 1202.59 km

Overview (*cont'd*) — Page 125

23. ≈ 470.37 cm

Bank of problems — Page 126

24. The volume of this silo is approximately 265.99 m^3.
25. The length of the lake is approximately 118.61 m.
26. *Several answers possible.*

Bank of problems (*cont'd*) — Page 127

27. The altitude of point B is approximately 1204.68 m.
28. Parachutist **A** must go in an angle measuring approximately 93.94° in relation to the north, and cover a distance of approximately 2508.81 m.

LES 12 | *Polytherapies*

1. Enumeration of all possible treatments.

a) Monotherapies
Number of possible monotherapies: 5 + 3 + 4 = 12 treatments

b) Bitherapies

	2 Medication **A**'s	2 Medication **B**'s	2 Medication **C**'s	1 Medication **A** and 1 Medication **B**	1 Medication **A** and 1 Medication **C**	1 Medication **B** and 1 Medication **C**
Enumerating possibilities	Combination of 2 elements out of 5	Combination of 2 elements out of 3	Combination of 2 elements out of 4	2 successive "draws"	2 successive "draws"	2 successive "draws"
Number of possible treatments	$\frac{5 \times 4}{2!} = 10$	$\frac{3 \times 2}{2!} = 3$	$\frac{4 \times 3}{2!} = 6$	$5 \times 3 = 15$	$5 \times 4 = 20$	$3 \times 4 = 12$

Number of possible biotherapies: 10 + 3 + 6 + 15 + 20 + 12 = 66 treatments.

c) Tritherapies

	3 Medication **A**'s	3 Medication **B**'s	3 Medication **C**'s	2 Medication **A**'s and 1 Medication **B**	2 Medication **A**'s and 1 Medication **C**
Enumerating possibilities	Combination of 3 elements out of 5	Combination of 3 elements out of 3	Combination of 3 elements out of 4	2 successive "draws" of 2 elements out of 4, then of 1 element out of 3	2 successive "draws" of 2 elements out of 5, then of 1 element out of 4
Number of possible treatments	$\frac{5 \times 4 \times 3}{3!} = 10$	$\frac{3 \times 2 \times 1}{3!} = 1$	$\frac{4 \times 3 \times 2}{3!} = 4$	$\frac{5 \times 4}{2!} \times 3 = 30$	$\frac{5 \times 4}{2!} \times 4 = 40$

	2 Medication **B**'s and 1 Medication **C**	1 Medication **A** and 2 Medication **B**'s	1 Medication **A** and 2 Medication **C**'s	1 Medication **B** and 2 Medication **C**'s	1 Medication **A**, 1 Medication **B** and 1 Medication **C**
Enumerating possibilities	2 successive "draws" of 2 elements out of 3, then of 1 element out of 4	2 successive "draws" of 1 element out of 5, then of 2 elements out of 3	2 successive "draws" of 1 element out of 5, then of 2 elements out of 4	2 successive "draws" of 1 element out of 3, then of 2 elements out of 4	3 successive "draws"
Number of possible treatments	$\frac{3 \times 2}{2!} \times 4 = 12$	$5 \times \frac{3 \times 2}{2!} = 15$	$5 \times \frac{4 \times 3}{2!} = 30$	$3 \times \frac{4 \times 3}{2!} = 18$	$5 \times 3 \times 4 = 60$

Number of possible tritherapies: 10 + 1 + 4 + 30 + 40 + 12 + 15 + 30 + 18 + 60 = 220 treatments.

2. Calculating the probabilities of administering treatments based on their effectiveness. If all the treatments have an equal chance of being administered, the probability of a category of treatments being administered is equal to the number of treatments of this category over the total number of existing treatments.

Situation 1

	Distribution	Number of possible treatments	Probability
Monotherapy	1 medication in either category	12	$\frac{12}{298}$
Bitherapy	2 medications in the same category	19	$\frac{19}{298}$
	2 medications of 2 different categories	47	$\frac{47}{298}$
Tritherapy	3 medications in the same category	15	$\frac{15}{298}$
	2 medications of one category and 1 medication from another category	145	$\frac{145}{298}$
	3 medications in 3 different categories	60	$\frac{60}{298}$
	Total	298	1 or 100%

If the 60 tritherapies that are composed of medication from three different categories have a 75% chance of being administered and that 25% of the chance that remains are distributed equally between the 238 other treatments, these other treatments have a $\frac{25}{238}$% probability of being administered.

Situation 2

	Distribution	Number of possible treatments	Probability (%)
Monotherapy	1 medication in either category	12	$\frac{25}{238} \times 12 \approx 1.26$
Bitherapy	2 medications in the same category	19	$\frac{25}{238} \times 19 \approx 2$
	2 medications of 2 different categories	47	$\frac{25}{238} \times 47 \approx 4.94$
Tritherapy	3 medications in the same category	15	$\frac{25}{238} \times 15 \approx 1.57$
	2 medications of one category and 1 medication from another category	145	$\frac{25}{238} \times 145 \approx 15.23$
	3 medications in 3 different categories	60	75
	Total	298	1 or 100

3. Calculation of the mean global decrease of the mortality rate.

This decrease corresponds to the mathematical expectation associated to each situation.

Mathematical expectation of Situation **1** (%) $= \frac{12}{298} \times 30 + \frac{19}{298} \times 40 + \frac{47}{298} \times 55 + \frac{15}{298} \times 45 + \frac{145}{298} \times 70 + \frac{60}{298} \times 82$

≈ 65.27

Mathematical expectation of Situation **2** (%) $= 0.0126 \times 30 + 0.02 \times 40 + 0.0494 \times 55 + 0.0157 \times 45 + 0.1523 \times 70 + 0.75 \times 82$

≈ 76.76

4. Conclusion.

According to this simulation, if each treatment has as many chances as another to be administered, the mean global decrease of the mortality rate linked to HIV will be approximately 65.27%, which would reduce the number of deaths by approximately 653 out of 1000 individuals. In the case where the probability of accessing the tritherapy based on medication from 3 different categories is 75%, the decrease of the number of deaths would be approximately 768 out of 1000 individuals. Approximately 115 additional lives by portions of 1000 people having received a seropositive diagnostic can be saved if Situation **2** prevails.

