June 2012 detailed solutions

PART A

This part of the examination consists of Questions 1 to 6.

Each question in this part of the examination is worth 4 marks.

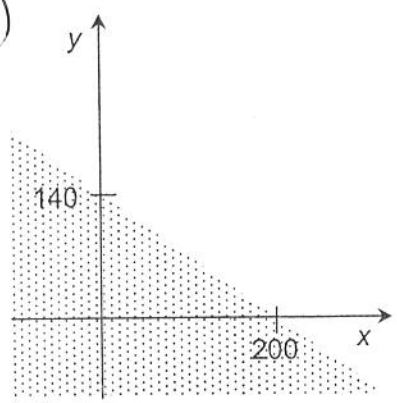
On the machine-scored answer sheet, use an HB pencil to fill in the box under the letter that corresponds to your answer.

1. Which of the following half-planes represents the solution set for the inequality $35x + 50y < 7000$?

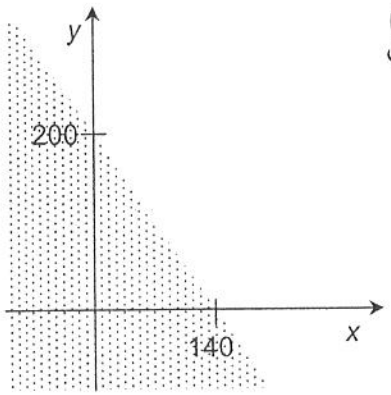
$50y < -35x + 7000$

$y < -0.7x + 140$ ↑ this is y-int

A)



C)



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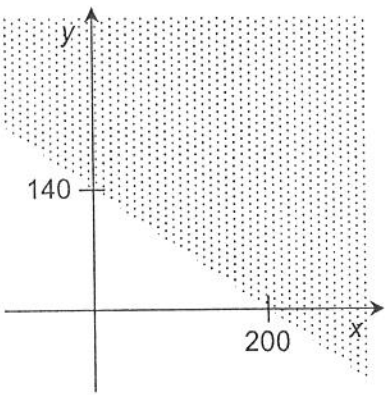
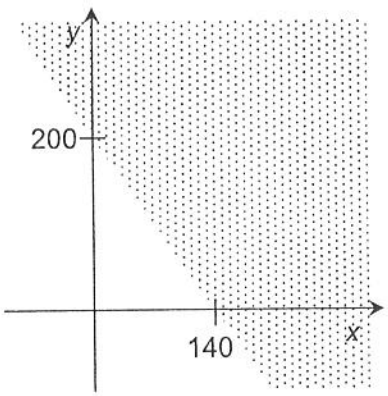
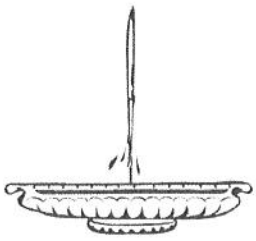


D)

