student book volume 1

Technical and Scientific

VISIONS

MATHEMATICS

Secondary Cycle Two Year Two

ANSWER KEY

Visions 1 to 3

LES ÉDITIONS CEC

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TABLE OF CONTENTS

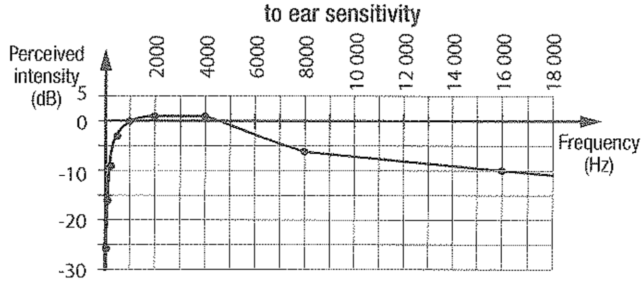
LES 1

A plan for a healthier living environment

Below is a procedure that can be used to design a noise map and recommend where the three types of sound barriers should be installed along the highway.

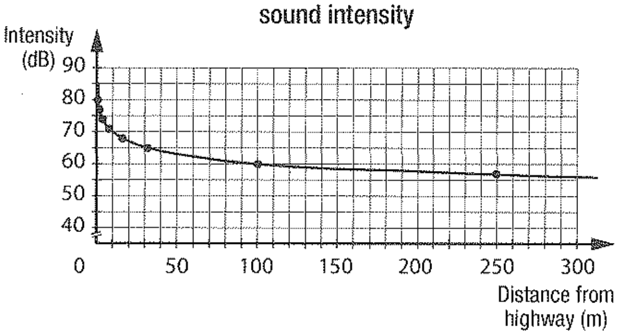
- Graphically represent the function relative to the adjustment of the sound intensity perceived by the human ear according to its frequency.

Graph ①
Ajustment relative to ear sensitivity



- Graphically represent the function relative to sound intensity according to the distance from the highway.

Graph ②
sound intensity



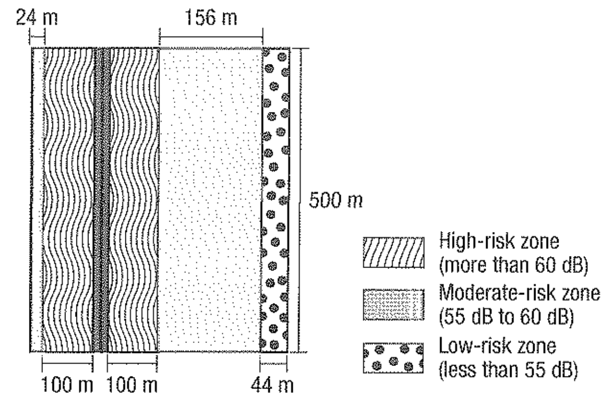
- Determine the adjustment of the sound intensity captured by the human ear in relation to the distance from the highway.

  Since the audible frequencies close to a highway are approximately 750 Hz, it is possible to see, with the help of Graph ① or the corresponding table of values, that the sound perceived by the human ear near a highway is 1 dB.

- Determine the distance at which the sound level is higher than 60 dB and lower than 55 dB.

  According to Graph ②, the intensity of the sound is higher than 60 dB between 0 m and 100 m from the highway and lower than 55 dB more than 256 m from the highway.

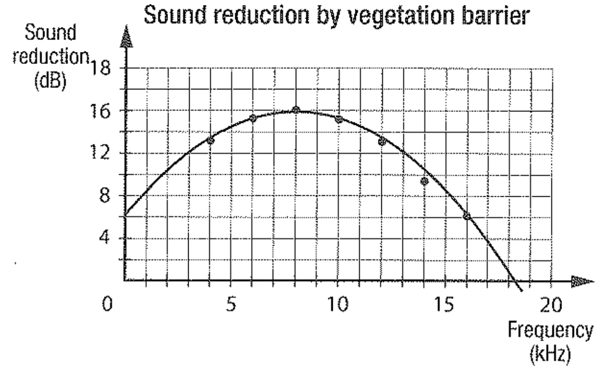
- Draw a noise map that respects these constraints.