LES 13 Working against HIV

Conjecture: Given that the objectives stated only address a limited part of the population, specifically children less than the age of 5 and adolescents between the ages of 15 to 24, Statement **2** seems the most realistic.

1. Calculation of initial life expectancy.
The life expectancy corresponds to the mathematical expectation associated to this situation. Since the ages are grouped in classes, you must use the middle value of these classes as the age reference. Therefore:

$$\begin{aligned}\text{Life expectancy for men} = {} & 0.185 \times 2.5 + 0.017 \times 10 + \\ & 0.065 \times 20 + 0.136 \times 30 + \\ & 0.154 \times 40 + 0.124 \times 50 + \\ & 0.104 \times 60 + 0.10 \times 70 + \\ & 0.115 \times 85 \\ \approx {} & 41.39 \text{ years.}\end{aligned}$$

In regards to women, you must calculate the experimental probability using the information in the table. Therefore:

$$\begin{aligned}\text{Life expectancy in women} = {} & \frac{45\,506}{246\,999} \times 2.5 + \frac{2833}{246\,999} \\ & \times 10 + \frac{14\,826}{246\,999} \times 20 + \\ & \frac{36\,882}{246\,999} \times 30 + \frac{28\,680}{246\,999} \\ & \times 40 + \frac{19\,946}{246\,999} \times 50 + \\ & \frac{22\,753}{246\,999} \times 60 + \frac{27\,808}{246\,999} \times \\ & 70 + \frac{47\,765}{246\,999} \times 85 \\ \approx {} & 44.78 \text{ years.}\end{aligned}$$

2. The effects of reaching the objectives of the millennium.

a) In boys less than the age of 5.

Total number of people	Number of deaths	Proportion of these deaths related to AIDS	Number of deaths related to AIDS	Number of deaths not related to AIDS
100	18.5	$\frac{1}{3}$	$\frac{1}{3} \times 18.5$ ≈ 6.17	$18.5 - 6.2$ $= 12.33$

If the probability of AIDS being the cause of death decreases by half, it becomes $\frac{1}{6}$. Therefore:

Total number of people	Number of deaths	Proportion of these deaths related to AIDS	Number of deaths related to AIDS	Number of deaths not related to AIDS
100	12.33	x	$12.33 + x$	$\frac{x}{12.33 + x} = \frac{1}{6}$

By solving this equation, the following is obtained:

$6x = 12.33 + x$

$x = 2.466$

$x \approx 2.47$ deaths related to AIDS, which gives a total number of $12.33 + 2.47 = 14.8$ deaths for a sample of 100 people.

The probability of death in this age group changes from 18.5% to 14.8%, which is a decrease of 3.7%.

These 3.7% must be distributed in one way or another in the older age groups.

b) In girls less than the age of 5.
The probability of AIDS being the cause of death is $\frac{3}{8}$, which represents 17 064.75 people. Using a reasoning similar to the one explained above, the following is found:

$$\frac{x}{28\,441.25 + x} = \frac{3}{16}$$

$$16x = 85\,323.75 + 3x$$

$x \approx 6563.37$ deaths linked to AIDS, which gives a total number of $28\,441.25 + 6563.37 = 35\,004.62$, approximately 35 005 deaths.

The number of deaths at this age has therefore decreased by 10 501, deaths that will be distributed in one way or another in the older age groups.

c) In boys aged 15 to 24 years.
The probability of the death being caused by AIDS is $\frac{1}{19}$ and will be $\frac{1}{19} \times \frac{3}{4} = \frac{3}{76}$ if it decreases by 25%.
Using a reasoning similar to the one explained above, the following is found:

$$\frac{x}{6.16 + x} = \frac{3}{76}$$

$x \approx 0.25$ deaths related to AIDS, which gives a total number of $6.16 + 0.25 = 6.41$ deaths for a sample of 100 people.

The probability of deaths in this age group changes from 6.5% to 6.41%, which is a decrease of 0.09%.

These 0.09% should be distributed in one way or another in the older age groups.

d) In girls aged 15 to 24 years.
The probability of the death being caused by AIDS is $\frac{3}{7}$, which represents 6354 people.
This probability would be $\frac{3}{7} \times \frac{3}{4} = \frac{9}{28}$ if it decreases by 25%.

Using a reasoning similar to the one previously explained, the following is found:

$$\frac{x}{8472 + x} = \frac{9}{28}$$

$x \approx 4013$ deaths related to AIDS, which gives a total number of $4013 + 8472 = 12\,485$ deaths.

The number of deaths at this age has therefore decreased by 2341 which will be distributed in one way or another in the older age groups.

Death in South Africa

	Men			Women		
Age	Probability that the death will occur at this age before the objectives (%)	Effects of attaining the objectives (%)	Probability that the death will occur at this age after reaching the objectives (%)	Number of deaths before the objectives	Effects of attaining the objectives	Number of deaths after attaining the objectives
[0, 5[	18.5	-3.7	**14.8**	45 506	-10 501	**35 005**
[5, 15[	1.7		**1.7**	2833		**2833**
[15, 25[	6.5	-0.09	**6.41**	14 826	-2341	**12 485**
[25, 35[	13.6		**13.6**	36 882		**36 882**
[35, 45[	15.4	+1.27	**16.67**	28 680	+4280	**32 960**
[45, 55[	12.4		**12.4**	19 946		**19 946**
[55, 65[	10.4	+1.26	**11.66**	22 753	+4281	**27 034**
[65, 75[	10.0		**10**	27 808		**27 808**
[75, 95[	11.5	+1.26	**12.76**	47 765	+4281	**52 046**
Total	100	0	**100**	246 999	0	**246 999**

3. Calculation of life expectancy after attaining the objectives of the millennium.

Life expectancy in men $= 0.148 \times 2.5 + 0.017 \times 10 + 0.0641 \times 20 + 0.136 \times 30 + 0.1667 \times 40 + 0.124 \times 50 + 0.1166 \times 60 + 0.10 \times 70 + 0.1276 \times 85$

≈ 43.61 years.