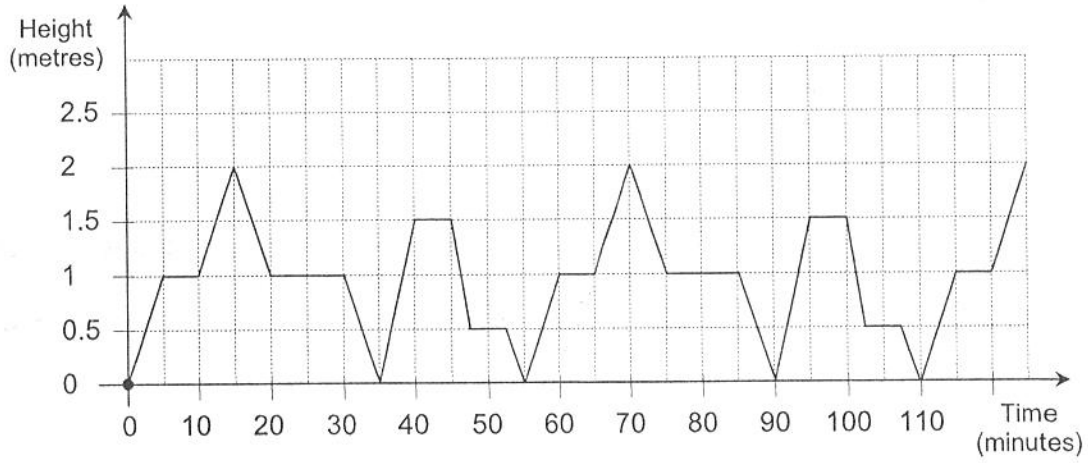


2. Water shoots out of the middle of a fountain.



The height of the jet of water varies.

The periodic function represented below can be used to determine the height of the jet of water in relation to the time elapsed from the moment the fountain was turned on.



Which of the following statements is true?

A) This function has no zeros.

B) The range of this function is $[0, 2]$. look @ y values

C) The period of this function is 35. → no it's 55

D) This function is negative over the interval $[15, 20]$.

3. Students in a school sold pencils and mugs to raise money for sporting activities.

They made a profit of \$1.15 for each pencil and a profit of \$3.25 for each mug.

The students sold 5 times as many pencils as mugs.

In all, they made a profit of \$1260.

$1.15x + 3.25y = 1260$

$x = 5y$

Let x : the number of pencils sold
y : the number of mugs sold

Which one of the following systems of equations could represent this situation?

A) $x = 5y$
$1.15x + 3.25y = 1260$

C) $y = 5x$
$1.15x + 3.25y = 1260$

B) $x = 5y$
$1.15x + 16.25y = 1260$
?

D) $y = 5x$
$5.75x + 3.25y = 1260$
?

4. The equation of a line is $6x + 2y - 15 = 0$.

What is the x-intercept of this line?

let $y = 0$

A) –7.5

C) 2.5

B) –2.5

D) 7.5

$6x + 2(0) - 15 = 0$

$6x = 15$

$x = 2.5$

5. Patricia was a lifeguard at a city pool last summer. On 24 sunny days, she recorded the number of swimmers and the maximum outdoor temperature. The table below shows the data she recorded.

NUMBER OF SWIMMERS ACCORDING TO THE MAXIMUM OUTDOOR TEMPERATURE

MAXIMUM OUTDOOR TEMPERATURE (°C)	NUMBER OF SWIMMERS				
	[5, 20[	[20, 35[	[35, 50[	[50, 65[	[65, 80[
[20, 22[	2	0	0	0	0
[22, 24[	0	2	0	0	0
[24, 26[	0	1	6	1	0
[26, 28[	0	0	1	6	0
[28, 30[	0	0	0	1	4

Which of the following statements best describes the linear correlation between the number of swimmers and the maximum outdoor temperature?

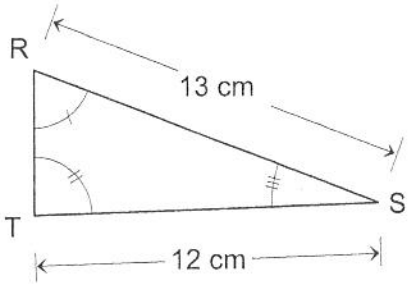
A) The linear correlation is negative and weak.

B) The linear correlation is negative and strong.

C) The linear correlation is positive and weak.

D) The linear correlation is positive and strong.

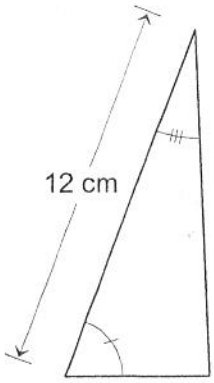
6. Consider triangle RST shown below.



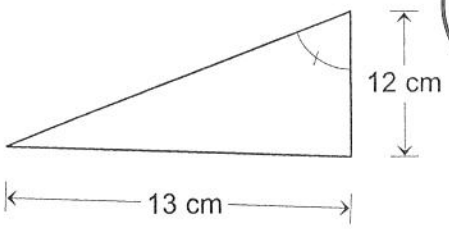
Which of the following triangles is definitely congruent to triangle RST?

