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REPRODUCIBLE SHEETS

VISION 4 From functions to modelling

Name: _____

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PRACTICE 4.1

Student Book, p. 20

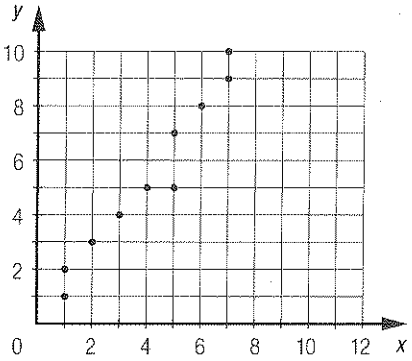
Number 3

3 For each of the graphs below, do the following:

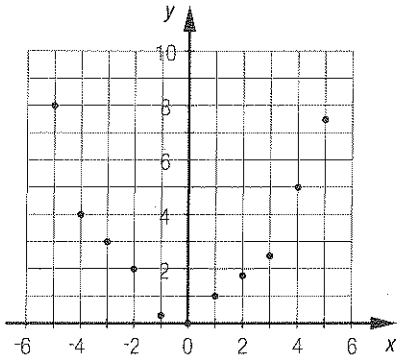
a) Determine what type of function best represents the scatter plot.

b) Draw a curve that best represents the data shown on the graph.

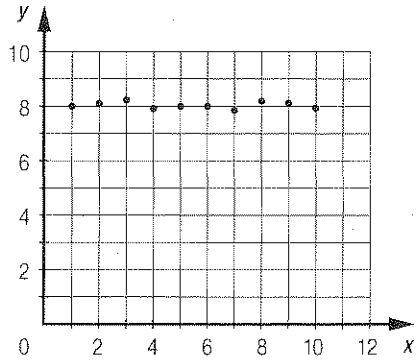
Graph ①



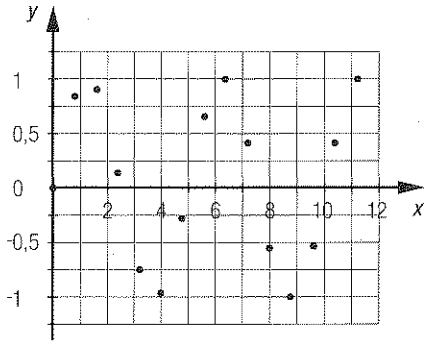
Graph ②



Graph ③



Graph ④



Name: _____

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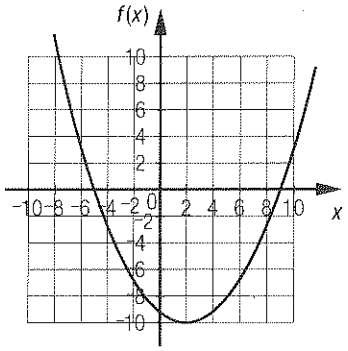
SUPPORT 4.1

Real functions

1 For each of the functions represented below, determine:

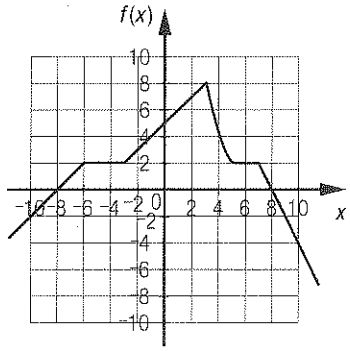
1) the domain and the range
2) the zero(s) and the initial value
3) the variation
4) the sign
5) the extrema
6) the type of function represented

a)



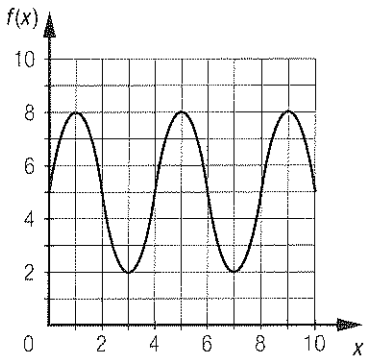
1) _____
2) _____
3) _____
4) _____
5) _____
6) _____

b)



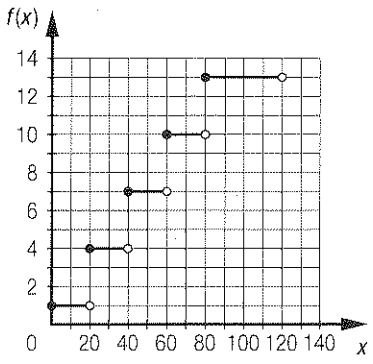
1) _____
2) _____
3) _____
4) _____
5) _____
6) _____

c)



1) _____
2) _____
3) _____
4) _____
5) _____
6) _____

d)



1) _____
2) _____
3) _____
4) _____
5) _____
6) _____

2 Match each situation in the left column with a function in the right column.

a) The change in an employee's hourly salary in relation to time. _____

b) The change in the area of a square based on the length of one of its sides. _____

c) The change in the number of cm in 1 m. _____

d) The height of the tides. _____

e) The change in the amount of interest to be paid on an unpaid credit card balance in relation to time. _____

1. Zero-degree polynomial function
2. First-degree polynomial function
3. Second-degree polynomial function
4. Inverse variation function
5. Periodic function
6. Exponential function
7. Step function

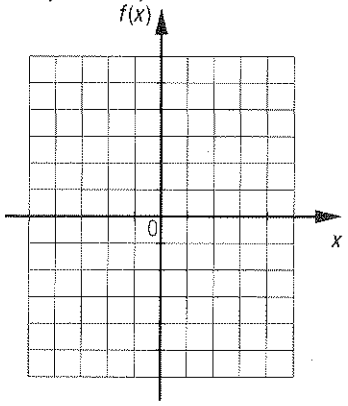
3 For each of the tables of values below, do the following:

1) Draw the corresponding scatter plot.
2) Draw a curve that best represents the points shown on the graph.
3) Determine what type of function serves as the best mathematical model for the situation.

a)

x	$f(x)$
-4	-9
-3	-12
-2	-18
-1	-36
1	36
2	18
3	12
4	9

1) and 2)

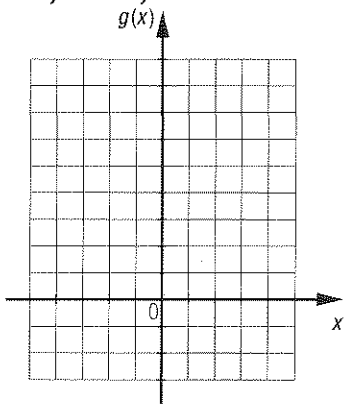


3) _____

b)

x	$g(x)$
-3	0.0625
-2	0.125
-1	0.25
0	0.5
1	1
2	2
3	4
4	8

1) and 2)

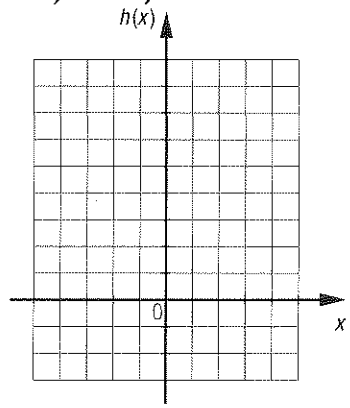


3) _____

c)

x	$h(x)$
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18
4	32

1) and 2)



3) _____

Name: _____

Group: _____ Date: _____

CONSOLIDATION 4.1

Real functions

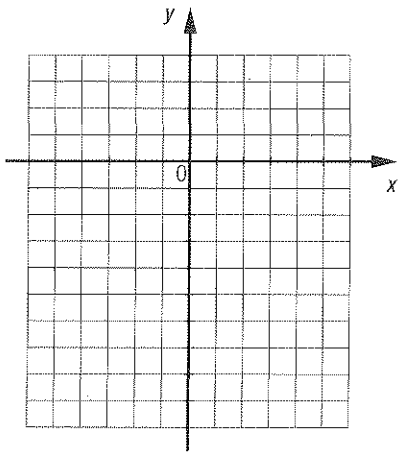
1 For each of the following tables of values:

1) Graphically represent the situation.
2) Determine what type of function represents the situation.

a)

x	-3	-2	-1	0	1	2
y	-5	2	-5	2	-5	2

1)

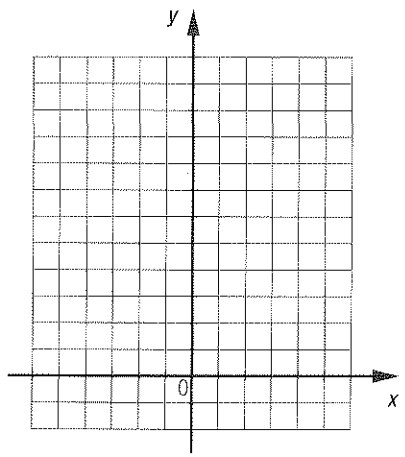


2) _____

b)

x	-2	-1	0	1	2	3
y	12	3	0	3	12	27

1)

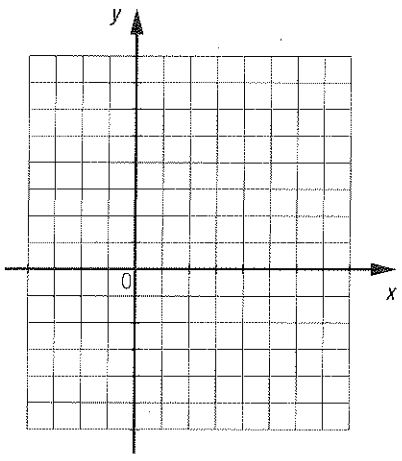


2) _____

c)

x	$[-5, -2[$	$[-2, 1[$	$[1, 4[$	$[4, 7[$	$[7, 10[$	$[10, 13[$
y	-10	-5	0	5	10	15

1)

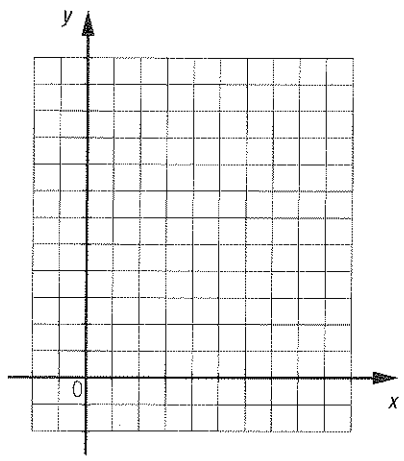


2) _____

d)

x	0.5	1	2	3	4	5	6	7	8
y	50	24	11	8.5	7	4.5	4	3.4	3

1)



2) _____

Name: _____

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2 The scatter plots below represents a young entrepreneur's profits during the first 24 months after opening his business. The curve represents the function on which he has chosen to model the changes in his company's profits in relation to time.

a) What type of function has the entrepreneur chosen to model his activities?

_____

b) For this function, determine:

1) the domain _____

2) the range _____

3) the variation _____

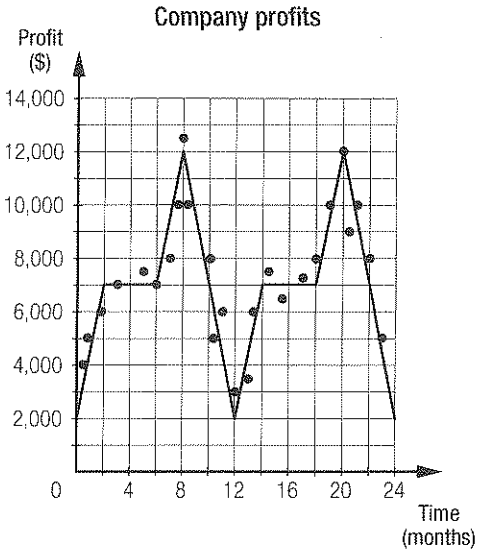
_____

_____

_____

4) the extrema _____

_____



3 Marc-Antoine participated in the Montréal Marathon. The graph below represents how far he ran during the marathon in relation to time.

a) How long did it take Marc-Antoine to complete the race?

_____

b) What distance was covered during the run?

_____

c) Provide a precise description of Marc-Antoine's run.

_____

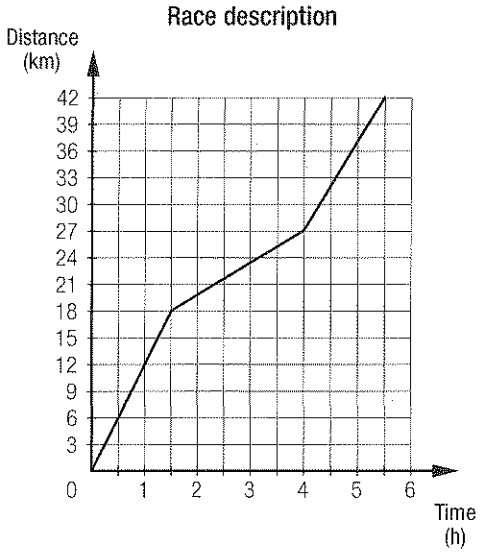
_____

_____

_____

d) What type of function is associated with this situation?

_____



Name: _____

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CONSOLIDATION 4.1

(cont'd)

4 A secondary school's student council is organizing a ski trip for winter break. Council members are considering the relation between the number of students who will take part in the trip and the transportation costs for each student. The following values were obtained: (5, 126), (6, 105), (7, 90), (8, 78.75), (9, 70), (10, 63), (12, 50.52), (14, 45), (15, 42), (18, 35).

a) Draw the scatter plot that represents this situation.