In regards to women, you must calculate the experimental probability by using the information in the table. Therefore:

Life expectancy in women $= \frac{35\,005}{246\,999} \times 2.5 + \frac{2833}{246\,999} \times 10 + \frac{12\,485}{246\,999} \times 20 + \frac{36\,882}{246\,999} \times 30 + \frac{32\,960}{246\,999} \times 40 + \frac{19\,946}{246\,999} \times 50 + \frac{27\,034}{246\,999} \times 60 + \frac{27\,808}{246\,999} \times 70 + \frac{52\,046}{246\,999} \times 85$

≈ 47.69 years.

4. Conclusion.

Although life expectancy is increasing in both men and women, after reaching the objectives of the millennium, this increase remains minimal.

For men, the life expectancy changes approximately from 41.39 years to 43.61 years, a 5.36% increase.

For women, the life expectancy changes approximately from 44.78 years to 47.69 years, a 6.5% increase.

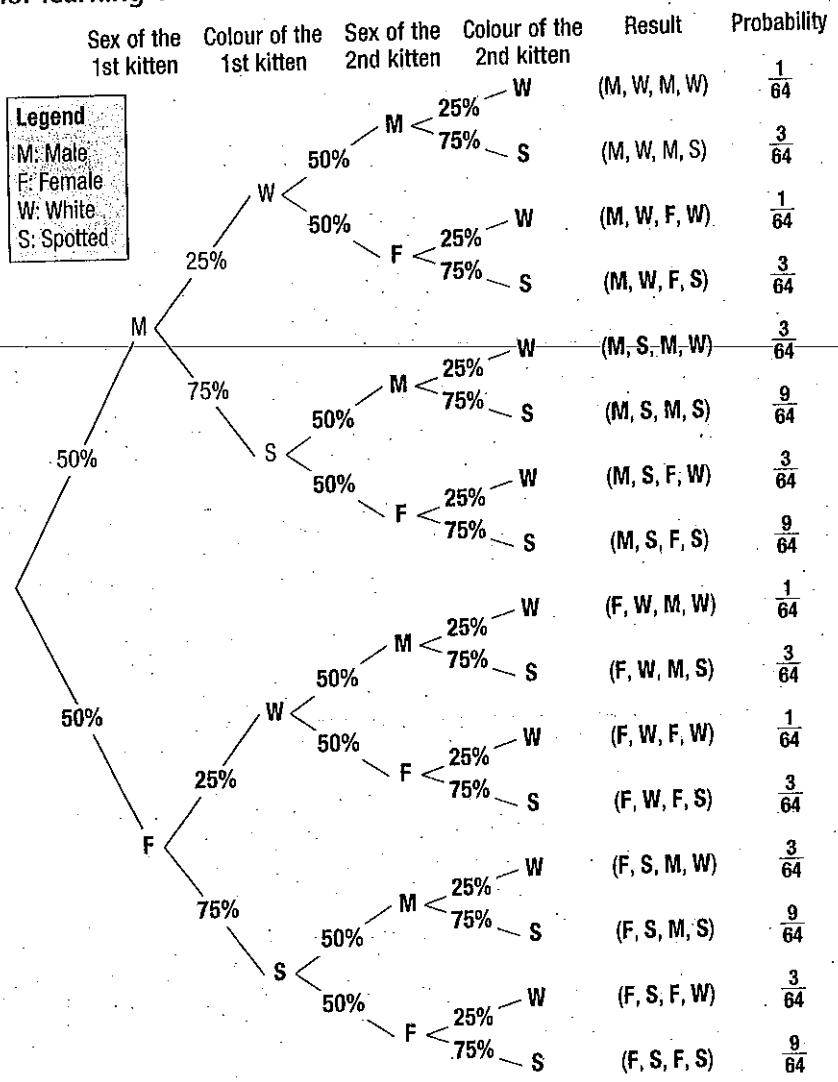
Statement **1**, based on which the objectives of the millennium attained would increase the life expectancy considerably is false, due to the word "considerably."

Statement **2** is a more realistic representation of this situation.

Page 130

Prior learning 1

a.



b. 1) $\frac{36}{64}$ or $\frac{9}{16}$, or 56.25%. 2) $\frac{16}{64}$ or $\frac{1}{4}$, or 25%. 3) $\frac{14}{64}$ or $\frac{7}{32}$, or 21.875%.

Page 131

Prior learning 2

a. 1) 8 people.
2) 5 people.
3) 9 people.

b. 1) $\frac{16}{46}$ or $\frac{8}{23}$.
2) $\frac{18}{46}$ or $\frac{9}{23}$.
3) $\frac{31}{46}$
4) $\frac{5}{46}$

Page 134

Knowledge in action

1. a) $\Omega = \{P, F\}$
b) $\Omega = \{1, 2, 3, 4, 5, 6\}$
c) $\Omega = \{\text{red, green, white, blue}\}$
d) $\Omega = \{\text{spades, hearts, diamonds, clubs}\}$

2. a) $\frac{4}{15}$ b) $\frac{2}{15}$ c) $\frac{2}{5}$
d) $\frac{8}{225}$ e) $\frac{4}{105}$

3. a) $\frac{7}{16}$ b) $\frac{1}{8}$ c) $\frac{1}{2}$
d) $\frac{3}{16}$ e) $\frac{3}{8}$

Knowledge in action (*cont'd*)