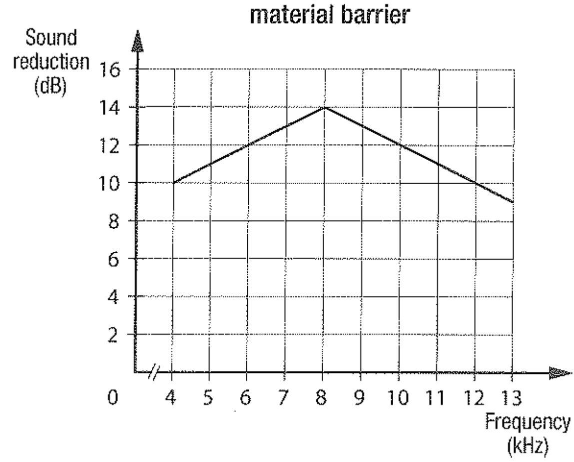
Analysis of various types of sound barriers

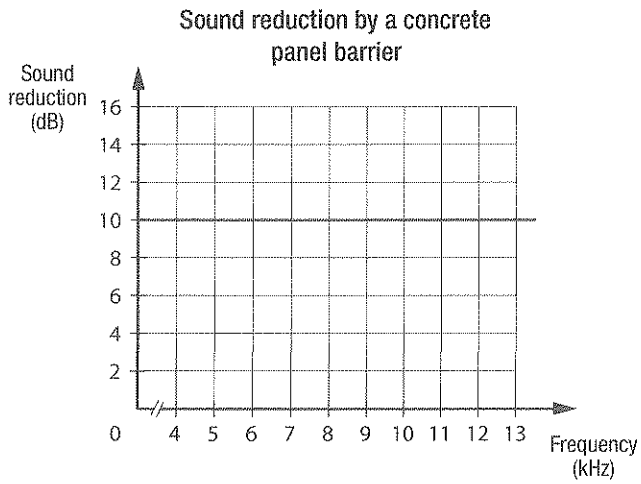
- Graphically represent the sound reduction for each type of wall according to frequency.

Sound reduction by vegetation barrier



Sound reduction by a recycled material barrier





- Determine the value of the sound reduction for each of the barriers when the frequency is near 750 Hz or 0.75kH$_2$.

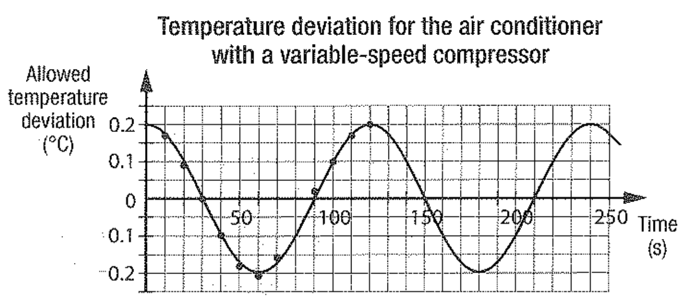
Vegetation barrier	Recycled material barrier	Concrete panel barrier
≈ 7.8 dB	10 dB	≈ 6.75 dB

The concrete-panelled barrier offers the best noise absorption for a frequency of approximately 750 Hz.

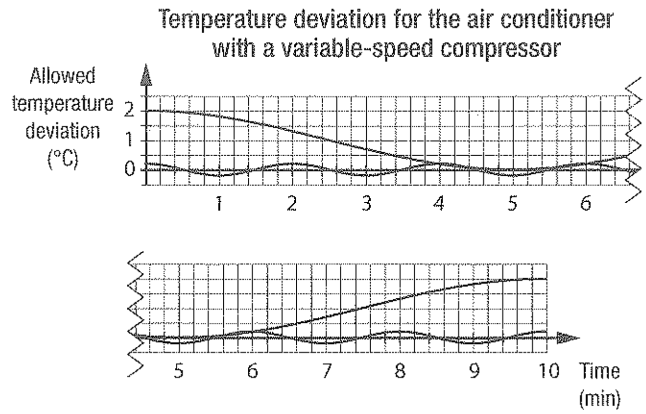
LES 2 Conserve and save

Below is a procedure that refutes the majority of the air conditioning contractor's claims.