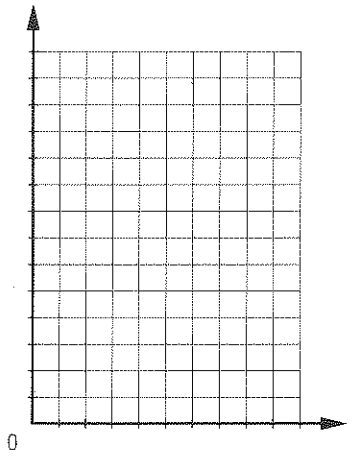
b) What kind of function can be used to model the situation? Explain your answer.

_____

_____

_____

_____



c) What are the total transportation costs?

_____

d) What is the minimum number of students who need to participate in the trip so that each student does not pay more than $10 for transportation?

_____

5 The scatter plot below represents the value of an investment over the course of 12 months.

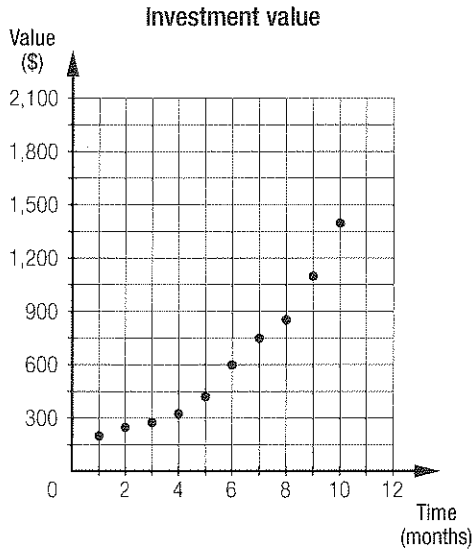
a) Draw the curve that best represents the data shown on the graph.

b) What kind of function can be associated with the curve drawn in a)?

_____

c) If the value of the investment continues to accumulate in the same way, what would its value be in the 11th month?

_____



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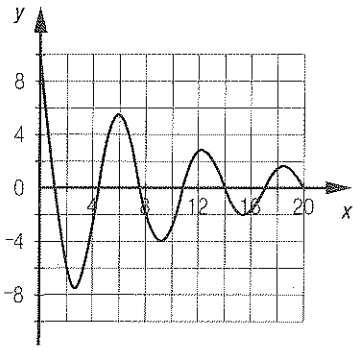
Group: _____ Date: _____

ENRICHMENT 4.1

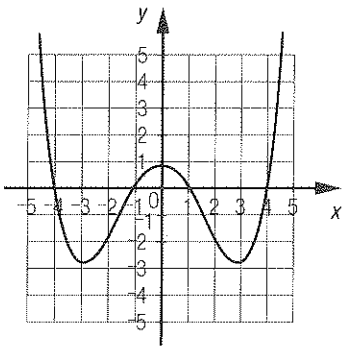
Real functions

1 The following are three types of functions that differ from those already studied:

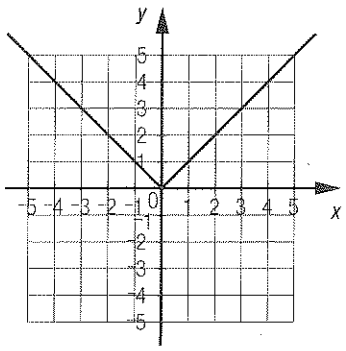
Bessel's function



Fourth-degree polynomial function



Absolute value function



For each of the tables of values below:

1) Draw the scatter plot and the curve that best represents the points shown on the graph.
2) Determine what type of function serves as the best mathematical model for this situation and justify your answer.
3) Approximate the value of y when x is equal to -2.5.

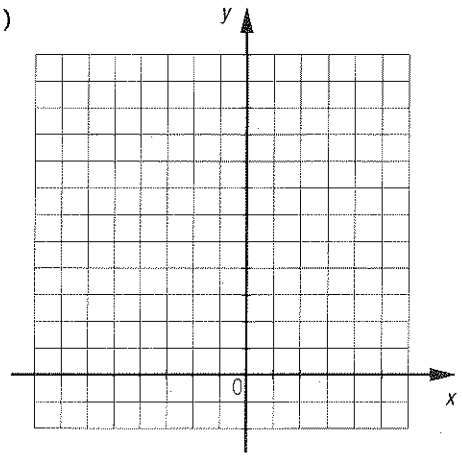
a)

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	11	7	4	2	1	1	1.5	2	1.5	1	1	2	4

b)

x	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
y	10	1	8	3	7	4	6.7	4.7	6.2	5	6	5.3	5.9

1)



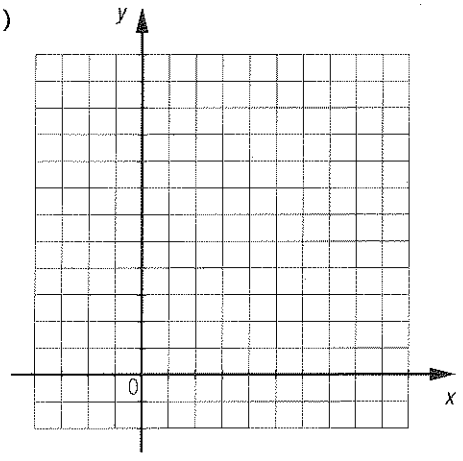
2) _____

_____

_____

3) _____

1)



2) _____

_____

_____

3) _____

Name: _____

Group: _____ Date: _____

SUPPORT 4.2

Second-degree polynomial functions

1 Solve the equation $y = ax^2$ considering that:

a) $x = 2.5$ and $a = 1.4$ _____

b) $x = 3$ and $a = 5$ _____

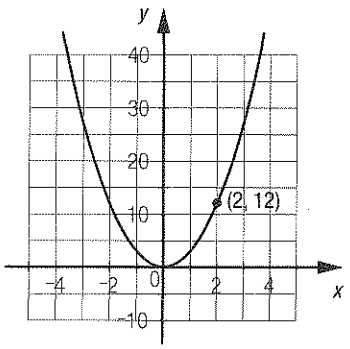
c) $x = 6$ and $y = 72$ _____

d) $x = 1.2$ and $y = 10.08$ _____

e) $y = 75$ and $a = 3$ _____

f) $y = 51.2$ and $a = 5$ _____

2 The adjacent function $f(x)$ is a second-degree polynomial function.



a) In what form is the rule that is associated with this function?

_____

b) If the curve of the function passes through point (2, 12), what is its equation?

_____

c) Solve the equation found in **b)**.

_____

d) Determine the rule associated with this function.

_____

e) Calculate:

1) $f(10)$ _____ 2) $f(-50)$ _____

3 The rule for a quadratic function is $f(x) = 5x^2$. Modify parameter **a** of this rule such that its curve:

a) is vertically compressed by a factor of four _____

b) is vertically stretched by a factor of two _____

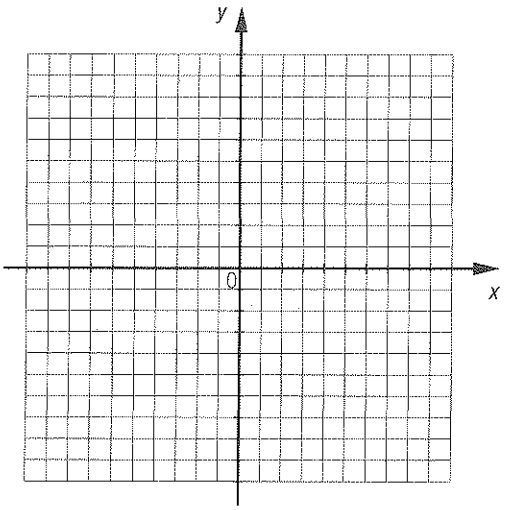
c) is a reflection about the x-axis _____

Name: _____

Group: _____ Date: _____

4 a) Construct a graph in a Cartesian plane which represents the following functions:

1) $f(x) = 2x^2$
2) $g(x) = -2x^2$
3) $h(x) = -0.25x^2$
4) $i(x) = 0.25x^2$



b) Describe the changes that the graph of a quadratic function undergoes when the sign of parameter **a** is changed.

_____

_____

_____

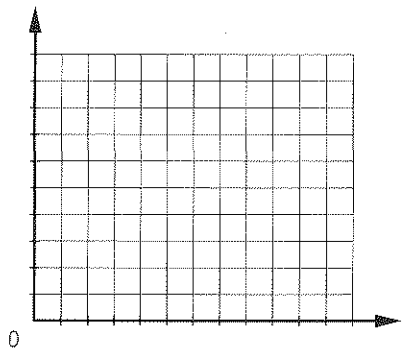
5 William has calculated the areas of several rectangles and in each case, the length of the rectangle is double its width. He formed the following ordered pairs based on the widths and the areas of the rectangles:

(0.5, 0.5), (1, 2), (1.5, 4.5), (2, 8), (2.5, 12.5), (3, 18), (3.5, 24.5), (4, 32)

a) Draw the graph which represents this situation.

b) With what type of function can you associate this situation?

_____



c) Draw the curve that best represents the points shown on the graph.

d) Identify the rule associated with the curve.

_____

e) Under these conditions:

1) What would the area of a rectangle be if it is 4.5 cm wide? _____

2) How wide would a rectangle be if it has an area of 30 cm^2? _____

Name: _____

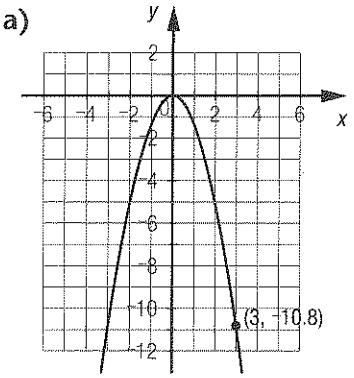
Group: _____ Date: _____

CONSOLIDATION 4.2

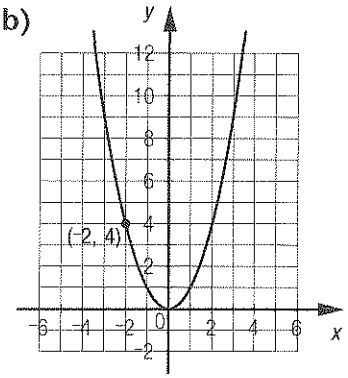
Second-degree polynomial functions

1 Determine the rule for each of the quadratic functions represented below:

a)



b)



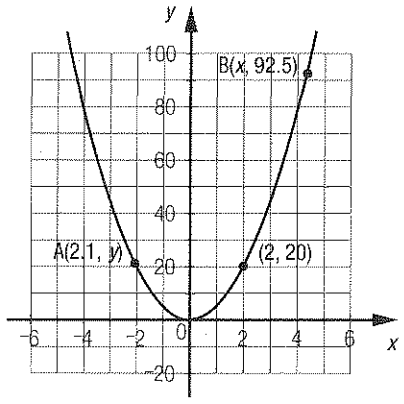
c)

x	y
−4	−48
−2	−12
0	0
2	−12
4	−48
6	−108

_____ _____ _____

2 Below is a graphical representation of a quadratic function:

a) Determine the *y*-coordinate of point A.



_____

b) Determine of the *x*-coordinate of point B.

_____

Name: _____

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CONSOLIDATION 4.2 *(cont'd)*

3 Match each of the rules below with the curve to which it corresponds.

a) $f(x) = \frac{1}{3}x^2$ _____

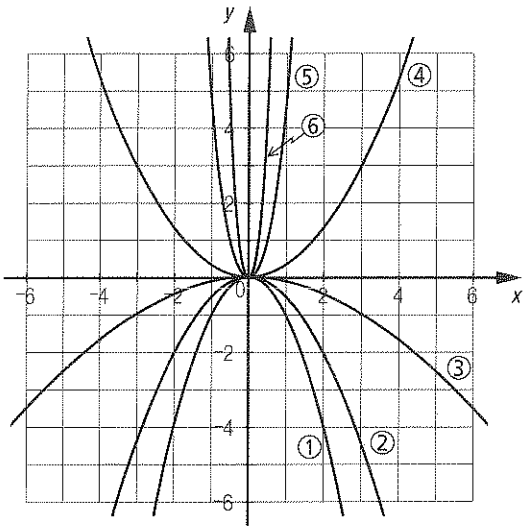
b) $g(x) = -0.5x^2$ _____

c) $h(x) = 20x^2$ _____

d) $i(x) = -x^2$ _____

e) $j(x) = 5x^2$ _____

f) $k(x) = -\frac{1}{10}x^2$ _____



4 An object falls from a roof top. The graph below represents the distance between the object and the starting point of its fall.

a) Does the dotted portion of the function adequately represent this situation? Justify your answer.

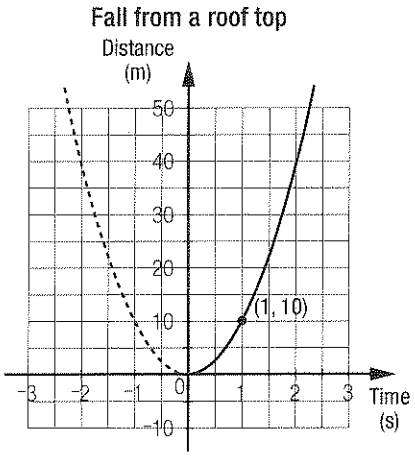
_____

_____

_____

_____

_____



b) Parameter **a** of the function corresponds to the gravitational pull on the object. If the gravitational pull on the Moon is about one-sixth that of the Earth's, how would the curve change if the object falls when on the Moon rather than on Earth?