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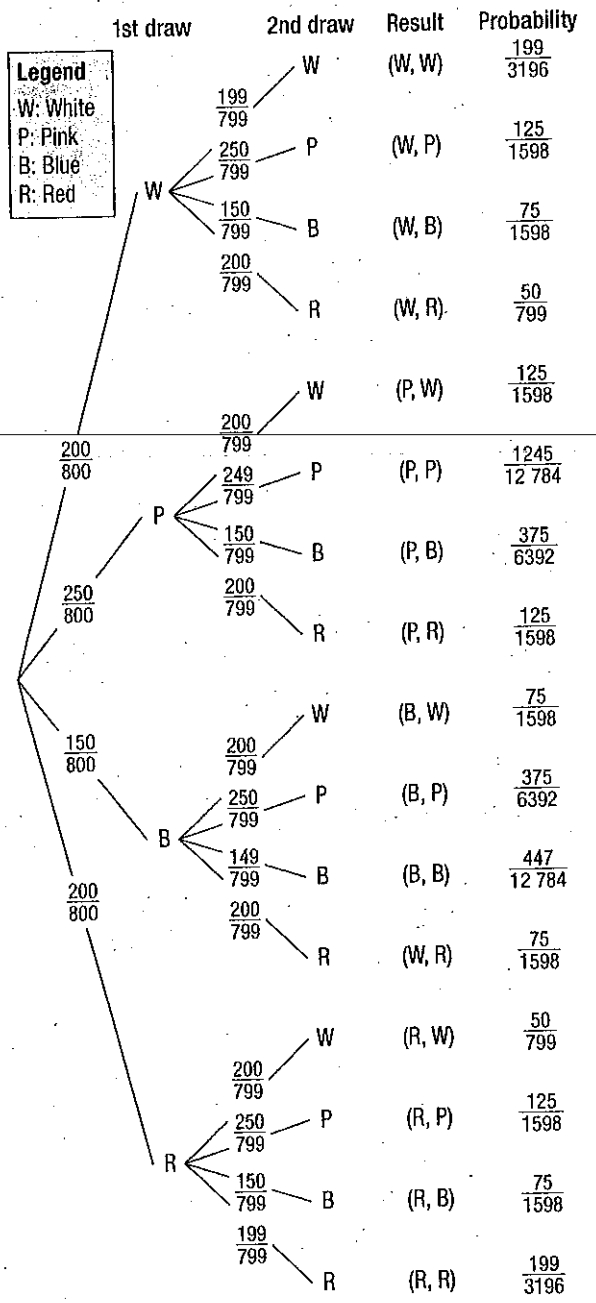
4. a)



b) 1) $\frac{199}{3196}$ 2) $\frac{821}{3196}$ 3) $\frac{2375}{3196}$

5. a) $\frac{1}{27}$ b) $\frac{1}{81}$ c) $\frac{1}{10\,000}$
d) $\frac{1}{270\,000}$ e) $\frac{1}{810\,000}$

6. 25%

7. a) $\frac{1}{22}$ b) $\frac{1}{38}$ c) $\frac{1}{836}$

SECTION 6.1 Enumerating possibilities

Problem

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$P(\text{JUPITER}) \approx 5.7 \times 10^{-5}$ or 0.0057%

Activity 1

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a. 1) 4 aces. 2) 13 hearts. 3) 1 ace of hearts.

b.

Draw	1	2	3	4	5	6	7
Number of cards remaining in the deck	52	51	50	49	48	47	46

8	9	10	11	12	13
45	44	43	42	41	40

c. 1) $\frac{1}{52}$ 2) $\frac{1}{51}$ 3) $\frac{1}{50}$

d. Approximately 3.95×10^{21} sets of 13 cards.

e. $\approx 2.53 \times 10^{-22}$

Activity 2

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a. 1) $\frac{1}{75}$ 2) $\frac{1}{75}$

b. 120 ways.

c. 1) $\frac{1}{2\,071\,126\,800}$ 2) $\frac{1}{17\,259\,390}$

d. 1) $\frac{1}{17\,259\,390}$

2) Yes, because by insisting that the balls be drawn in order, this limits the number of favourable outcomes and consequently, the probability.

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a. The factorial function calculates the product of all whole numbers strictly positive, less than or equal to the number. It calculates the number of possible permutations in a distinct set of elements.

b. 1) 12 ways.

2) The number located on the left represents the number of elements present in the initial set, and the number located on the right represents the number of elements that was chosen in the initial set.

c. 1) 36 ways.
2) The number located on the left represents the number of elements present in the initial set, and the number located on the right represents the number of elements that is chosen in the inital set.

d. In Screen **2**, the order is taken into consideration, which is not done in Screen **3**.

e. By comparing the answers displayed on the calculator with the answers obtained by the methods seen in class, you can notice that the calculator executes arrangements and combinations without replacement.

f. 1) 54 627 300 teams.
2) Approximately 1.13×10^{12} access codes.

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Practice 6.1

1. a) 24 b) 40 320
c) 3 628 800 d) 3 628 801
e) 1 f) 15
g) 604 800 h) 348

2. a) $\approx 1.16 \times 10^{10}$ b) 67 910 864

3. a) $5! = 120$ ways. b) $3! = 6$ ways.
c) $2! = 2$ ways. d) $6! = 720$ ways.
e) $4! = 24$ ways. f) $13! = 6\ 227\ 020\ 800$ ways.

4. a) 165 765 600 words. b) $26^6 = 308\ 915\ 776$ words.

5. $12! = 479\ 001\ 600$ ways.

6. 24 ways.

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Practice 6.1 (*cont'd*)

7. $16! \approx 2.09 \times 10^{13}$ ways.

8. a) 1) 24 assortments. 2) 256 assortments.
b) 1) $\frac{1}{8}$ 2) $\frac{3}{4}$ 3) $\frac{1}{4}$

9. a) 362 880 ways. b) 362 880 ways.

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Practice 6.1 (*cont'd*)

10. a) 48 meals. b) 1) $\frac{1}{48}$ 2) $\frac{1}{16}$

11. a) 12 870 teams. b) 40 320 ways.

12. 657 720 choices.