A)

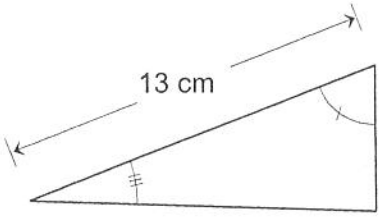
C)



B)



D)



PART B

This part of the examination consists of Questions 7 to 10.

Each question in this part of the examination is worth 4 marks.

Write each of your answers in the space provided on page 3 of your *Student Booklet*.

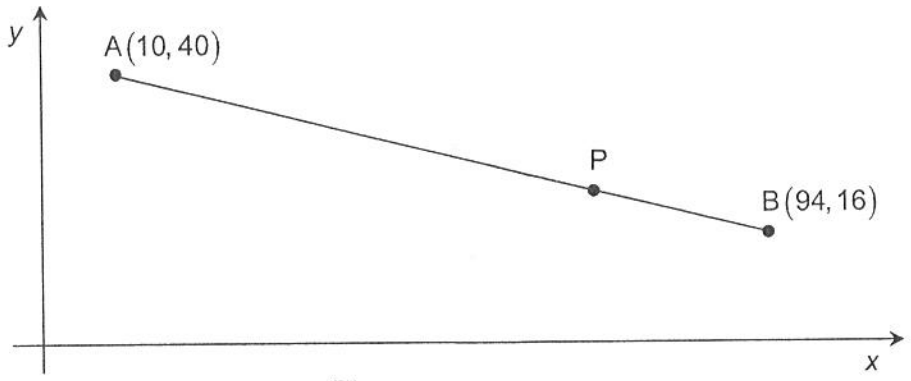
7. What is the solution to the following system of equations?

$$x + y = 96$$
$$11.5x + 8.25y = 1026$$

$x+y=96$
$72+y=96$
$y=24$ $(72, 24)$

$y=-x+96$
$11.5x+8.25(-x+96)=1026$
$11.5x-8.25x+792=1026$
$3.25x=234$
$x=72$

8. Point P is on line segment AB represented in the Cartesian plane below.



From point A, point P is located $\frac{3}{4}$ of the way along line segment AB.

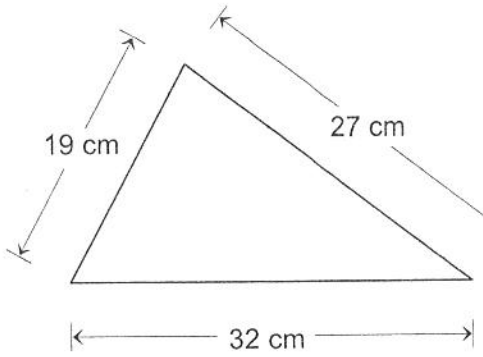
What are the coordinates of point P?

$x_P = 10 + \frac{3}{4}(94-10)$
$= 10 + \frac{3}{4}(84)$
$= 10 + 63$
$= 73$

$y_P = 40 + \frac{3}{4}(16-40)$
$= 40 + (-18)$
$= 22$

$P(73, 22)$

9. Consider the triangle shown below.



Hero's formula.