_____

c) Determine the rule of the function associated with the situation:

1) on Earth _____ 2) on the Moon _____

d) If an object falls from a height of 50 m, how long will it take for the object to reach the ground if the object falls:

1) on Earth _____ 2) on the Moon _____

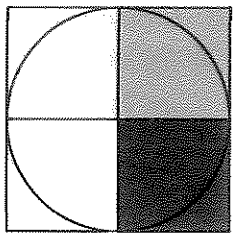
Name: _____

Group: _____ Date: _____

CONSOLIDATION 4.2

(cont'd)

5 A tile maker makes circular tiles by cutting quarter circles out of small ceramic squares of different colours. The tile maker calculates how much ceramic she will waste using the function $f(x) = (4 - \pi)x^2$ where x represents the length (in cm) of a side of a ceramic square and $f(x)$, the amount of ceramic wasted as a result of the tile's design.



a) What would the wastage be for each design if the tile maker uses ceramic tiles which measure 12 cm on each side?

_____

b) What is the measure of the side of the largest ceramic square that the tile maker is able to use if she doesn't want to waste more than 250 cm^2 of ceramic on each design?

_____

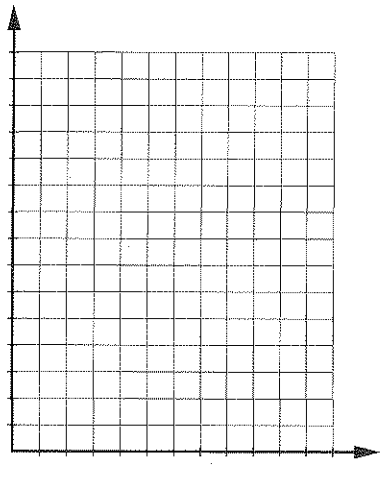
6 Consider the relation between the length of the base and the area of several similar isosceles triangles. The ordered pairs which express this relation are as follows:

(0.8, 1.0), (1.2, 2.2), (1.5, 3.5), (1.7, 5.7), (2.1, 5.9), (2.5, 8.3), (2.8, 11.1), (3.0, 12.6)

a) Draw the scatter plot which represents this situation.

b) Draw the curve that best represents the points shown on the graph.

c) Determine the rule associated with this curve.



_____

d) If a triangle has an area of 50 cm^2, what would the length of its base be?

_____

Name: _____

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ENRICHMENT 4.2

Second-degree polynomial functions

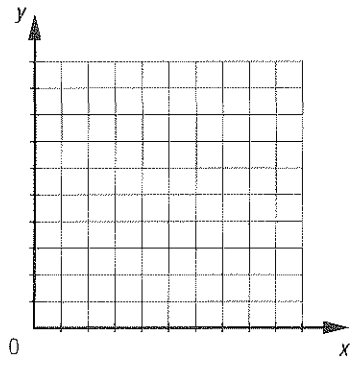
1 The adjacent table of values shows the relation between variables x and y.

x	0	1	2	3	4	5	6	7
x^2								
y	0	0.6	2.4	5.4	9.6	15	21.6	29.4

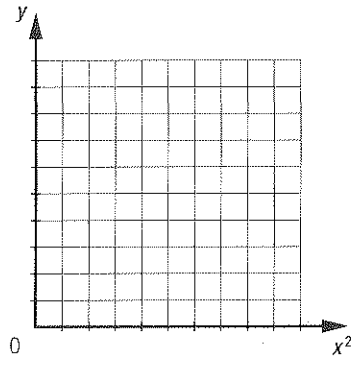
a) Complete the table of values.

b) Draw a scatter plot to show the relation between:

1) y and x



2) y and x^2



c) Draw a curve which best represents the points shown in each of the scatter plots.

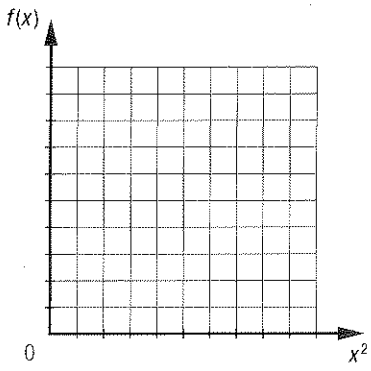
d) Using these graphical representations, explain why you can state that y is directly proportional to the square of x.

_____

_____

2 Using reasoning similar to that applied in **1 d)**, determine whether a second-degree polynomial function expressed by the rule $y = ax^2$ is the best model for representing the situation below. Justify your answer using the graph provided.

x	$f(x)$
0	0
1	1
2	3
3	6
4	10
5	18
6	34
7	59



_____

_____

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_____

_____

Name: _____

Group: _____ Date: _____

ACTIVITY 1 4.3

Student Book, p. 36

Worst-kept secret

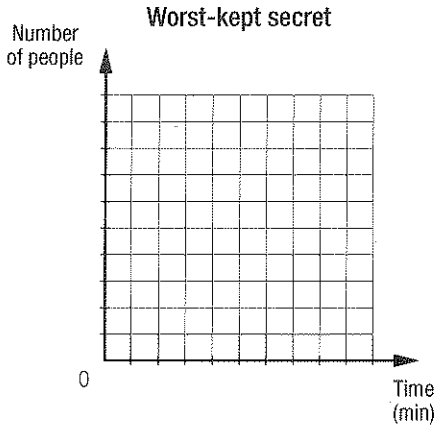
In the situation illustrated in the *Student Book*, each person who is told the secret tells another person a minute later, so the total number of people who know the secret doubles every minute.

a. Complete the adjacent table.

b. Draw a graphical representation of this situation.

Worst-kept secret

Time (min)	Calculation	Number of people aware of the secret
0	1×2^0	1
1	$1 \times 2 = 1 \times 2^1$	2
2	$1 \times 2 \times 2 = 1 \times 2^2$	4
3	$1 \times 2 \times 2 \times 2 = 1 \times 2^3$	8
4		
5		
6		
7		
8		
9		
...	...	...
t		



c. What type of function best represents this situation?

_____

d. How many minutes does it take for the secret to be known by:

1) a group of 32 students _____

2) a small town with a population of 16 384 people _____

3) a big city with a population of 524 288 people _____

e. How many people know the secret after:

1) 20 min? _____

2) 25 min? _____

3) 33 min? _____

Name: _____

Group: _____ Date: _____

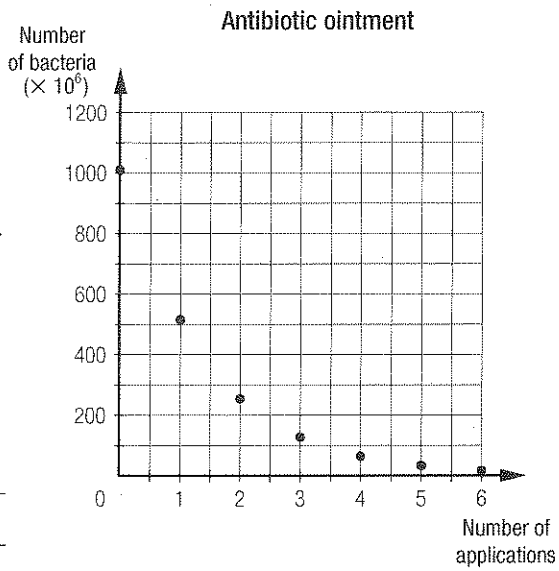
ACTIVITY 2 4.3

Student Book, p. 37

Clinical tests

A technician is testing the effectiveness of an antibiotic ointment by applying a certain amount of antibiotic ointment to a bacterial culture every 24 h. She then observes the number of bacteria through a microscope.

The adjacent graph shows the number of bacteria in the culture in relation to the number of applications of the antibiotic ointment.



a. In your own words, describe the change in the number of bacteria in relation to the number of applications of the antibiotic ointment.

_____

_____

_____

b. Complete the table below.

Antibiotic ointment

Number of applications	Calculation	Number of bacteria ($\times 10^6$)
0	1024×0.5^0	1024
1	$1024 \times 0.5 = 1024 \times 0.5^1$	512
2	$1024 \times 0.5 \times 0.5 = 1024 \times 0.5^2$	256
3	$1024 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^3$	
4		
5		
6		
7		
...	...	...
n		

c. What is the initial value in the situation? _____

d. By what number should you multiply the number of bacteria in order to determine the number of bacteria present after the next application? _____

e. The graph of the mathematical model related to this situation is a curve. What can be said about the distance between this curve and the *x*-axis as *n* increases?

_____

_____

Name: _____

Group: _____ Date: _____

PRACTICE 4.3

Student Book, p. 47

Number 20

20 A couple wants to invest $25,000.

> These are two medium-term investment plans, Plan **A** with an annual interest rate of 6% compounded annually, and Plan **B** with an annual interest rate of 6% compounded every six months.

a) According to Plan **A**, what will the value of this investment be at the end of one year if no withdrawals are made? _____

b) What percentage of the initial investment does the amount calculated in **a)** amount to? _____

c) Based on Plan **B**, what will the value of the investment be at the end of one year, if no withdrawals are made? _____

d) What percentage of the initial investment does the amount calculated in **c)** amount to? _____

e) Complete the following table.

Plan A				Plan B			
Time (month)	Time (years)	Calculation	Value of investment ($)	Time (month)	Time (years)	Calculation	Value of investment ($)
0	0	$25\,000(1.06)^0$	25,000	0	0	$25\,000(1.03)^0$	25,000
				6	0.5	$25\,000(1.03)^1$	
12	1			12	1	$25\,000(1.03)^2$	
				18	1.5		
24	2			24	2		
				30	2.5		
36	3			36	3		
				42	3.5		
48	4			48	4		
...	...	...	...	...	...	...	...
	x				*x*		

f) Which of the two investments is more profitable? Explain your answer.

_____

_____

Name: _____

Group: _____ Date: _____

SUPPORT 4.3

Exponential functions

1 The following are the rules for three exponential functions:

$$f(x) = 1.5(3)^x \qquad g(x) = 2(5)^x \qquad h(x) = -5\left(\frac{1}{4}\right)^x$$

a) Determine:

1) $f(0)$ _____ 2) $g(0)$ _____ 3) $h(0)$ _____

b) Using the results found in **a)**, explain what parameter **a** represents in an exponential function expressed in the form $f(x) = a(\text{base})^x$.

_____

2 The tables of values below are associated with exponential functions.

$f(x) = 2(3)^x$

x	$f(x)$	
1	6	
		$\times 3$
2	18	
		$\times 3$
3	54	
		$\times 3$
4	162	

(+1 between each pair of rows)

$g(x) = 128\left(\frac{1}{4}\right)^x$

x	$g(x)$	
1	32	
		$\times \frac{1}{4}$
2	8	
		$\times \frac{1}{4}$
3	2	
		$\times \frac{1}{4}$
4	0.5	

(+1 between each pair of rows)

$h(x)$

x	$h(x)$
1	5
2	25
3	125
4	625

a) Explain what the base represents in the exponential function $f(x) = a(\text{base})^x$.

_____

_____

b) Determine the base for the exponential function $h(x)$. _____

3 The function represented in the adjacent graph is in the form $f(x) = a(\text{base})^x$.

a) Determine the value of parameter **a**.

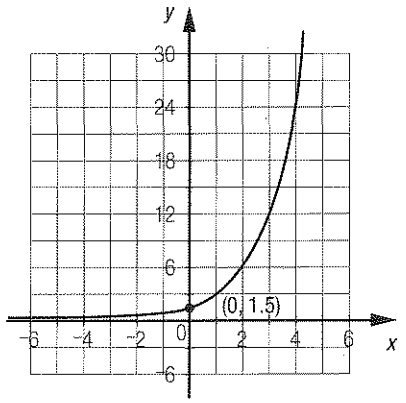
_____

b) Considering that the point (3,12) is on the curve, what equation can you form?

_____

c) By solving the equation provided in **b)**, determine the rule of this function.

_____



Name: _____

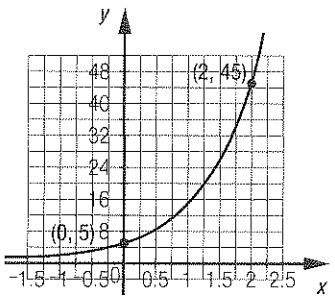
Group: _____ Date: _____

SUPPORT 4.3

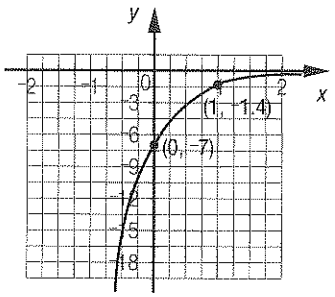
(cont'd)

4 a) Determine the rule that is associated with each graphical representation below:

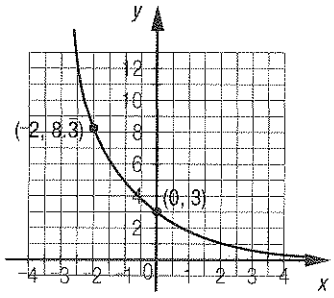
Graph ①



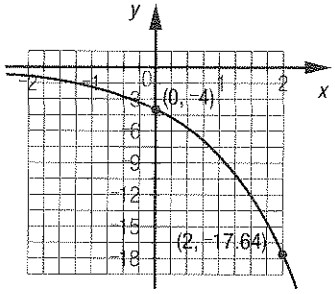
Graph ②



Graph ③



Graph ④



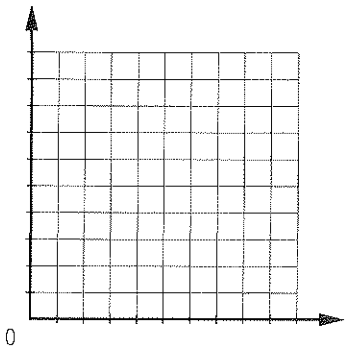
b) Using the graphical representations and the rules found in **a)**, describe the commonalities found between:

1) the curves for which the base is between 0 and 1

2) the curves for which the value of parameter **a** is negative

5 A person deposits $5 into a bank account. Each year the amount doubles.

a) Draw the graph which shows how this situation changes over the course of the first 10 years.



b) What is the rule associated with this situation?

c) Determine how much money will be in the account in 15 years if no withdrawals are made before then.