13. a) $\frac{1}{308\ 915\ 776}$
b) The level of security is increased because even if there are less numbers than letters, the number of possible positions of the number in the access code causes there to be more available access codes.

14. a) 4096 ways. b) 720 ways.

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Practice 6.1 (*cont'd*)

15. a) $\frac{1}{1341}$ b) $\frac{128}{1341}$ c) $\frac{1213}{1341}$

16. a) Approximately 2.91×10^{10} selections.
b) 1) $\frac{1}{16}$ 2) $\frac{3}{8}$

17. a) 40 000 000 telephone numbers.
b) $\frac{1}{64\ 000\ 000\ 000\ 000}$

18. $\frac{1}{12\ 167}$

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Practice 6.1 (*cont'd*)

19. a) 720 arrangements.
b) 64 arrangements.
c) 46 080 arrangements.
d) 1) $\frac{1}{6}$ 2) $\frac{1}{4}$ 3) $\frac{1}{12}$
4) $\frac{1}{720}$ 5) $\frac{1}{32}$

20. a) 1) 362 880 ways.
2) 80 640 arrangements.
b) 40 320 ways.

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Practice 6.1 (*cont'd*)

21. $\frac{1}{28}$

22. a) 1) 20 358 520 hands.
2) Approximately 6.35×10^{11} hands.
b) 1) 25 827 165 hands.
2) Approximately 1.11×10^{12} hands.

23. a) 64 000 combinations.
b) No. Arrangement should be talked about more since the order is important.

24. $\frac{1}{252}$

25. a) 362 880 ways. b) 362 880 ways.
c) 362 880 ways.

SECTION 6.2 Suggested probability and odds

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Problem

Yes, this way of sharing the chores is fair for everyone.

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Activity 1

a.

Predictions for the Alouettes vs. Roughriders game

	Analyst ①	Analyst ②	Analyst ③	Analyst ④	Analyst ⑤	Analyst ⑥
Probability of Alouettes victory	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{4}$
Probability of an Alouettes defeat	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{4}$
Odds for an Alouettes victory	3:2	3:1	1:1	3:2	3:2	3:1
Odds against an Alouettes victory	2:3	1:3	1:1	2:3	2:3	1:3

b. Analysts ①, ④ and ⑤ are saying the same thing, and Analysts ② and ⑥ are saying the same thing.

c. The odds allows you to compare the probabilities relative to two issues of an event, whereas the probability allows you to quantify the accomplishment of an event in relation to a universe of possibilities.

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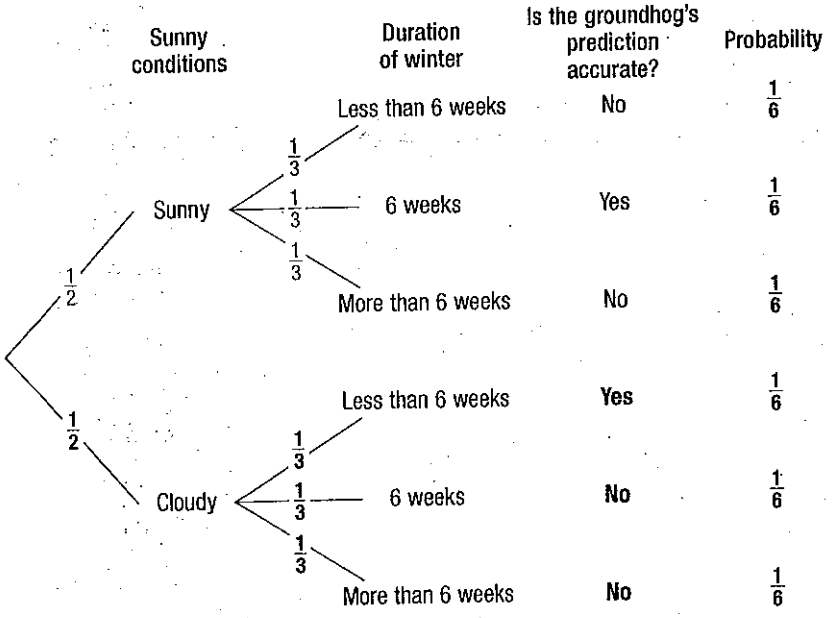
Activity 2

a. $\approx 37.38\%$

b.



c. 1) No. 2) Yes.

d. *Several answers possible. Example:* The groundhog's predictions are based on odds since the probability that the groundhog's predictions are accurate by chance (33.3%) is very close to the mean of the percentages of the years when the groundhog's predictions were correct.

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a. 1) 0, 1, 25 2) 1, 6, 35 3) 1, 12, 200

b. 1) $\frac{54}{100}$ or $\frac{27}{50}$. 2) $\frac{46}{100}$ or $\frac{23}{50}$.

c. 1) *Several answers possible.*
2) *Several answers possible.*

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Practice 6.2

1. a) Subjective. b) Theoretical.
c) Experimental. d) Subjective.
e) Subjective. f) Theoretical.

2. a) 2:5 b) 5:2
c) 1) \$12.50 2) \$2

3. It is the second individual (the man) who is right, because the Tornadoes have 3 chances of losing and 5 chances of winning.

4. a) \$8 b) \$18

Practice 6.2 (*cont'd*) Page 155

5. a) 18% b) 88%
   c) 42% d) 12%

6. Hanna is wrong. It will definitely rain, but the duration nor the quantity can be predicted.

7. 78:22 or 39:11.

8. a) This indicates that there is a chance out of two that the event is occurring.
   b) Obtaining tails by tossing a coin.

9. a) 1 against 5 or 1:5. b) 35 against 1 or 35:1.
   c) 1 against 35 or 1:35. d) 0 against 1 or 0.

10. a) $\frac{1}{6}$
    b) *Several answers possible.*
    c) *Several answers possible.*

Practice 6.2 (*cont'd*) Page 156

11. The best investment is Investment **C**, because it offers the highest probability (80%) of producing a return of 150%.