- Graphically represent the allowed temperature deviation for the air conditioner with a variable-speed compressor as a function over time.



- Associate the curve with a periodic function.
- Compare the graphical representations of the allowed temperature deviation for both models.



- Verify if the temperature adjustments occurs five times more often with the variable-speed compressor model than with the standard model.

  The period of the curve associated with the standard air conditioner is 10.

  The period of the curve associated with the air conditioner with a variable-speed compressor is 2.

  The temperature is therefore adjusted five times more often on the variable-speed compressor model than it is on the standard model.

- Verify if the allowed temperature deviation is five times less with the variable-speed compressor model than it is with the standard model.

  For the standard air conditioner, the mean allowed temperature deviation is 1°C.

  For the variable-speed compressor air conditioner, the mean allowed temperature deviation is 0.2°C.

  The allowed temperature deviation is therefore five times less with the variable-speed compressor model than it is with the standard air conditioner model.

- Verify if the variable-speed compressor model cuts power consumption by half compared to the standard model by comparing the power consumption of each air conditioner model over a period of 10 min.

Standard air conditioner

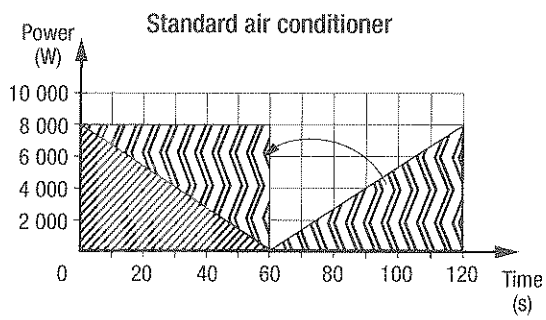
For 10 min:

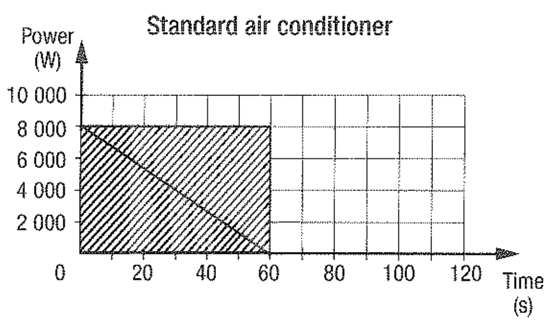
Energy = power × time
= 12 000 × 5 + 0 × 5 = 60 000 W

The energy consumption is therefore 100 W/s.

Air conditioner with a variable-speed compressor

By using the graphical representation, it is possible to note that the energy consumption of this air conditioner corresponds to that of an air conditioner that functions 60 s for every cycle of 120 s at a power level of 8000 W.





It is therefore possible to calculate the energy consumption of the air conditioner over a period of 10 min as follows:

Energy = 5 × power × time
= 5 × 8000 × 1 + 0 × 60 = 40 000 W

The energy consumption is therefore approximately 66.7 W/s.

The air conditioner with a variable-speed compressor does consume less energy, but it does not consume half the amount of the standard model.

REVISION 1

Prior learning 1

Page 4

a. The share value of Company **A** was $100 and that of Company **B** was $40.

b. The minimum value was $40.

c. The maximum value was $220.

d. 1) The value increased over the periods (months) [0, 3], [4, 6], [7, 8] and [10, 12] months.
   2) The value decreased over the periods (months) [3, 4], [6, 7] and [8, 10] months.

e. The share value of Company **A** was $70 and that of Company **B** was $180.

f. No, because the *x*-coordinate 70, in the graphical representation of the inverse, is associated with more than one value.

g. The share value decreased during the first three months, increased slightly the following two months, remained stable for one month, increased the two following months, remained stable for two months and finally increased the last two months.

h. Company **B** showed the best stock performance because the share value was $40 at the beginning of year and reached a value of $120 after 12 months. This $80 increase corresponds to an increase of 200% per share. The share value for Company **A** was $100 at the start of the year and increased to $110 after 12 months, which is a rise of $10 and corresponds to an increase of 10% per share.