Name: _____

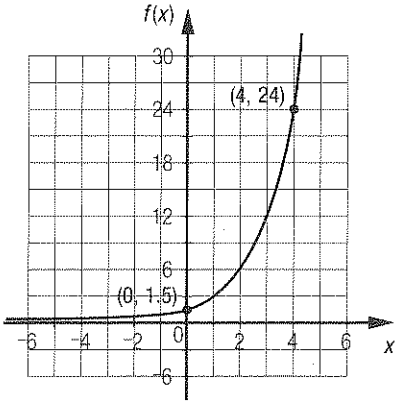
Group: _____ Date: _____

CONSOLIDATION 4.3

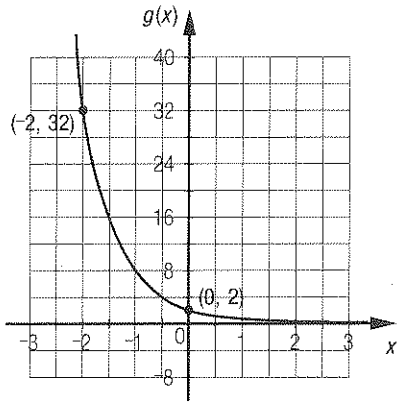
Exponential functions

1 Determine the rule for the following exponential functions:

a)



b)



$f(x) =$ _____ $g(x) =$ _____

c)

x	$h(x)$
0	-4
1	-8
2	-16
3	-32
4	-64

d)

x	$i(x)$
-2	$\frac{1}{27}$
-1	$\frac{1}{9}$
2	3
4	27
5	81

$h(x) =$ _____ $i(x) =$ _____

2 The growth of a certain number of bacteria is under laboratory observation. The growth can be modelled by the function $f(x) = 250(2)^x$ where $f(x)$ represents the number of bacteria and x represents the number of hours which have passed since the observation began.

a) How many bacteria were there at the beginning of the observation period? _____

b) How many bacteria were there after 30 min? _____

c) How many bacteria were there after 2 h? _____

d) How long will it take before there are 4000 bacteria? _____

Name: _____

Group: _____ Date: _____

CONSOLIDATION 4.3

(cont'd)

3 Match each of the rules below to its corresponding curve.

a) $f(x) = -10\left(\frac{1}{3}\right)^x$ _____

b) $g(x) = -20(3)^x$ _____

c) $h(x) = 20\left(\frac{1}{3}\right)^x$ _____

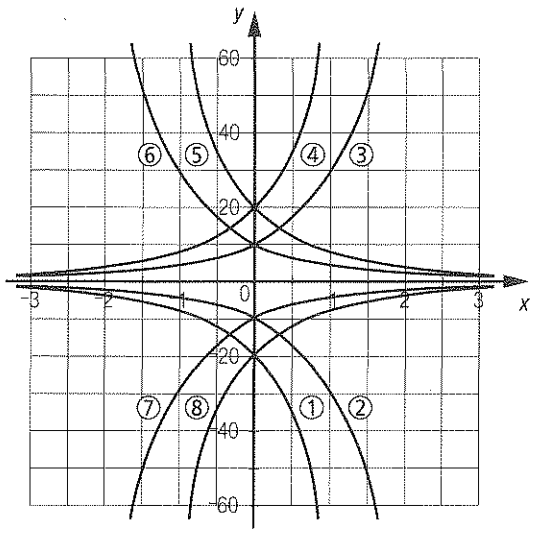
d) $i(x) = 10(3)^x$ _____

e) $j(x) = -10(3)^x$ _____

f) $k(x) = 10\left(\frac{1}{3}\right)^x$ _____

g) $l(x) = 20(3)^x$ _____

h) $m(x) = -20\left(\frac{1}{3}\right)^x$ _____



4 One particular property of "superballs" is their ability to bounce by using a large quantity of the energy they have absorbed. Fred bounces a ball several times. After each bounce, the maximum height the ball reaches decreases by 20% in relation to the ball's last bounce.

a) If Fred drops the ball from a height of 3 m, find the rule which allows you to calculate the maximum height of the ball as a function of the number of bounces.

_____

b) Determine the maximum height of the ball after 4 bounces if Fred drops the ball from a height of 1.5 m.

_____

c) From what height has Fred dropped the ball if after 3 bounces the maximum height that the ball reaches is 61.4 cm?

_____

d) According to the model associated to this situation, after how many bounces will the ball stop bouncing? Explain your answer.

_____

_____

Name: _____

Group: _____ Date: _____

CONSOLIDATION 4.3

(cont'd)

5 A financial institution offers a credit card with an annual interest rate of 18%. In reality, cardholders pay an interest rate of 1.5% compounded monthly.

a) What is the real rate of interest paid after one year on a $1,000 purchase?

_____

b) This financial institution offers two new credit cards.

Card **1**: 24% interest rate, compounded yearly

Card **2**: 1.9% interest rate, compounded monthly

Which card is more advantageous to the cardholder? Explain your answer.

_____

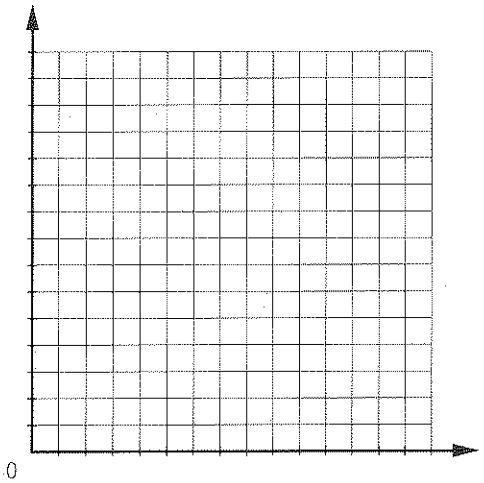
_____

_____

_____

6 The ordered pairs below represent the value of an investment in relation to the number of years since the investment was made.

(0, 5000), (3, 7600), (5, 11 900),
(7, 13 300), (9, 15 000), (10, 20 200),
(11, 23 200), (13, 32 800), (15, 44 900)



a) After you have represented this situation with a scatter plot, draw the curve which best represents the data shown in the graph.

b) Determine the rule that allows you to calculate the value of the investment as a function of the number of years since the investment was made.

_____

c) What is the function's base, and to what does it correspond to in this situation?

_____

d) If the value of the investment continues to grow in the same way, what will it be worth after 18 years?

_____

Name: _____

Group: _____ Date: _____

ENRICHMENT 4.3

Exponential functions

1 An organism's time of death can be revealed by measuring the amount of carbon-14 it contains. An organism's carbon-14 content decreases exponentially after it dies. Considering that an organism's carbon-14 content is 0.012 u if it is measured 800 years after it has died and the quantity is 0.009 u if it is measured 5600 after it has died, determine the time of death for an organism if its carbon-14 content is 1.0×10^{-6} u.

2 Show, algebraically, that there is no exponential function of the form $y = a(\text{base})^x$ that exists when the base is a positive real number and the curve passes through two points having y-coordinates with opposite signs.

Name: _____

Group: _____ Date: _____

ACTIVITY 1 4.4

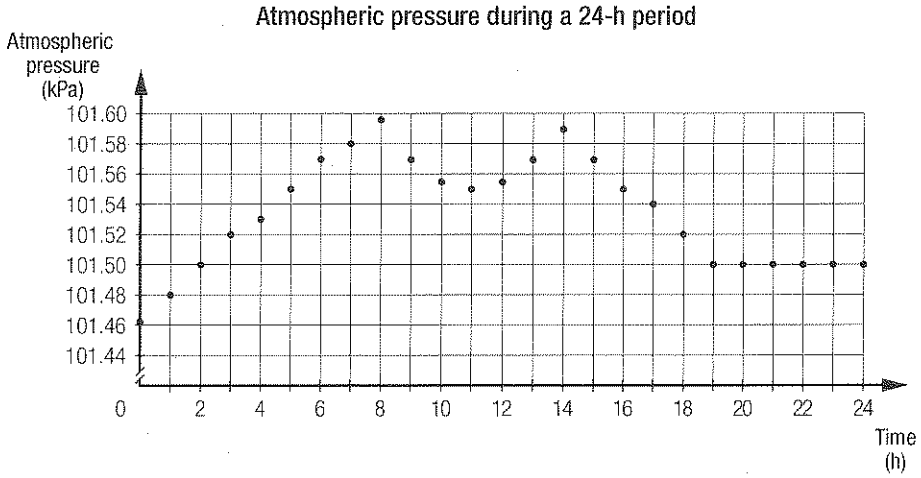
Student Book, p. 49

What's to come?

Weather forecasts are possible through interpretation of satellite images, radar images and data collected from a vast network of weather stations. The data collected by these stations include temperature, relative humidity, atmospheric pressure, wind speed and direction.

The study of variations in atmospheric pressure allows meteorologists to track weather systems such as atmospheric lows that cause bad weather.

The graph below shows the atmospheric pressure recorded at the same location during the course of one day.



a. The scatter plot representing this situation is made up of distinct parts. What type of function can you associate with each of the following intervals?

1) [0, 8] h _____ 2) [8, 14] h _____

3) [14, 19] h _____ 4) [19, 24] h _____

b. What type of function can you associate with this situation?

_____

c. Draw the curve which best fits each segment of the function.

d. According to the model obtained in **c.**, determine the atmospheric pressure at:

1) 2:30 a.m. _____ 2) 8:25 a.m. _____

3) 2:45 p.m. _____ 4) 6:15 p.m. _____

Name: _____

Group: _____ Date: _____

SUPPORT 4.4

Step, periodic and piecewise functions

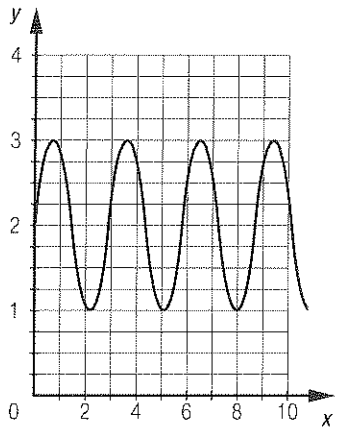
1 a) Among the functions represented below, identify:

1) the ones that are represented by a repeating pattern _____

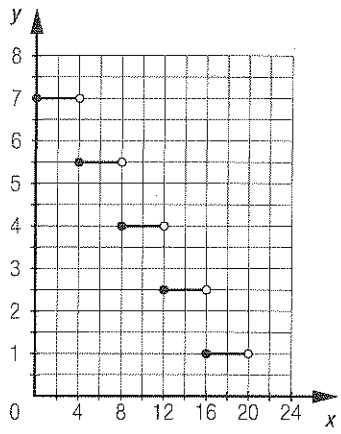
2) the ones that are represented by horizontal segments _____

3) the ones that are represented by several functions _____

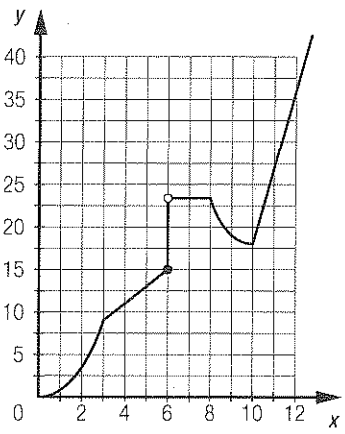
A



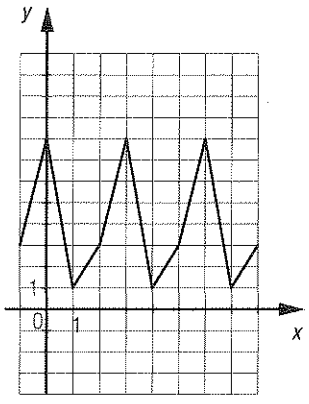
B



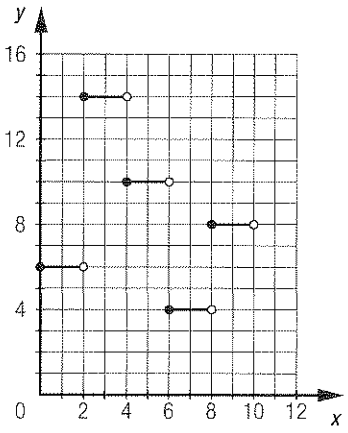
C



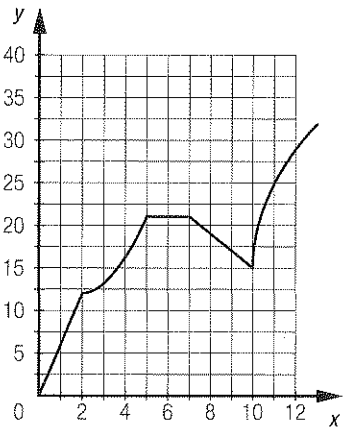
D



E



F



b) Determine the type of function shown in **a)**.