$p = 39$

$A = \sqrt{39(39-19)(39-27)(39-32)}$

$= \sqrt{39(20)(12)(7)}$

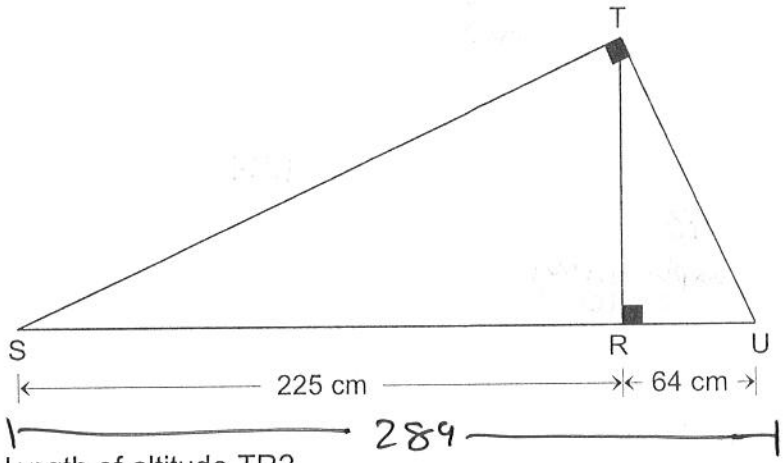
$= \sqrt{65520}$

$= 255.97$

To the nearest cm^2, what is the area of this triangle?

$256\ cm^2$.

10. Altitude TR was drawn in right triangle STU shown below.



metric relations

What is the length of altitude TR?

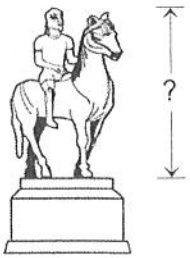
$h^2 = mn$

$h^2 = 225 \cdot 64$

$\sqrt{h^2} = \sqrt{14400}$

$h = 120\ cm$

11. STATUE

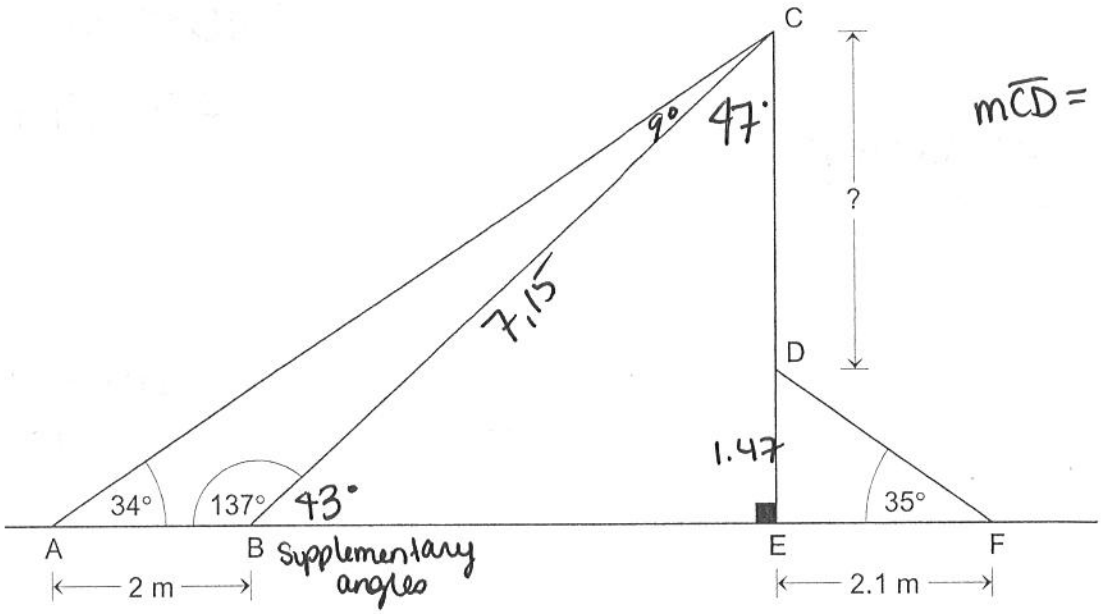


A statue is secured to a base. Joan determined the measurements in the diagram below to find the height of the statue without the base.

In this diagram,

- line segment CD represents the statue
- line segment DE represents the base
- line AF represents the ground

TRIG



$m\overline{CD} = m\overline{CE} - m\overline{DE}$

Supplementary angles

To the nearest tenth of a metre, what is the height of the statue without the base?