Knowledge in action

Page 8

1. **a)** Independent variable: The number of days the car is rented.

   Dependent variable: The cost of renting a car.

   b) Independent variable: The mass of the chicken.

   Dependent variable: The time required to cook a chicken.

   c) Independent variable: The number of members in a family.

   Dependent variable: The weekly grocery bill.

   d) Independent variable: The mass of a parcel.

   Dependent variable: The postal mailing cost.

e) Independent variable: The time of the day.

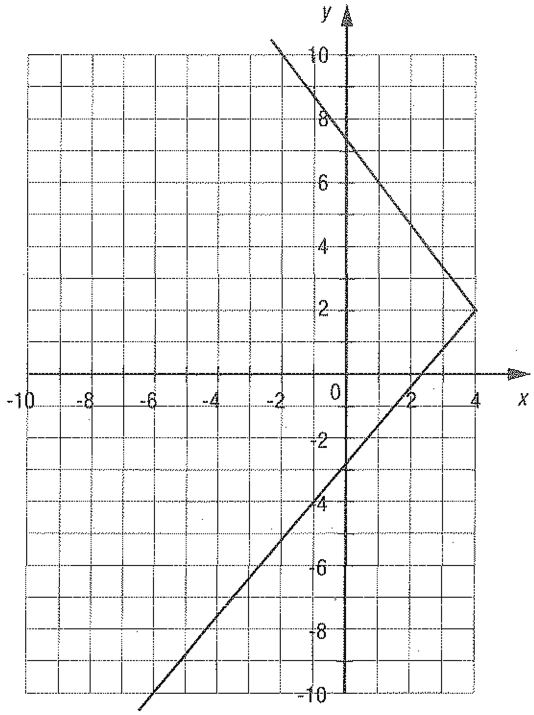
Dependent variable: The moon's position in the sky.

f) Independent variable: The number of donors at a blood drive.

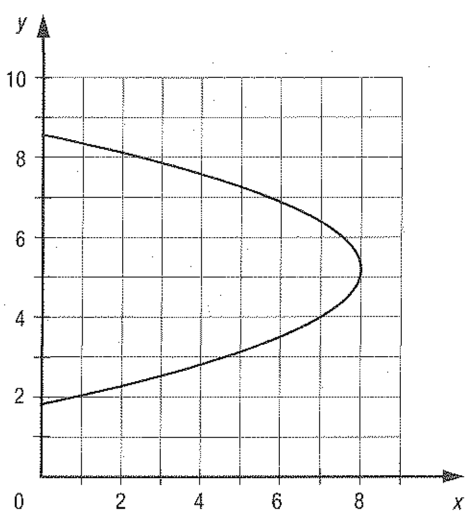
Dependent variable: The amount of blood collected.

2. Ⓐ and Ⓑ.

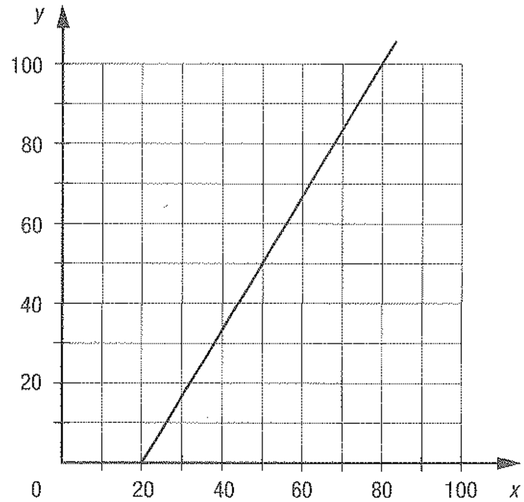
3. a) Graph ①



Graph ②



Graph ③



b) The inverse of Graph ① is not a function.