A _____ **B** _____ **C** _____

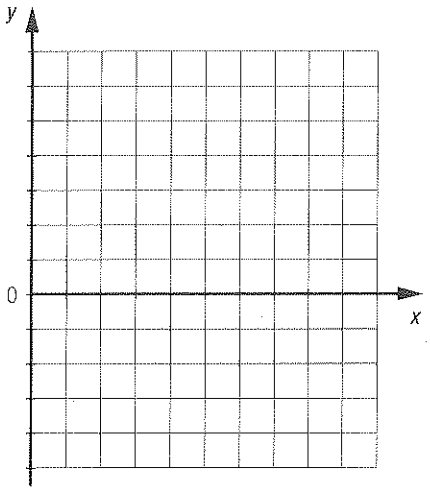
D _____ **E** _____ **F** _____

Name: _____

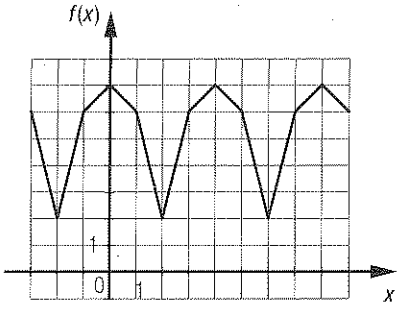
Group: _____ Date: _____

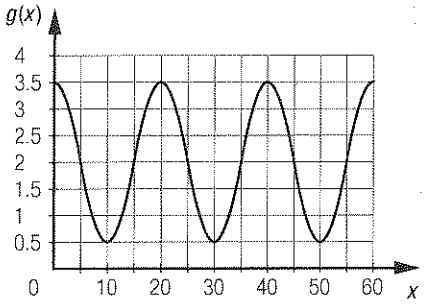
2 In the adjacent Cartesian plane, draw the following piecewise function.

Interval	Rule
[0, 5]	$f(x) = 0.5x^2$
[5, 8[	$f(x) = 12.5$
[8, 15[	$f(x) = -x + 5$



3 Below are two periodic functions:





a) Determine period P for:

$f(x)$ _____ $g(x)$ _____

b) Use the graphical representations to determine the value of:

1) $f(1)$ _____ $f(3 + P)$ _____ $f(3 - P)$ _____

2) $g(5)$ _____ $g(5 + P)$ _____ $g(5 + 2P)$ _____

c) Determine the value of:

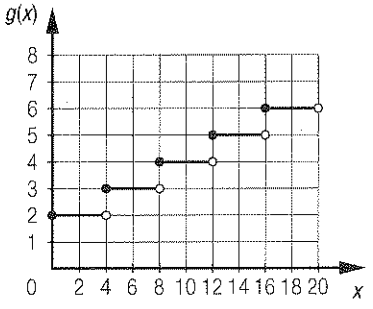
1) $f(17)$ _____ $f(-11)$ _____ $f(40)$ _____

2) $g(-30)$ _____ $g(105)$ _____ $g(-200)$ _____

4 For the adjacent step function, determine:

a) the range _____

b) the critical values _____



Name: _____

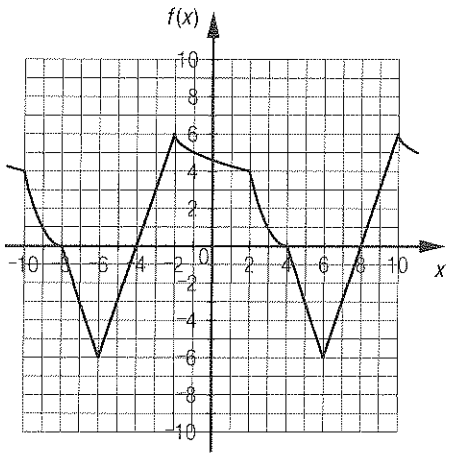
Group: _____ Date: _____

CONSOLIDATION 4.4

Step, periodic and piecewise functions

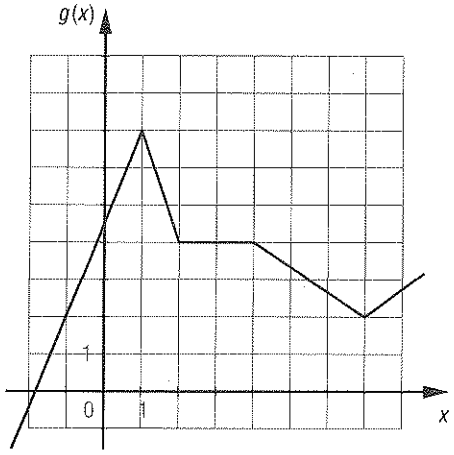
1 Determine the type of function associated with each of the following curves:

a)



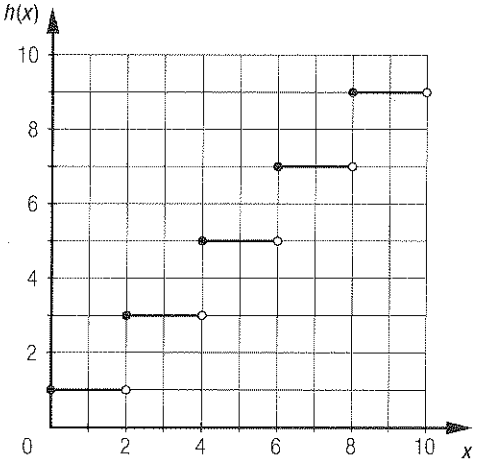
_____

b)



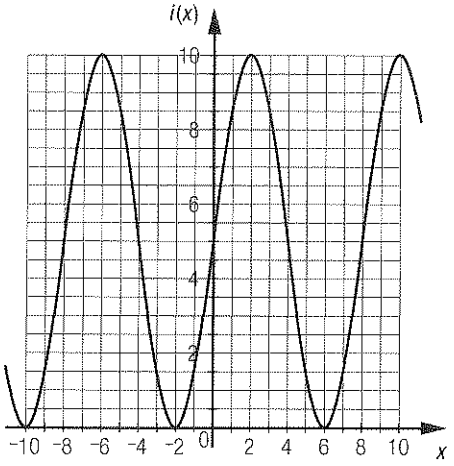
_____

c)



_____

d)



_____

2 Identify the type of functions which best represent the following situations:

a) A discount store that offers a $5 gift card for every $100 in purchases.

_____

b) A car's speed during rush hour.

_____

c) The cardiac activity of a patient that is being monitored by an electrocardiogram.

_____

Name: _____

Group: _____ Date: _____

CONSOLIDATION 4.4

(cont'd)

3 A cell phone company offers a calling plan which has a base fee of $24.90 for each month and a calling charge of $0.15 for each minute. There are no charges other than the monthly fee if a user makes phone calls that last less than one minute each month.

a) Draw a graph which represents this situation.

b) What type of function can be used to model this situation?

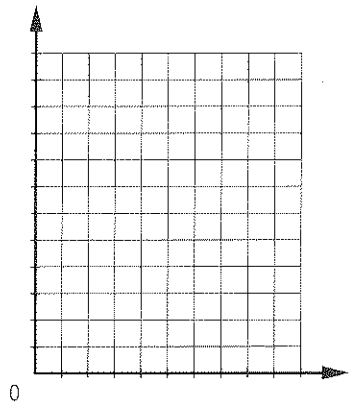
_____

c) What would the cost be for a customer who uses the telephone for 50 min 22 s during the course of a month?

_____

d) How much time does a customer spend on the phone if her monthly phone bill comes to $38.85?

_____



4 The scatter plot below represents Maxime's mountain bike race.

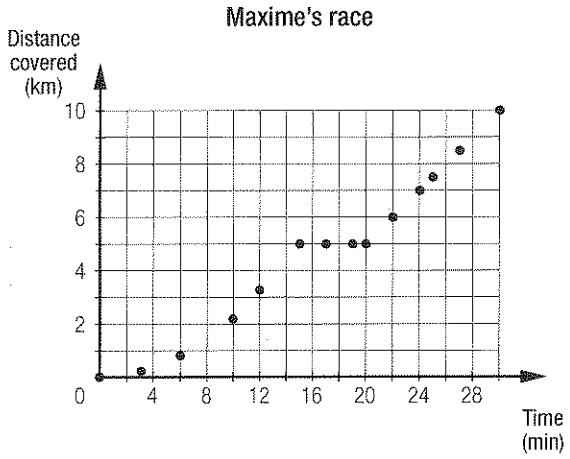
a) What type of function can be used to model this situation? Justify your answer.

_____

_____

_____

_____



b) Draw the curve which best corresponds to each segment of the scatter plot.

c) Complete the table below.

Interval (min)	Distance covered at the end of this interval (km)	Type of function	Rule
	5		
[15, 20[		Zero-degree polynomial	
[20, 30]			$f(x) = 0.5x - 5$

d) How much distance had Maxime covered after:

1) 8 min? _____ 2) 26 min? _____

Name: _____

Group: _____ Date: _____

CONSOLIDATION 4.4

(cont'd)

5 A factory foreman tallies up the number of work-related accidents annually since the factory was opened. He then models this situation using the function represented below.

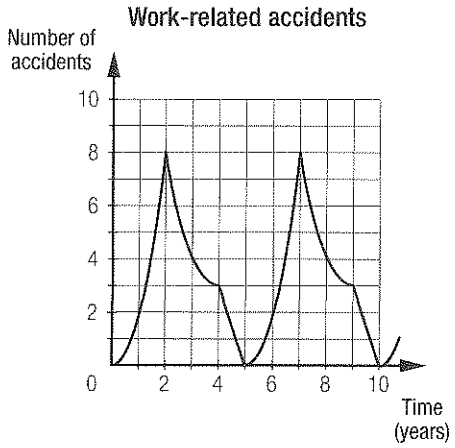
a) Describe this situation.

_____

_____

_____

_____



b) According to this model, how many work-related accidents occurred during:

1) year 10? _____

2) year 12? _____

3) year 24? _____

6 The adjacent table of values provides data about the mean yearly temperature of a region in relation to the number of years since the temperature was first recorded.

Temperature variations

Time (years)	Mean yearly temperature (°C)	Time (years)	Mean yearly temperature (°C)
0	15.5	8	13.5
1	17.2	9	12.0
2	12.1	10	15.3
3	13.2	11	16.9
4	11.9	12	12.2
5	15.6	13	13.0
6	17.5	14	12.1
7	12.2	15	15.5

Predict what the region's mean yearly temperature will be in 35 years. Justify your answer.

_____

_____

_____

_____

Name: _____

Group: _____ Date: _____

ENRICHMENT 4.4

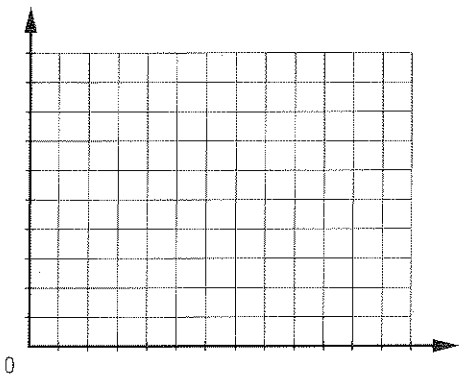
Step, periodic and piecewise functions

1 The atomic mass is a measure of the mass of an atom. The table below indicates the atomic masses for the first 25 elements of the periodical table.

Atomic mass of the first 25 elements

Element	Atomic number	Atomic mass (10^{-12} m)	Element	Atomic number	Atomic mass (10^{-12} m)
Hydrogen	1	37	Silicon	14	118
Helium	2	50	Phosphorus	15	108
Lithium	3	152	Sulfur	16	106
Beryllium	4	111.3	Chlorine	17	99
Boron	5	86	Argon	18	95
Carbon	6	77	Potassium	19	232
Nitrogen	7	74	Calcium	20	197
Oxygen	8	73	Scandium	21	162
Florine	9	71.7	Titanium	22	147
Neon	10	65	Vanadium	23	134
Sodium	11	186	Chromium	24	128
Magnesium	12	160	Manganese	25	127
Aluminum	13	143.1			

a) Draw a scatter plot in order to represent the relation between the atomic numbers and the atomic masses of the first 25 elements.



b) Determine which type of function can best model this situation.

_____

c) The atomic mass of the elements is characterized as "periodic." Does the word "periodic" have the same meaning as it would in reference to functions? Explain your answer.

_____

_____

_____

_____

Name: _____

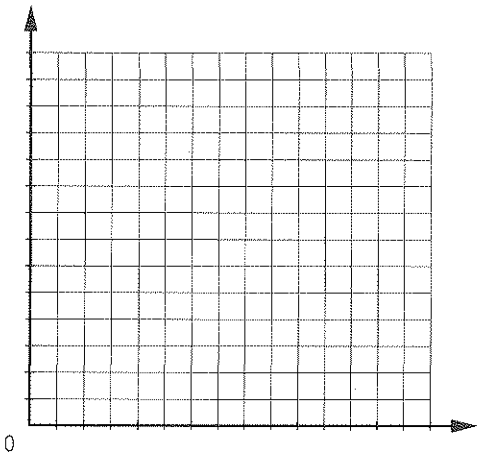
Group: _____ Date: _____

SNAPSHOT 4

1 **KINETIC ENERGY** Kinetic energy is associated with the movement of objects and depends, among other factors, upon an object's speed. The following ordered pairs provide information about the kinetic energy of a car (in J) in relation to the car's speed (in km/h).

(0, 0), (5, 784), (10, 3126), (15, 7056), (20,12 540), (25,19 600).

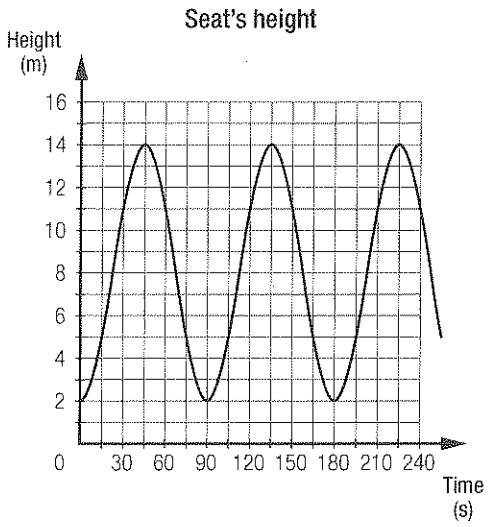
In order to complete an accident report in which the car's speed is a factor, the police want to determine the kinetic energy of the car. Considering that that the speed of the car increased from 40 km/h to 50 km/h, calculate the car's kinetic energy.