12. a) $\frac{4}{7}$ b) 4:3 c) $\frac{3}{7}$ d) 3:4

13. a) The theoretical approach, because it is obtained by the enumeration of favourable and possible cases.
    b) The subjective approach, because personal judgement can intervene.
    c) The experimental approach, because it can be obtained following a large number of trials.
    d) The theoretical approach, because it is obtained by the enumeration of favourable and possible cases.
    e) The experimental approach, because you can pick out a large number of clubs, then enumerate the ones with four leaves.

14. a) 1) $\frac{5}{8}$ or 62.5%.
    2) This probability is subjective.
    b) 1) \$9 2) \$25

Practice 6.2 (*cont'd*) Page 157

15. A 4, B 3, C 2, D 1

16. a) 1) $\frac{3}{8}$ 2) $\frac{5}{7}$
    b) 1) 3:5 2) 5:2

17. a) 1) Side **1**: 13% Side **2**: 13.2%
    Side **3**: 12.8% Side **4**: 13.5%
    Side **5**: 34.5% Side **6**: 13%
    2) It consists of experimental possibilities, because they were obtained based on the results of a large number of tosses.
    b) Throwing the die a large number of times, counting the number of times each side has been obtained, calculating the experimental probability of obtaining each of these sides and comparing these possibilities with theoretical probabilities.

18. Martine is not taking into consideration the strengths and weaknesses of the teams. It would be surprising that all the teams have as many odds of winning. Therefore, the distribution of odds should not only be based on the number of participating teams.

Practice 6.2 (*cont'd*) Page 158

19. a) It is a theoretical probability since it relies on the fact that the ball, not being able to hit the nail, would necessarily fall to the right or left side of the nail and that is has as many odds of falling on either side.
    b) No, although you can find the probability, you cannot calculate the number of balls that will be collected in Bin **4**.
    c) Yes, however the calculation is complex, because you must determine the number of paths leading to Bin **4** and the total number of possible paths.
    d) It is an experimental probability since it stems from the outcomes of a large number of repetitions in the experiment.

20. a) No, because there is no information on the phenomenon.
    b) Yes. It is possible to use the experimental approach. However, the outcome must be interpreted with a lot of nuances since you must also know the geographic distribution of the victims, as well as a large number of other factors.

Practice 6.2 (*cont'd*) Page 159

21. $\approx$ 20.74%

22. a) This probability is experimental because approximately as many newborn boys as newborn girls are enumerated.

b)

Composition of the family	Subjective probability	Theoretical probability
G, F, G, F in this order	*Several answers possible.*	$\frac{1}{16}$
2 boys and 2 girls in any order	*Several answers possible.*	$\frac{6}{16}$
4 boys	*Several answers possible.*	$\frac{1}{16}$

23. $\frac{1}{720}$

24. 4 white marbles, 16 blue marbles, 8 red marbles, 20 black marbles and 12 yellow marbles.

SECTION 6.3 Mathematical expectation

Problem — Page 160

Yes, because on average, each house should bring in $2,300.

Activity 1 — Page 161

a. No.

b. Portfolio **A**: $1,000
Portfolio **B**: $850
Portfolio **C**: $400
Portfolio **D**: $550

c. Portfolio **A**.

d.

Portfolio E	
Profit ($)	**Probability (%)**
2,000	20
1,500	20
1,200	**20**
0	10
-1,000	30

Activity 2 — Page 162

a. 1) Situation **A**.
2) Situation **C**.
3) Situation **B**.

b. 1) Decrease the value of the potential loss.
2) Increase the value of the potential gain.
3) Increase the probability of a gain occurring.
4) Decrease the probability of a loss incurring.

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a. 1) The losses associated to the 0.75 probability have gone from -$15 to -$20. In cell C4, the contribution of this loss to the value of this mathematical expectation has gone from -$11.25 to -$15. In addition, this mathematical expectation of this experiment has gone from -$2.30 to -$6.05.

2) The probabilities relative to the cells B2 and B5 have gone respectively from 0.1 to 0.01 and 0.01 to 0.1. The contributions to this value of the mathematical expectation of the losses associated to these probabilities have gone from $2.00 to $0.20 and from $1 to $10. In addition, the mathematical expectation of this experiment has gone from -$2.30 to $4.90.

b. The values of Column **C** correspond to the contributions of each loss and each gain to the value of the mathematical expectation.

c. *Several answers possible.*

d. When calculating the mathematical expectation associated to a situation, you must make sure that the sum of all the probabilities of all the possible outcomes of this situation is 1 or 100%. In other words, you make sure to take all the outcomes of the situation into consideration.

e. 1) *Several answers possible. Example:*

	A	B	C	D
2	Net gains and probabilities			
3	Experiment 2			
4	**Net gain ($)**	**Probability (*P*)**	**Product ($) (*P* × net gain)**	
5	20	0.1	2	
6	50	0.05	2.5	
7	-15	0.75	-11.25	
8	200	0.01	2	
9	100	0.02	2	
10	45	0.03	1.35	
11	35	0.04	1.4	
12				
13	Sum of probabilities:			1
14	Mathematical expectation ($):			0

2) *Several answers possible. Example:*

	A	B	C	D
1				
2	Net gains and probabilities			
3	Experiment 2			
4	Net gain ($)	Probability (*P*)	Product ($) (*P* × net gain)	
5	20	0.04	0.80	
6	50	0.05	2.50	
7	-15	0.71	-10.65	
8	100	0.01	1.00	
9	65	0.02	1.30	
10	25	0.09	2.25	
11	35	0.08	2.80	
12				
13	Sum of probabilities			1
14	Mathematical expectation ($)			0

Practice 6.3

Page 165

1. a) 5.5 **b)** 0.95 **c)** -4.86
d) -0.25 **e)** 3

2. a) $x = \frac{16}{3}$ **b)** $x \approx -23\ 411.76$
c) $x = 0.88$ **d)** $x = \frac{2}{9}$

3. This game is not fair. In order for it to be fair, the cost of participating must cost approximately $1.48.

Practice 6.3 (*cont'd*)

Page 166

4. a) ≈ -$0.13 **b)** ≈ -$1.13 **c)** ≈ $0.87
d) So that the public lotteries can make profit.