The inverse of Graph ② is not a function.

The inverse of Graph ③ is a function.

Knowledge in action (*cont'd*) **Page 9**

4. a) Function ①: domain: [0, 12] months
range: $[-500, 1750]

Function ②: domain: [0, 6.5] s
range: [0, 68] m

b) Function ①: increasing over [3, 7] months
∪ [9, 12] months
decreasing over[0, 3] ∪ [7, 9] months.

Function ②: increasing over [0, 6.5] s.

c) Function ①: positive over [0, $2.\bar{3}$] months
∪ [4, 12] months
negative over [$2.\bar{3}$, 4] months.

Function ②: positive over [0, 6.5] s.

d) Function ①: minimum: −$500
maximum: $1,750

Function ②: minimum, 0 m maximum: 68 m.

e) Function ①: $1,750, function ②: 0 m.

f) Function ①: $2.\bar{3}$ and 4 months.

Function ②: 0 s.

5. a) The lowest temperature is −3°C.

b) The initial temperature is 3°C.

c) 1) at 4:00 a.m.
2) at 12:00 p.m.

d) The temperature is 0°C at 2:00 a.m. and 6:00 a.m.

e) 1) The temperature is negative over the time interval [2, 6] h.
2) The temperature is positive over the time interval [0, 2] h ∪ [6, 12] h.

6. It will take Dina 2 years and 5 months.

SECTION 1.1 From real life to a model

Problem — Page 10

Several answers possible. Example: It is possible to represent the ordered pairs in a Cartesian plane in order to determine the relation or the type of function associated with this situation. By substituting y with 216.8 for the rule $y = \frac{129x - 159}{73}$, where x represents the distance covered (in km) and y is the time (in min), a distance of approximately 123.92 km is obtained.

Activity 1 — Page 11

a. 1) 1.5 hours or 1 hour and 30 min.

2) *Several answers possible. Example:* The maximum duration occurs when the speed of the vehicle is very close to 0 km/h.

3) *Several possible answers. For example:* The minimum duration occurs when the speed of the vehicle is very high. However, it should be noted that the speed limit in Québec never exceeds 100 km/h.

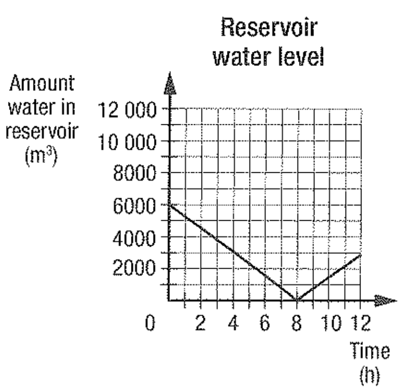
b. 1) The height of the tide is approximately 3.25 m.
2) The tidal range is 8 m.

c. **Situation ①**: As the speed increases, the duration of the trip decreases.

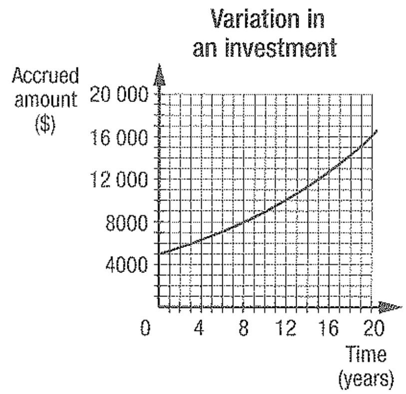
Situation ②: The tide rises over a period of six hours, then goes down over a period of six hours and so on.

Activity 1 (*cont'd*) — Page 12

d. **Situation ③**



Situation ④



e. 1) There was 6000 m^3 of water.

2) The hourly displacemnt from the pump is 750 m^3/h.

3) This operation lasted 16 hours.

f. 1) The accrued amount will be approximately $14,271.