Name: _____

Group: _____ Date: _____

2 **FERRIS WHEEL** The ferris wheel is one of the amusement park rides that young children like best. The adjacent graph shows the height of one of the seats in relation to the ground as a function of time. How high will the seat be 525 s after the ride's start? Justify your answer.



Name: _____

Group: _____ Date: _____

SNAPSHOT 4

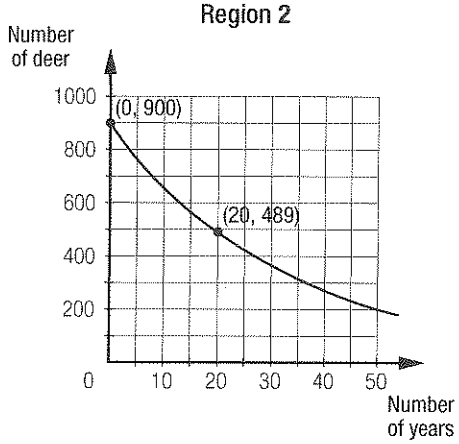
(cont'd)

3 **DEER** A wildlife technician is studying the changes in deer populations in three different regions. She represents each of these by a function.

Region 1

Time (years)	Population remaining
0	1170
1	1145
2	1122
5	1100

Region 2



Region 3

$f(x) = 850(0.99)^x$ where x represents time in years and $f(x)$ represents the remaining population.

Her supervisor asks her to indicate the region in which the deer population is most rapidly decreasing. Determine this region and justify your answer.

Name: _____

Group: _____ Date: _____

SNAPSHOT 4

(cont'd)

4 **THE RIGHT AMOUNT OF TIME** Xavier makes a bank investment with the intention of withdrawing the entire amount in a certain number of years. Below are three investment options available to him.

Investment 1: $7,000 at a 2.3% rate of interest compounded annually

Investment 2: $6,500 at a 2.5% rate of interest compounded annually

Investment 3: $5,000 at a 3.5% rate of interest compounded annually

In order to advise Xavier on which investment to make, determine the number of years needed to maximize his investment given the proposed choices.

Name: _____

Group: _____ Date: _____

5 **LET'S EAT!** Three cabinet-makers produce round tables. They calculate the sales price using the functions below.

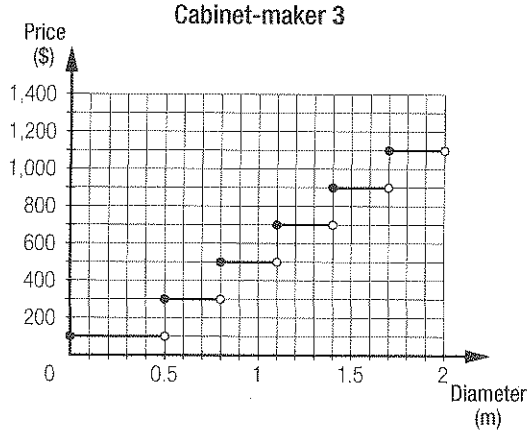
Cabinet-maker 1

$f(x) = 422.25x^2$ where x represents the measure of the table's diameter (in m) and $f(x)$, the table's price (in $).

Cabinet-maker 2

Diameter (m)	Price ($)
0	0
0.6	155.59
0.7	211.79
0.8	276.59
0.9	349.99

Cabinet-maker 3



Can it be stated that Cabinet-maker 3 proposes the best price for a round table which is 1.50 m in diameter? Justify your answer.

Name: _____

Group: _____ Date: _____

6 **FIT AS A FIDDLE** The change in Isabelle's heart rate during a workout can be modelled by a piecewise function in which x represents the amount of time that has gone by since the beginning of the workout and $f(x)$, her number of heart beats for each minute. The following information relate to the function:

Isabelle's workout

Time (min)	Function
[0, 10]	$f(x) = 75(1.05)^x$
[10, 15]	Her heart rate decreases by 5 heart beats every minute.
[15, 18]	$f(x) = ax^2$

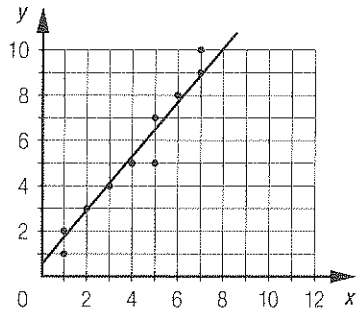
Isabelle would like to increase her heart rate to 180% of its initial value. Does she reach her objective? Justify your answer.

Practice 4.1

Page 1

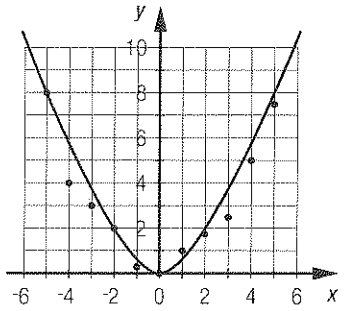
3. a) With a first-degree polynomial function.

b) Graph ①



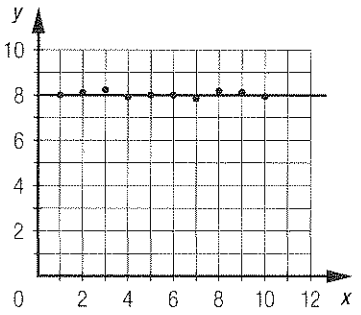
a) With a second-degree polynomial function.

b) Graph ②



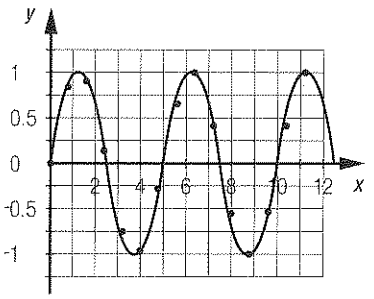
a) With a zero-degree polynomial function.

b) Graph ③



a) With a periodic function.

b) Graph ④



Support 4.1

Page 2

1. a) 1) Domain f: $\mathbb{R}$; range f: $[-10, +\infty[$
   2) Zero(s): $\{-5, 9\}$; initial value: -9
   3) Increasing: $[2, +\infty[$; decreasing: $]-\infty, 2]$
   4) Positive: $]-\infty, -5] \cup [9, +\infty[$; negative: $[-5, 9]$
   5) Minimum: -10
   6) A second-degree polynomial function.

   b) 1) Domain f: $\mathbb{R}$; range f: $]-\infty, 8]$
   2) Zero(s): $\{-8, 8\}$; initial value: 5
   3) Increasing: $]-\infty, 3] \cup [5, 7]$;
   decreasing: $[-6, -3] \cup [3, +\infty[$
   4) Positive: $[-8, 8]$; negative: $]-\infty, -8] \cup [8, +\infty[$
   5) Minimum: 8
   6) A piecewise function.

   c) 1) Domain f: $[0, 10]$; range f: $[2, 8]$
   2) Zero(s): $\{\}$; initial value: 5
   3) Increasing: $[0, 1] \cup [3, 5] \cup [7, 9]$;
   decreasing: $[1, 3] \cup [5, 7] \cup [9, 10]$
   4) Positive: $[0, 10]$
   5) Minimum: 2; maximum: 8
   6) A periodic function.

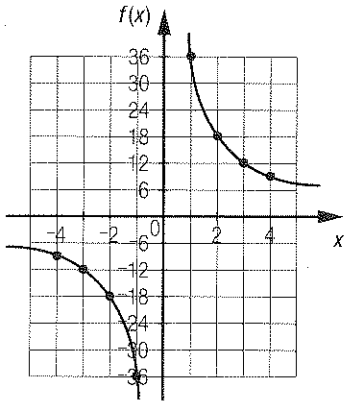
   d) 1) Domain f: $[0, 120[$;
   range f: $\{1, 4, 7, 10, 13\}$
   2) Zero: none; initial value: 1
   3) Increasing: $[0, 120[$
   4) Positive: $[0, 120[$
   5) Minimum: 1; maximum: 13
   6) A step function.

Support 4.1 (*cont'd*)

Page 3

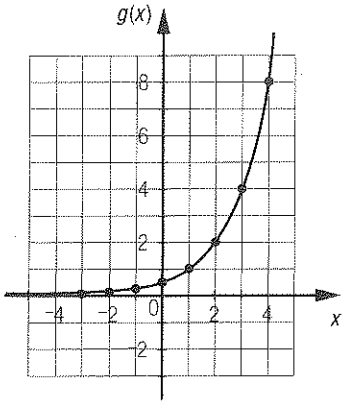
2. a) 7 b) 3 c) 1 d) 5 e) 6

3. a) 1) and 2)



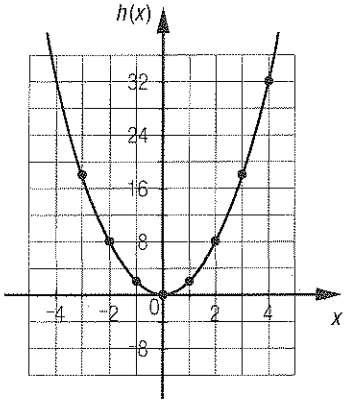
3) Inverse variation function.

b) 1) and 2)



3) Exponential function.

c) 1) and 2)

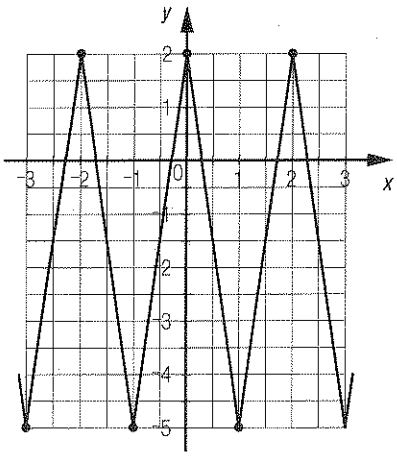


3) Second-degree polynomial function.

Consolidation 4.1

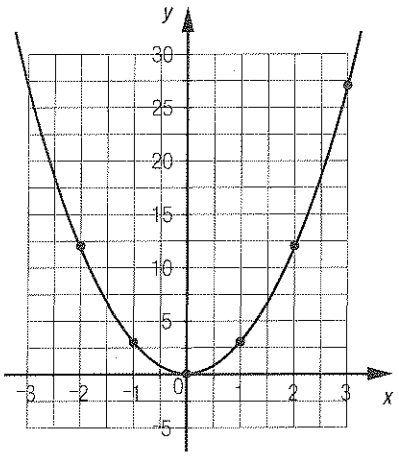
Page 4

1. a) 1)



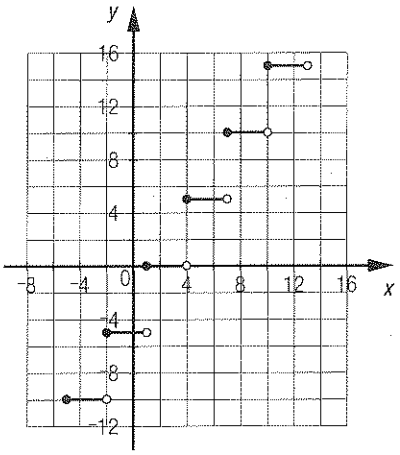
2) Periodic function.

b) 1)



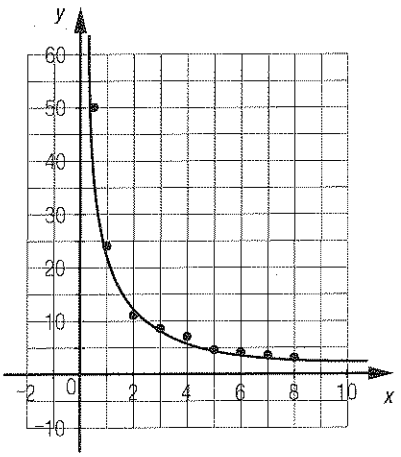
2) Second-degree polynomial function.

c) 1)



2) Step function.

d) 1)



2) Inverse variation function.

Consolidation 4.1 (*cont'd*)

Page 5

2. a) Periodic function.

b) 1) [0, 24]

2) [2000, 12 000]

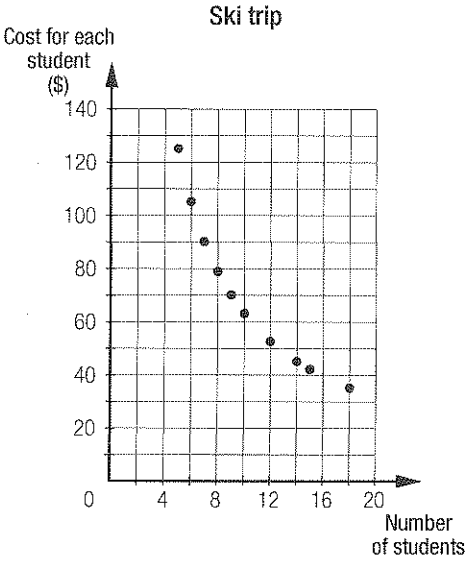
3) Increasing: [0, 8] ∪ [12, 20]; decreasing: [2, 6] ∪ [8, 12] ∪ [14, 18] ∪ [20, 24]

4) Minimum : 2000; maximum: 12 000

3. a) 5 h 30 min
   b) 42 km
   c) Marc-Antoine runs at a steady pace of 12 km/h for 1½ h. He then reduces his speed, maintaining a tempo of 3.6 km/h for 2½ h. Lastly, he runs at a speed of 10 km/h for 1½ h.
   d) Piecewise function.