5. a) 1) $3.30 a week.
2) Approximately $7.89 each week.
3) Approximately $7.11 each week.

b) 1) $4.70
2) ≈ $3.93
3) ≈ $2.88

Practice 6.3 (*cont'd*)

Page 167

6. a) 1) 50% or 0.5. 2) 50% or 0.5.
b) 0
c) Yes.
d)

Number of tosses	1	2	3	4	5	6	7	8	9	10
Maude's holdings ($)	Probability									
11	–	–	–	–	–	–	–	–	–	≈ 0.0010
10	–	–	–	–	–	–	–	–	≈ 0.0020	–
9	–	–	–	–	–	–	–	≈ 0.0039	–	≈ 0.0088
8	–	–	–	–	–	–	≈ 0.0078	–	≈ 0.0156	–
7	–	–	–	–	–	≈ 0.0156	–	≈ 0.0273	–	≈ 0.0342
6	–	–	–	–	≈ 0.0313	–	≈ 0.0469	–	≈ 0.0527	–
5	–	–	–	0.0625	–	≈ 0.0781	–	≈ 0.0781	–	≈ 0.0732
4	–	–	0.125	–	0.125	–	≈ 0.1094	–	≈ 0.0938	–
3	–	0.25	–	0.1875	–	≈ 0.1406	–	≈ 0.1094	–	≈ 0.0879
2	0.5	–	0.25	–	≈ 0.1563	–	≈ 0.1094	–	≈ 0.0820	–
1	–	0.25	–	0.125	–	≈ 0.0781	–	≈ 0.0547	–	≈ 0.0410
0	0.5	–	0.125	–	0.0625	–	≈ 0.0391	–	≈ 0.0273	–

e) 1) 0.5 or 50%. 2) 0 3) 0.125 or 12.5%.
4) 0.625 or 62.5%. 5) ≈ 0.7539 or ≈ 75.39%.

Practice 6.3 *(cont'd)*

6. f)

Number of tosses	1	2	3	4	5	6	7	8	9	10
Jonathan's holdings ($)	Probability									
11	–	–	–	–	–	≈ 0.0156	–	≈ 0.0313	–	≈ 0.0439
10	–	–	–	–	≈ 0.0313	–	≈ 0.0547	–	≈ 0.0703	–
9	–	–	–	0.0625	–	≈ 0.0938	–	≈ 0.1094	–	≈ 0.1172
8	–	–	0.125	–	≈ 0.1563	–	≈ 0.1641	–	≈ 0.1641	–
7	–	0.25	–	0.25	–	≈ 0.2344	–	≈ 0.2188	–	≈ 0.2051
6	0.5	–	0.375	–	0.3125	–	≈ 0.2734	–	≈ 0.2461	–
5	–	0.5	–	0.375	–	0.3125	–	≈ 0.2734	–	≈ 0.2451
4	0.5	–	0.375	–	0.3125	–	≈ 0.2734	–	≈ 0.2441	–
3	–	0.25	–	0.25	–	≈ 0.2344	–	≈ 0.2148	–	≈ 0.1953
2	–	–	0.125	–	≈ 0.1563	–	≈ 0.1563	–	≈ 0.1465	–
1	–	–	–	0.0625	–	≈ 0.0781	–	≈ 0.0781	–	≈ 0.0732
0	–	–	–	–	≈ 0.0313	–	≈ 0.0391	–	≈ 0.0391	–

g) 1) $\frac{1}{32}$ or 3.125%. 2) 0 3) $\frac{5}{128}$ or ≈ 3.91%.
4) 0 5) $\frac{7}{64}$ or ≈ 10.94%.

h) The player that begins playing with the least amount of money, because the one who has more will have more chances of gaining back what he lost.

i) The person playing at the casino, because she is the one who had the least amount of money from the start.

7. a) Yes, because the mathematical expectation is $2.57 for every share.

b) $48,110.40

Practice 6.3 *(cont'd)*

8. a) Expected gain $= \frac{1}{x}(0.5xy - y) - \frac{(x-1)}{x}y$

b) No, because when simplifying the rule, you obtain $-0.5y$, which is a negative value since $y > 0$.

9. a) 0

b) 1) It can't be known.
2) The player with the most tokens.

10. a) $50,000

b) This person has 5 out of 6 odds of losing his or her $100,000 bet.

c) Because there are too many odds of losing his or her $100,000 bet.

d) 1) ≈ $73,333.33
2) 5 out of 6 odds of losing his or her $10,000 bet.

e) In this example, 5 out of 6 odds of losing represents too much of a risk of losing his or her initial bet, and few people have the means of repeating this experiment in order to see the mathematical expectation come true.

SPECIAL FEATURES 6

Chronicle of the past

1. $\frac{64}{123}$ or ≈ 52.03%.

2. 18 times.

3. The obtained probability is becoming closer to an experimental probability, because this formula relies on the observation of natural phenomenon.

4. 1

In the workplace

1. a) The expected gain of Investment **A** is 5%.
The expected gain of Investment **B** is -1%.
The expected gain of Investment **C** is -2.5%.

b) 1) The expected gain of Investments **A** and **B** is 2%.
2) The expected gain of Investments **A** and **C** is 1.25%.
3) The expected gain of Investments **B** and **C** is -1.75%.

c) To invest in Investment **A**.

2. **a)** ≈ \$6,139.13
b) ≈ \$11,057.27
c) ≈ \$12,881.13
d) 1) ≈ \$8,203.48
2) ≈ \$8,831.81
3) ≈ \$11,930.93

Overview — Page 174

1. 479 001 600 ways.