$\triangle DEF$: $m\overline{DE}$:

$\tan 35 = \frac{x}{2.1}$

$x = 2.1 \tan 35$

$= 1.47$

$\triangle ABC$: $\frac{2}{\sin 9^\circ} = \frac{x}{\sin 34}$

$\frac{x \sin 9}{\sin 9} = \frac{2 \sin 34}{\sin 9}$

$x = 7.15$

$\triangle BCE$: $m\overline{CE}$:

$\sin 43 = \frac{x}{7.15}$

$x = \sin 43 (7.15)$

$= 4.88$

$4.88 - 1.47$

$= 3.41$ m

12. WELL WATER

Jim bought a cottage a few years ago. We need to figure out when he bought it

Since then Jim has been analyzing the water in the well at his cottage every year on June 1.

Data collected over past years was used to establish that function f described below represents the number of atypical bacteria in the well water in relation to the amount of time elapsed since Jim bought the cottage.

$f(x) = 16(1.5)^x$ exponential.

where x: number of years elapsed since Jim bought the cottage
$f(x)$: number of atypical bacteria per 100 mL of water

In 2012, the analysis revealed that the well water contained 54 atypical bacteria per 100 mL.

In what year will the analysis reveal for the first time that there are more than 200 atypical bacteria per 100 mL of water?

$y = 16(1.5)^x$

$\frac{54}{16} = \frac{16(1.5)^x}{16}$

$3.375 = 1.5^x$

guess & check for values of x

$x = 3$

$\therefore$ 3 years later = 2012

So Jim bought cottage in 2009.

$\frac{200}{16} < \frac{16(1.5)^x}{16}$

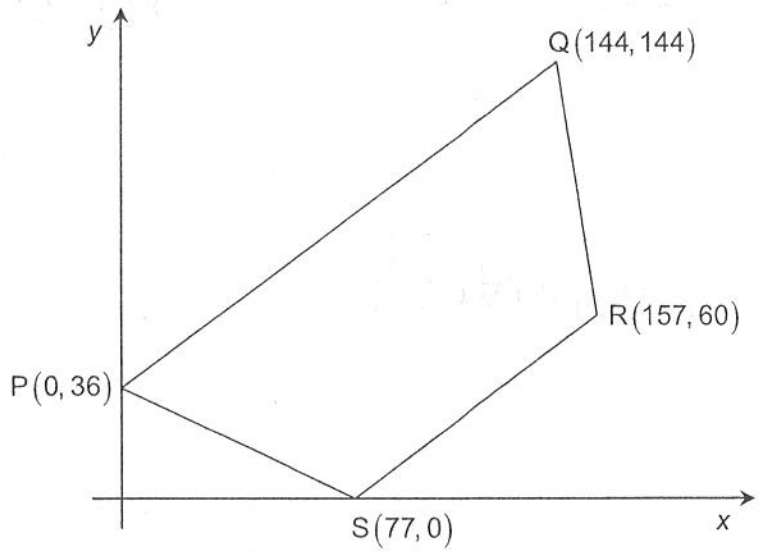
$12.5 = 1.5^x$

$7 = x$

In 2016.

13. ISOSCELES TRAPEZOID

Consider quadrilateral PQRS represented in the Cartesian plane below.



analytic geometry

Show that quadrilateral PQRS is an isosceles trapezoid.

2 sides are ≅

2 sides are // (same slope)

$m\overline{PS} = m\overline{QR}$

$\sqrt{(77-0)^2 + (0-36)^2}$

$= \sqrt{7225}$

$= 85$ ✓

$\sqrt{(157-144)^2 + (60-144)^2}$

$= \sqrt{7225}$

$= 85$ ✓

SR // PQ ∴ same slope

$\frac{60-0}{157-77}$ $\quad$ $\frac{144-36}{144-0}$

$\frac{60}{80}$ $\quad$ $\frac{108}{144}$

$= \frac{3}{4}$ $\quad$ $= \frac{3}{4}$ ✓

14. SCHOLARSHIPS

A music school awards scholarships to students who have excelled over the past school year.