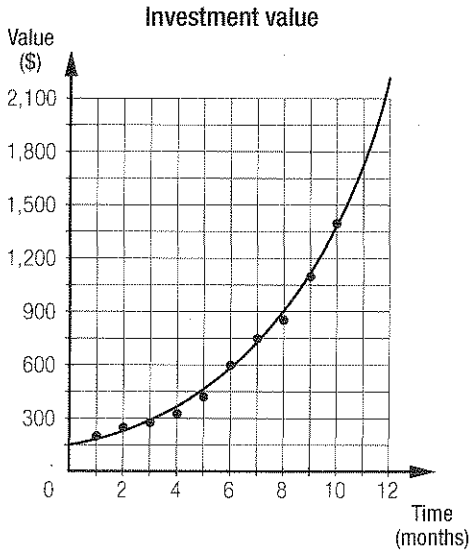
Consolidation 4.1 (*cont'd*) — Page 6

4. a)



   b) By an inverse function because it's a curve which decreases and will never cross the *x* or *y* axes.
   c) $630
   d) 63 students.

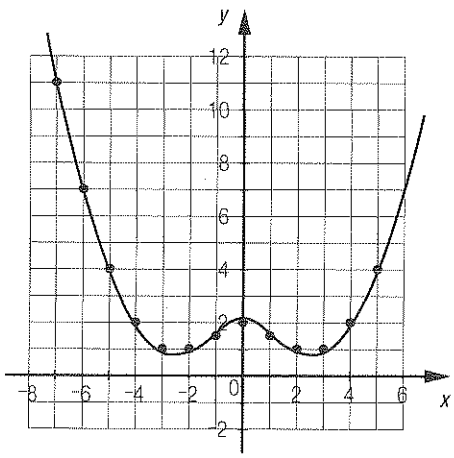
5. a)



   b) With an exponential function.
   c) ≈ $1,750

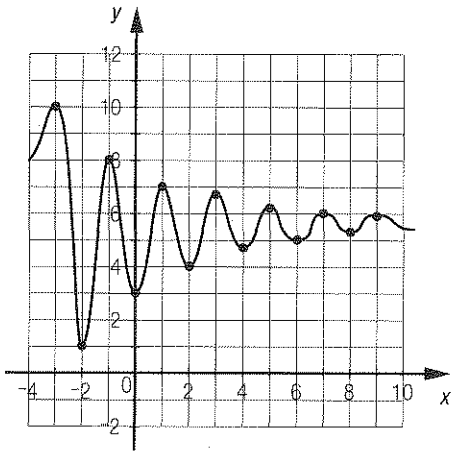
Enrichment 4.1 — Page 7

1. a) 1)



   2) A fourth-degree polynomial function is the best choice of model because the scatter plot reveals a trend associated with this type of function.
   3) $y \approx 0.8$

   b) 1)



   2) Bessel's function is the best choice of model because the scatter plot shows a tendency associated with this type of function.
   3) $y \approx 4.5$

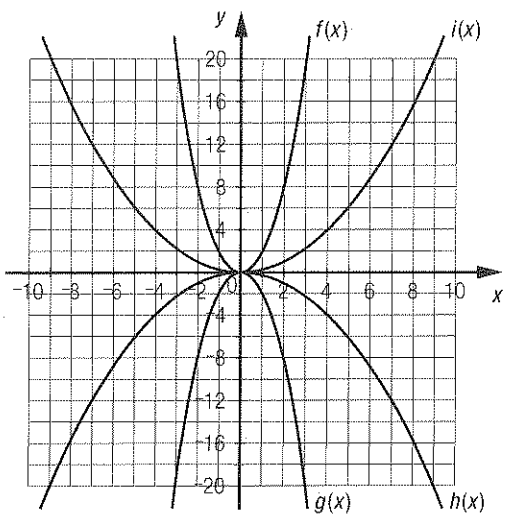
Support 4.2 — Page 8

1. a) $y = 8.75$ b) $y = 45$ c) $a = 2$
   d) $a = 7$ e) $x = \pm 5$ f) $x = \pm 3.2$
2. a) $f(x) = ax^2$ b) $12 = a(2)^2$
   c) $a = 3$ d) $f(x) = 3x^2$
   e) 1) $f(10) = 300$ 2) $f(-50) = 7500$

3. a) Divide parameter a by 4.
   $a = 5 \div 4 = 1.25$
   b) Multiply parameter a by 2.
   $a = 5 \times 2 = 10$
   c) Change the sign of parameter a.
   $a = 5 \times -1 = -5$

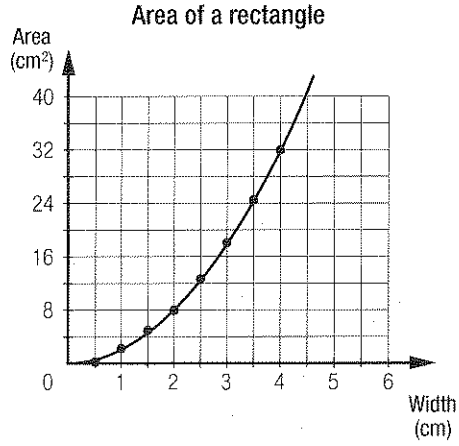
Support 4.2 (*cont'd*) — Page 9

4. a)



   b) The graph undergoes a reflection about the *x*-axis.

5. a) and c)



   b) A second-degree polynomial function.
   d) $y = 2x^2$
   e) 1) 40.5 cm² 2) $\sqrt{15} \approx 3.87$ cm

Consolidation 4.2 — Page 10

1. a) $y = -1.2x^2$ b) $y = x^2$ c) $y = -3x^2$

2. a) A(2.1, 22.05)
   b) B(≈ 4.3, 92.5)

Consolidation 4.2 (*cont'd*) — Page 11

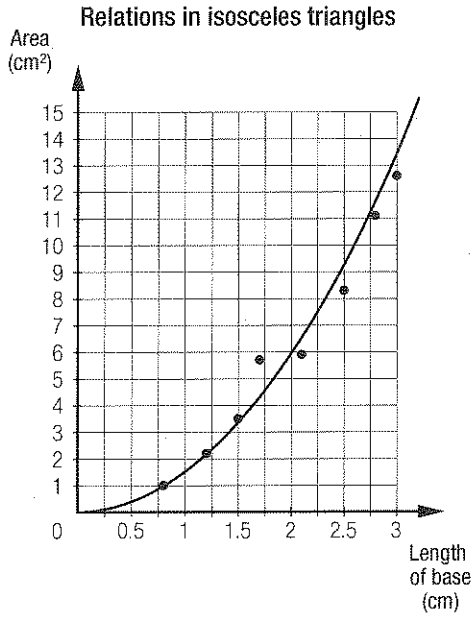
3. a) ④ b) ② c) ⑥
   d) ① e) ⑤ f) ③

4. a) No, because there cannot be negative time.
   b) The vertical compression of the curve would be 6 times greater.
   c) 1) $f(x) = 10x^2$
   2) $f(x) = \frac{5}{3}x^2$
   d) 1) After about 2.24 s.
   2) After about 5.48 s.

Consolidation 4.2 (*cont'd*) — Page 12

5. a) ≈ 123.61 cm² b) ≈ 17.07 cm

6. a) and b)



   c) $f(x) = 1.5x^2$ d) ≈ 5.77 cm

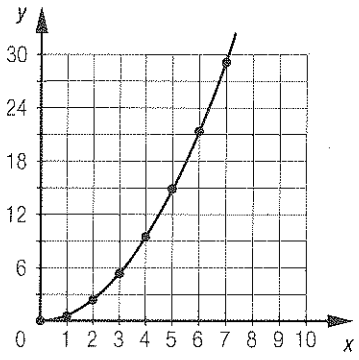
Enrichment 4.2 — Page 13

1. a)

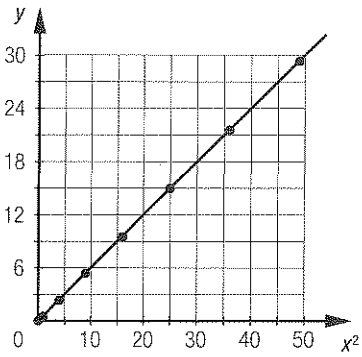
x	0	1	2	3	4	5	6	7
x^2	0	1	4	9	16	25	36	49
y	0	0.6	2.4	5.4	9.6	15	21.6	29.4

b) and c)

1)

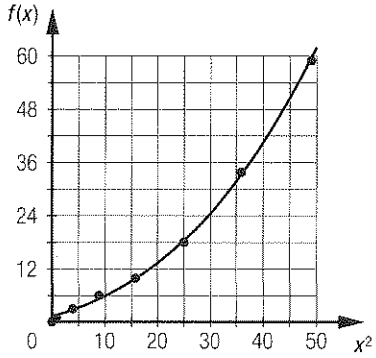


2)



d) The scatter plot representing the relation between y and x^2 is a straight line that passes through the origin. This type of graphical representation is characteristic of a directly proportional situation.

2.



The function whose rule takes the form $y = ax^2$ is not the best model for this situation since the curve of the relation of y and x^2 is not a straight line passing through the origin. Therefore, if a function is to be modelled by the rule $y = ax^2$, y must be directly proportional to x^2.

Activity 1

Page 14

a.

Worst-kept secret

Time (min)	Calculation	Number of people aware of the secret
0	1×2^0	1
1	$1 \times 2 = 1 \times 2^1$	2
2	$1 \times 2 \times 2 = 1 \times 2^2$	4
3	$1 \times 2 \times 2 \times 2 = 1 \times 2^3$	8
4	$1 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^4$	16
5	$1 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^5$	32
6	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^6$	64
7	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^7$	128
8	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^8$	256
9	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1 \times 2^9$	512
...	...	...
t	1×2^t	

b.

c. An exponential function because the rate of change gets increasingly larger.

d. 1) 5 min 2) 14 min 3) 19 min

e. 1) 1 048 576 people.
2) 33 554 432 people.
3) 8 589 934 592 people.

Activity 2

Page 15

a. The number of bacteria decreases by half at each application.

b. Antibiotic ointment

Number of applications	Calculation	Number of bacteria ($\times 10^6$)
0	1024×0.5^0	1024
1	$1024 \times 0.5 = 1024 \times 0.5^1$	512
2	$1024 \times 0.5 \times 0.5 = 1024 \times 0.5^2$	256
3	$1024 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^3$	128
4	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^4$	64
5	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^5$	32
6	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^6$	16
7	$1024 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 1024 \times 0.5^7$	8
...	...	...
n	1024×0.5^n	

c. 1024 million bacteria.

d. By 0.5.

e. The distance between the curve and the x-axis continually decreases but it will never be 0.

Practice 4.3

Page 16

20. a) $26,500 b) 106%

c) $26,522.50 d) 106.09%

e)

Plan A			
Time (months)	Time (years)	Calculation	Value of investment ($)
0	0	$25\,000(1.06)^0$	25,000
12	1	$25\,000(1.06)^1$	26,500
24	2	$25\,000(1.06)^2$	28,090
36	3	$25\,000(1.06)^3$	29,775.40
48	4	$25\,000(1.06)^4$	31,561.92
...	...	...	...
	x	$25\,000(1.06)^x$	

Plan B			
Time (months)	Time (years)	Calculation	Value of investment ($)
0	0	$25\,000(1.03)^0$	25,000
6	0.5	$25\,000(1.03)^1$	25,750
12	1	$25\,000(1.03)^2$	26,522.50
18	1.5	$25\,000(1.03)^3$	27,318.18
24	2	$25\,000(1.03)^4$	28,137.72
30	2.5	$25\,000(1.03)^5$	28,981.85
36	3	$25\,000(1.03)^6$	29,851.31
42	3.5	$25\,000(1.03)^7$	30,746.85
48	4	$25\,000(1.03)^8$	31,669.25
...	...	...	...
	x	$25\,000(1.03)^{2x}$	

f) Plan B is the most advantageous because the second level of interest at 3% is calculated on an amount to which 3% has already been added.

Support 4.3

Page 17

1. a) 1) $f(0) = 1.5$ 2) $g(0) = 2$ 3) $h(0) = -5$

b) Parameter a of the exponential function of the form $f(x) = a(\text{base})^x$ represents the initial value.

2. a) The base of the exponential function of the form $f(x) = a(\text{base})^x$ represents the multiplication factor acting on the y-coordinate for each chance of 1 increment undergone by the x-coordinate.

b) The base of $h(x)$ is 5.

3. a) $a = 1.5$

b) $12 = 1.5(\text{base})^3$

c) $y = 1.5(2)^x$

Support 4.3 (*cont'd*)

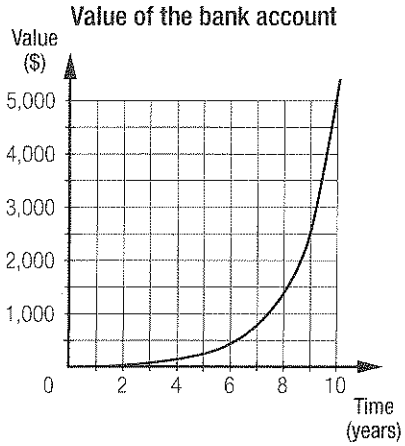
Page 18

4. a) Graph ①: $y = 5(3)^x$
Graph ②: $y = -7(0.2)^x$
Graph ③: $y = 3(0.6)^x$
Graph ④: $y = -4(2.1)^x$

b) 1) Curves for which the base is between 0 and 1 come closer to the *x*-axis as the value of the independent variable increases.