2. **a)** 1) $\frac{8}{243}$ 2) $\frac{2}{51}$ **b)** $\frac{4}{17}$
c) 1) 8:235 2) 2:49 3) 4:13

3. **a)** Subjective probability. **b)** Theoretical probability.
c) Experimental probability. **d)** Experimental probability.
e) Subjective probability. **f)** Theoretical probability.

Overview (*cont'd*) — Page 175

4. **a)** 67 600 000 serial numbers.
b) 20 442 240 serial numbers.
c) 65 000 000 serial numbers.
d) 19 656 000 serial numbers.

5. \$0.20/carton.

6. **a)**

Durability of light bulbs

Number of days after which the bulb stopped working (days)	Number of bulbs	Probability that a bulb will not work during this interval of time
[10, 20[	12	$\frac{12}{1050} = \frac{2}{175}$
[20, 30[	50	$\frac{50}{1050} = \frac{1}{21}$
[30, 40[	47	$\frac{47}{1050}$
[40, 50[	18	$\frac{18}{1050} = \frac{3}{175}$
[50, 60[	76	$\frac{76}{1050} = \frac{38}{525}$
[60, 70[	83	$\frac{83}{1050}$
[70, 80[	141	$\frac{141}{1050} = \frac{47}{350}$
[80, 90[	168	$\frac{168}{1050} = \frac{4}{25}$
[90, 100[	213	$\frac{213}{1050} = \frac{71}{350}$
[100, 110[	122	$\frac{122}{1050} = \frac{61}{525}$
[110, 120[	74	$\frac{74}{1050} = \frac{37}{525}$
[120, 130[	13	$\frac{13}{1050}$
[130, 140[	24	$\frac{24}{1050} = \frac{4}{175}$
[140, 150[	9	$\frac{9}{1050} = \frac{3}{350}$

b) It involves experimental probabilities since they are obtained based on the outcomes of an experiment repeated several times.
c) Approximately 81.94 days.
d) The mathematical expectation corresponds to the mean duration of the life of a bulb of this model. It can also be called, in this context, the lifespan of a bulb.

Overview (*cont'd*) — Page 176

7. **a)** $\frac{1}{16}$ **b)** $\frac{1}{16}$

8. **a)** $\frac{1}{479\,001\,600}$ **b)** $\approx 4.36 \times 10^{-18}$
c) $\approx 7.53 \times 10^{-79}$

Overview (*cont'd*) — Page 177

9. **a)** 1) 87.2% 2) 46% 3) ≈ 93.09%
b) 1) 2:3 2) 9:1

10. **a)** Participant **1**: -\$2.50 Participant **2**: \$2.50
Participant **3**: -\$2.50 Participant **4**: \$2.50
b) The mathematical expectation corresponds to the mean sum that each person would have won if this situation were repeated a large number of times under the same conditions.

Overview (*cont'd*) — Page 178

11. 840 different board of directors.

12. **a)** 14% **b)** 36% **c)** 45%

13. **a)** 1) ≈ 0.03% 2) ≈ 3.82%
b) $\frac{1}{78}$

14. **a)** $\frac{1}{7}$ **b)** $\frac{1}{365}$ **c)** $\frac{1}{12}$

Overview (*cont'd*) — Page 179

15. **a)** -\$1.80
b) 1) 100 tickets. 2) \$2,000 3) \$0.20

16. **a)** 59 040 pin tumbler locks.
b) 531 432 pin tumbler locks.

17. 48 sundaes.

18. **a)** 56 combinations. **b)** $\frac{1}{175\,616}$

19. **a)** $\frac{1}{20}$
b) No. The odds against are larger.

Overview (*cont'd*)

20. a)

Number of tosses until heads	1	2	3	4	5	6	7	8	9	10	100	300	Total
Winning amount ($)	1	2	4	8	16	32	64	128	256	512	2^{99}	2^{299}	–
Cost of participating ($)	5	5	5	5	5	5	5	5	5	5	5	5	–
Net gain ($)	-4	-3	-1	3	11	27	59	123	251	507	$2^{99} - 5$	$2^{299} - 5$	–
Probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1024}$	$\frac{1}{2^{100}}$	$\frac{1}{2^{300}}$	–
Net gain × probability	-2	-0.75	-0.125	≈ 0.188	≈ 0.344	≈ 0.422	≈ 0.461	≈ 0.480	≈ 0.490	≈ 0.495	0.5	0.5	≈ 1.005

b) The mathematical expectation of this lottery is infinite, because each new term (probability × net gain) will add 0.5 to its value.

c) The paradox reveals that, despite a mathematical expectation that seems infinite, the number of times that must be played to see the mathematical expectation happen is so large, that we are always losing.

21. $\frac{1}{10^{14}}$

Bank of problems

22. The owners should buy Machine **3**. In fact, the calculation of the mathematical expectation shows that this machine can cause an expectation of a mean profit of $120 for every piece, which is greater than the profit that the other two machines can generate.

23. The client can expect to win $7.50 by spinning the wheel. It would therefore be profitable to add articles whose prices would total, at maximum, $7.50, which implies that the initial amount must be greater than or equal to $42.50. However, with the mathematical expectation only pertaining to long term, this reasoning is only valid if the client participates a large number of times in the draw.

24. Gabrielle's statement is false because the calculation shows that the events are equally probable. This stems from the fact that the event "having 2 boys and 2 girls" such as Gabrielle describes it, must be done in a given order. However, the probability of having 2 girls and 2 boys in any order is larger than the probability of having 4 girls. Gabrielle would have been right if she had not specified the order in which the intermediary events occur.

Bank of problems (*cont'd*)

25. P(defect of Generator **A**) $= \frac{1}{10\,000}$

P(defect of Generator **B**) $= \frac{1}{20\,000}$

26. No. The mining company, even by carrying out a large number of operations of explorations under the same conditions, can expect to lose on average $590,000 each time.

27. The amino acids must be coded with sets of at least 3 bases of nitrogen to have enough possible arrangements to code the 20 existing amino acids.