- This year, there are 8 piano students at the school. The year-end mark of each piano student is given below.

  86 86 86 88 88 94 96 96

- The mean of these 8 marks is 90.

MD= $\frac{4+4+4+2+2+4+6+6}{8}$

$= 4$

In order for piano students to receive a scholarship, their year-end mark must meet the following two criteria.

CRITERION A

- In order to meet criterion A, the piano student's year-end mark must satisfy the following inequality:

  Piano student's mark $\geq 86 +$ *Mean deviation of the marks of the 8 piano students*

≥ 90 ∴ 94, 96, 96

CRITERION B

- In order to meet criterion B, the student's year-end mark must meet the following requirement:

  Taking into account the marks of all the students in the music school, the percentile associated with the student's mark must be greater than 90.

- This year, there are 119 students at the school. The following stem-and-leaf plot shows their year-end marks.

YEAR-END MARKS OF THE 119 STUDENTS IN THE MUSIC SCHOOL

Tens	Units	
5	0, 0, 0, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5, 5, 5, 6, 7, 8	22
6	0, 0, 0, 2, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, 7, 8, 8, 8, 9, 9, 9, 9, 9, 9	25
7	0, 1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 7, 7, 7, 7	25
8	0, 1, 1, 1, 1, 2, 2, 3, 4, 4, 5, 6, 6, 6, 6, 7, 7, 8, 8, 8, 8, 8, 8, 8, 9	25
9	0, 0, 0, 1, 2, 2, 3, 4, 5, 5, 5, 5, 5, 6, 6, 6, 7, 8, 8, 8, 9, 9	

How many piano students will receive a scholarship this year?

Percentile of 94 : $\frac{104 + \frac{1}{2}}{119}$ = 88th percentile

Percentile of 96 : $\frac{110 + \frac{3}{2}}{119}$ = 94th percentile

∴ 2 students (96)

16. Drawing Bills

A box contains $5 bills and $10 bills.

A game of chance involves placing a bet and then drawing a bill from the box.

Players get to keep the bill they draw from the box.

Players can choose between two versions of this game.

VERSION A

The box contains fifteen $5 bills and ten $10 bills.

Players must bet $6.

VERSION B

The box contains nine $5 bills and eleven $10 bills.

Players must bet $8.

How many versions of this game are to the player's advantage?

Version A :

$ME = \frac{15}{25}(-1) + \frac{10}{25}(4)$

$= -\frac{15}{25} + \frac{40}{25}$

$= \frac{25}{25}$

$= 1$ (advantage)

Version B :

$ME : \frac{9}{20}(-3) + \frac{11}{20}(2)$

$= -\frac{27}{20} + \frac{22}{20}$

$= \frac{-5}{20}$

$= -0.25$ (disadvantage)

1 version !!.

15. ANTHONY'S JERSEY

Here is the discount advertised at a sporting goods store.

DISCOUNT
Get \$4 off for every \$25 you spend before taxes.

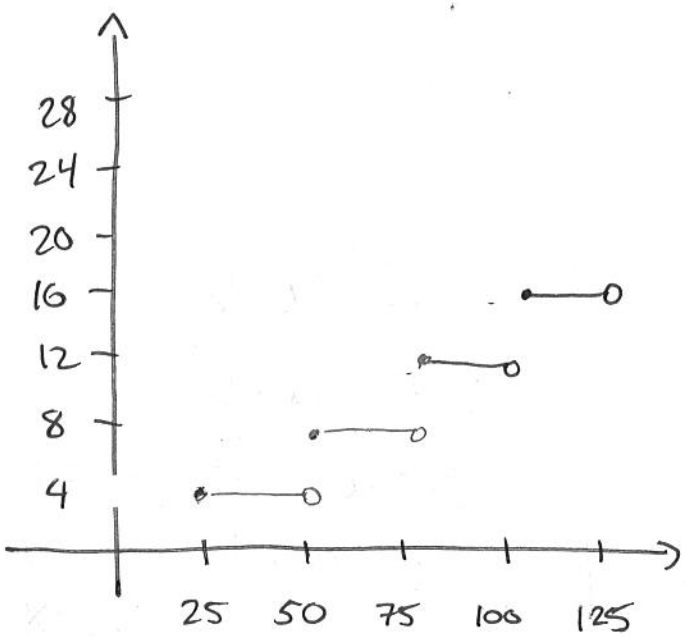
step function.