2) Curves for which the value of parameter a is negative are reflected about the *x*-axis.

5. a)



b) $f(x) = 5(2)^x$

c) $163,840

Consolidation 4.3 — Page 19

1. a) $f(x) = 1.5(2)^x$ b) $g(x) = 2(0.25)^x$
   c) $h(x) = -4(2)^x$ d) $i(x) = \frac{1}{3}(3)^x$
2. a) 250 bacteria. b) About 353 bacteria.
   c) 1000 bacteria. d) After 4 h.

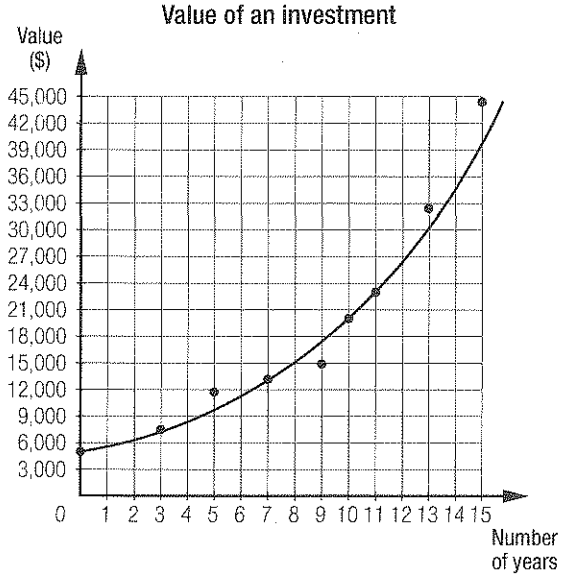
Consolidation 4.3 (*cont'd*) — Page 20

3. a) ⑦ b) ① c) ⑤ d) ③
   e) ② f) ⑥ g) ④ h) ⑧
4. a) $f(x) = 3(0.8)^x$
   b) 61.44 cm
   c) ≈ 1.20 m
   d) According to the model, the ball will never stop bouncing because this situation is typical of an exponential function having an asymptote at $y = 0$. In reality, the ball will eventually stop bouncing.

Consolidation 4.3 (*cont'd*) — Page 21

5. a) An approximate interest rate of 19.56%.
   b) Card 1 is the most advantageous because the annual interest rate on Card 2 is approximately 25.34%.

6. a)



b) $f(x) = 5000(1.15)^x$

c) The base of this function is 1.15. It represents the yearly increase of the investment's value.

d) ≈ $61,877.27

Enrichment 4.3 — Page 22

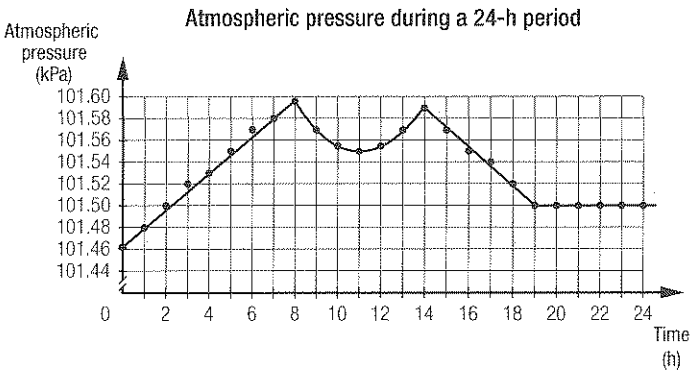
1. The equation for the exponential model $y \approx 0.01259(0.999940068)^x$.
   Constructing a table of values gives an answer of approximately 157 000 years.
2. Set-up two points with coordinates (m, n) and (c, d) such that $\frac{d}{n} < 0$.
   If the points belong to an exponential function of the form $y = a(\text{base})^x$, it is possible to write two equations:
   $n = a(\text{base})^m$ and $d = a(\text{base})^c$.
   The proportion is $\frac{d}{n} = \frac{a(\text{base})^c}{a(\text{base})^m}$, and the equation $(\text{base})^{c-m} = \frac{d}{n} \Rightarrow (\text{base})^{c-m} < 0$.
   This equation has no real solution as there is no positive real number raised to either a positive or negative exponent that yields a negative as a solution.

Activity 1 — Page 23

a. 1) With a first-degree polynomial function.
   2) With a second-degree polynomial function.
   3) With a first-degree polynomial function.
   4) With a zero-degree polynomial function.

b. With a piecewise function.

c.



d. 1) ≈ 101.51 kPa 2) ≈ 101.59 kPa
3) ≈ 101.58 kPa 4) ≈ 101.515 kPa

Support 4.4

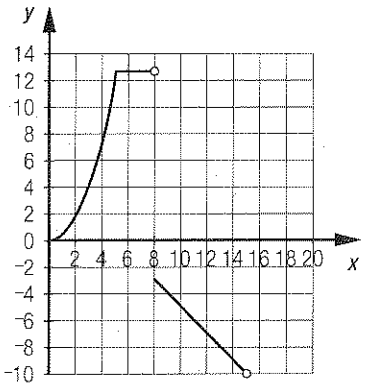
Page 24

1. a) 1) A, D 2) B, E 3) C, F
   b) A Periodic function.
   B Step function.
   C Piecewise function.
   D Periodic function.
   E Step function.
   F Piecewise function.

Support 4.4 (*cont'd*)

Page 25

2.



3. a) $f(x)$: P = 4 $g(x)$: P = 20
   b) 1) $f(1) = 6$ $f(3 + P) = 6$ $f(3 - P) = 6$
   2) $g(5) = 2$ $g(5 + P) = 2$ $g(5 + 2P) = 2$
   c) 1) $f(17) = 6$ $f(-11) = 6$ $f(40) = 7$
   2) $g(-30) = 0.5$ $g(105) = 2$ $g(-200) = 3.5$

4. a) Range g: {2, 3, 4, 5, 6}
   b) Critical values: {4, 8, 12, 16}

Consolidation 4.4

Page 26

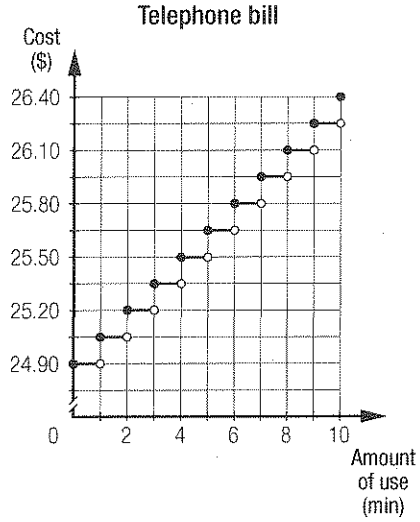
1. a) Periodic function.
   b) Piecewise function.
   c) Step function.
   d) Periodic function.

2. a) Step function.
   b) Piecewise function.
   c) Periodic function.

Consolidation 4.4 (*cont'd*)

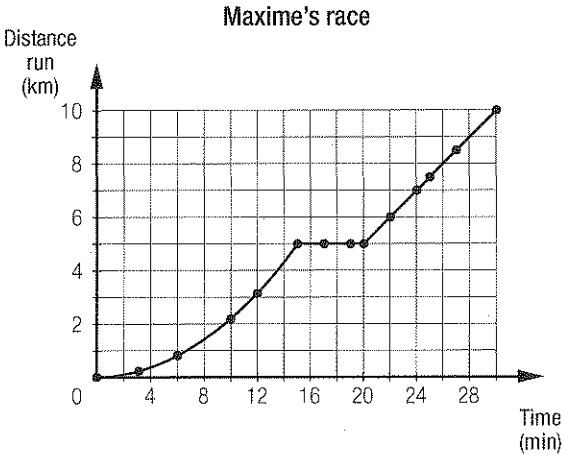
Page 27

3. a)



   b) Step function.
   c) $32.40
   d) 93 min or more, but less than 94 min.

4. a) Piecewise function. To be more precise, 3 different curves are required in order to model the situation.
   b)



c)

Interval (min)	Distance covered at the end of this interval (km)	Type of function	Formula
[0, 15[	5	Second-degree polynomial	$f(x) = \frac{1}{45}x^2$
[15, 20[	5	Zero-degree polynomial	$f(x) = 5$
[20, 30[	10	First-degree polynomial	$f(x) = 0.5x - 5$

d) 1) $\approx$ 1.42 km 2) 8 km

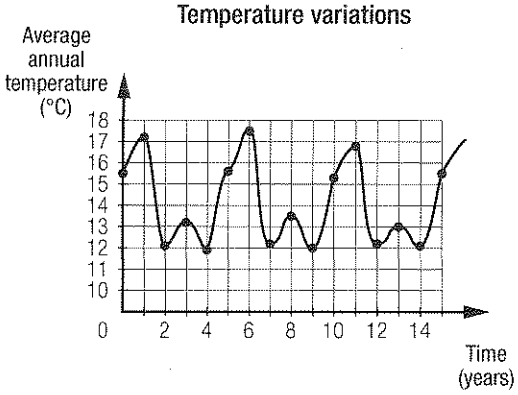
Consolidation 4.4 (*cont'd*)

Page 28

5. a) The first two years, the number of accidents increased rapidly then declined until reaching 0 accidents after 5 years. Thereafter, the number of accidents began to vary once again as it did during the first five years.

   b) 1) No accidents.
   2) 8 accidents.
   3) 3 accidents.

6.

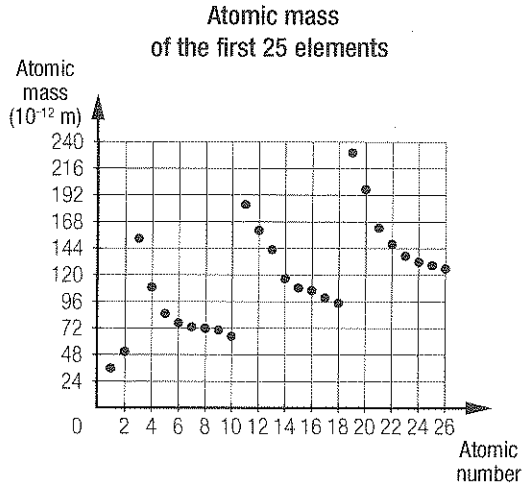


The scatter points indicate a periodic trend of 5 years. By drawing a curve which respects this trend, or by continuing the table of values, it is possible to determine that the average temperature should be about 12°C after 35 years.

Enrichment 4.4

Page 29

1. a)



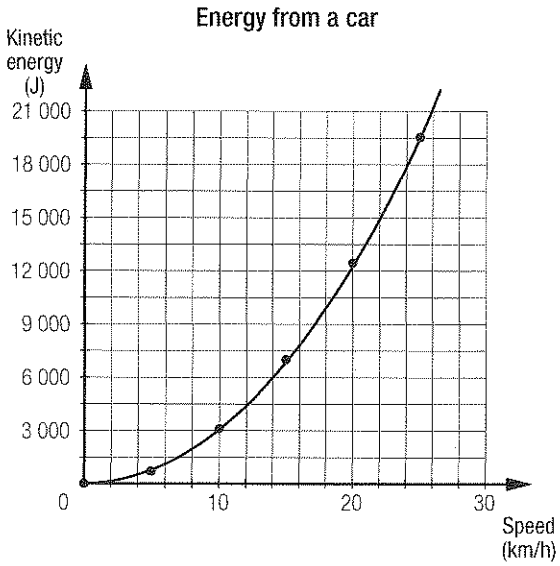
   b) Piecewise function.

   c) No, because in the context of functions, the word "periodic" means that the function is constituted by the repetition of a single pattern which is not the case here. In the context of atomic mass, the word "periodic" means that trends are repeated with relative regularity.

Snapshot 4

Page 30

1. The equation for the curve which represents this situation is $f(x) = 31.4x^2$. The increase in the car's kinetic energy would be about 28 260 J.



Snapshot 4 (*cont'd*) **Page 31**

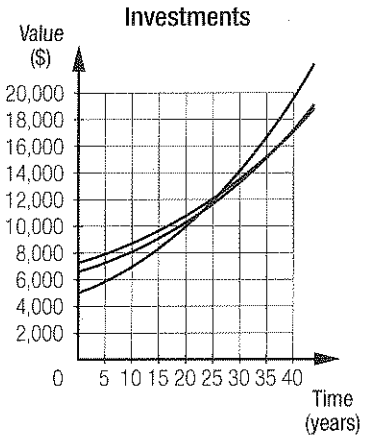
2. Because the function has a period of 90 s, $90 \times 5 = 450$ and $525 - 450 = 75$, the chair will be located at the same height as at 75 s after the start of the period. The chair will be 5 m off the ground.

Snapshot 4 (*cont'd*) **Page 32**

3. The population will decline more rapidly in Region **2** because it is in this region that the base of the exponential function is the smallest (base ≈ 0.97).

Snapshot 4 (*cont'd*) **Page 33**

4. The value of Investment **1** is the most significant of the three during the first 28 years. Investment **3** becomes more advantageous from the 29th year. Investment **2** is at no time the most advantageous of the three.



Snapshot 4 (*cont'd*) **Page 34**

5. Cabinet-maker **3** proposes the best price at $900. Cabinet-maker **1**'s price is $950.06 and Cabinet-maker **2**'s price is $972.44.

Snapshot 4 (*cont'd*) **Page 35**

6. Isabelle's initial heart rate is 75 beats/min. 180% of this value corresponds to 135 beats/min. At the end of her workout, Isabelle's heart rate is 139 beats/min. She has reached her objective.