Greg bought a jersey at this store and got a discount of \$12.

Anthony bought a jersey and a cap at the same store. He got a discount of \$16. The price of the cap was \$19.99.

The price of the jersey that Anthony bought was the same as the price of the jersey that Greg bought.

What are the possible prices, before taxes, of the jersey Anthony bought?



\$12 discount :
[75, 100[

\$16 discount :
[100, 125[

check both ends.

100 − 19.99
= 80.01

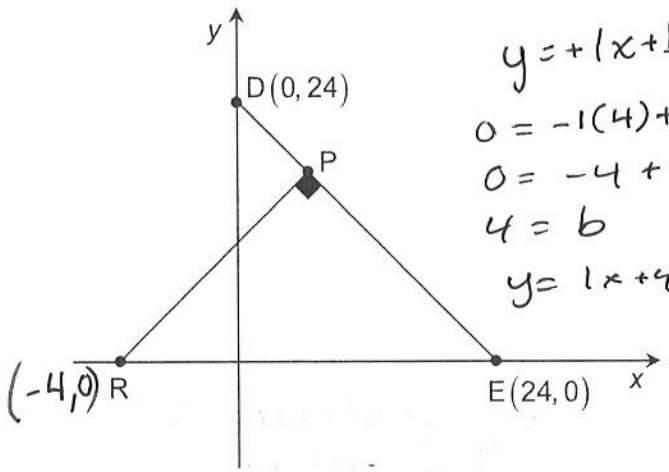
Between [80.01, 100[

15. COORDINATES

In the Cartesian plane:

- line segment DE is drawn such that the coordinates of its endpoints are $D(0, 24)$ and $E(24, 0)$
- a point P is chosen on line segment DE, but P is not one of the endpoints of segment DE
- line segment PR is drawn perpendicular to segment DE, with endpoint R located on the *x*-axis

Below are two possible representations of segment PR described above.



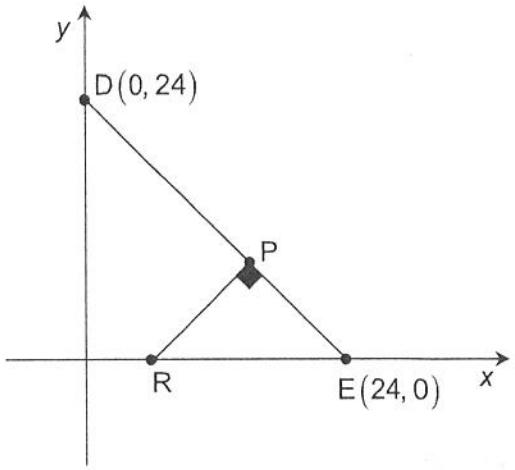
$y = +1x + b$

$0 = -1(4) + b$

$0 = -4 + b$

$4 = b$

$y = 1x + 4$



Formulate a conjecture describing the relationship between the *x*-coordinate of point R and the coordinates of point P for segments corresponding to the above description of segment PR.

Slope of $\overline{DE} = \frac{0-24}{24-0}$

$= -1$

∴ lines are ⊥

∴ slope of $\overline{PR} = 1$

$\overline{RP}$: $\frac{y-0}{x_2-x_1} = 1$

$\frac{y}{x_2-x_1} = 1$

$R(x_1, 0)$

$P(x_2, y)$

Because slope = 1

you need $\frac{y_2-y_1}{x_2-x_1} = 1$

∴ ex: $\frac{15}{15}$

y coordinate = $x_2 - x_1$

The only way to get 1 as slope is to divide a # by itself