2) The accrued amount will reach $16,000 during the 22nd year.

g. Yes. Each value of the independent variable is associated with no more than one value of the dependent variable.

h. **Situation ③**: The quantity of water in the reservoir decreases at a constant rate for eight hours and increases afterward at the same rate.

Situation ④: The invested sum increases more and more rapidly.

Activity 2 — Page 13

a. Curve ③.

b. Curve ①: first-degree polynomial function $y = 10x + 10$

Curve ②: second-degree polynomial function $y = 2x^2$

Curve ③: exponential function $y = 15.5(1.25)^x$

c. An exponential function.

d. 1) ≈ $4,102.82

2) ≈ $12,520.80

e. *Several answers possible. Example:* No, the price of a barrel of crude oil follows the law of supply and demand. With all the efforts to reduce greenhouse gases, it would be surprising if the demand became that strong.

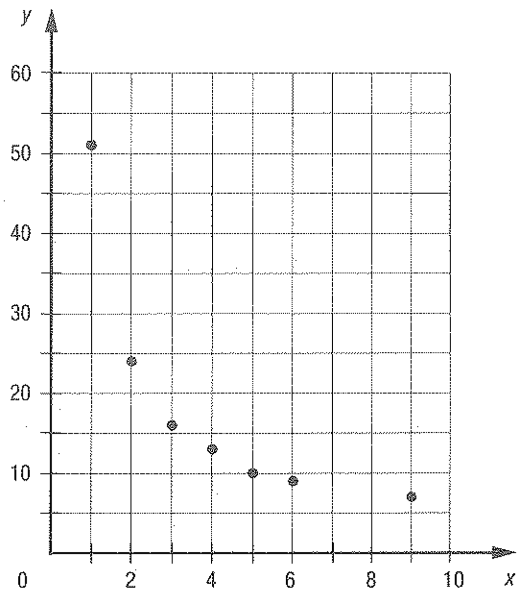
Technomath

Page 14

a. A second-degree polynomial function.

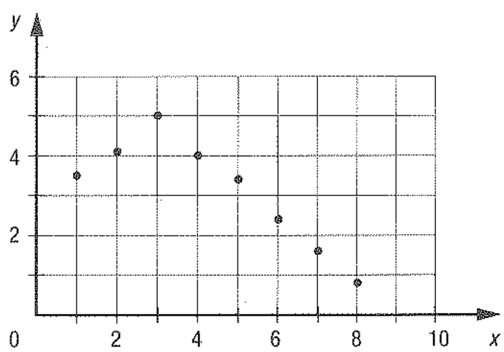
b. 150

c. 1) **Table of values ①**



The trend is represented by a decreasing curve.

Table of values ②



The trend is represented by the graph of two straight lines connected by a maximum.

2) **Table of values ①:** inverse variation function
Table of values ②: absolute value function

Practice 1.1

Page 17

1. a) Second-degree polynomial function.
   b) Inverse variation function.
   c) Piecewise function.
   d) Periodic function.
   e) First-degree polynomial function.
   f) Step function.

Practice 1.1 (*cont'd*)

Page 18

2. A 4, B 1, C 3, D 2, E 5

3. a) **Function ①:** domain: ℝ
   range: [-1, ∞[
   Function ②: domain:]$-$∞, 0[∪]0, ∞[
   range:]$-$∞, 0[∪]0, ∞[
   Function ③: domain: ℝ; range:]0, ∞[

   b) **Function ①:** increasing over the interval [-2, ∞[
   decreasing over the interval]$-$∞, -2]
   Function ②: decreasing over the interval
   ]$-$∞, 0[∪]0, ∞[
   Function ③: increasing in ℝ

   c) **Function ①:** positive over the interval
   ]$-$∞, -3] ∪ [-1, ∞[
   negative over the interval [-3, -1]
   Function ②: positive over the interval]0, ∞[
   negative over the interval]$-$∞, 0[
   Function ③: positive over ℝ

   d) **Function ①:** minimum of -1
   Function ②: none
   Function ③: none

   e) **Function ①:** 3
   Function ②: none
   Function ③: 1

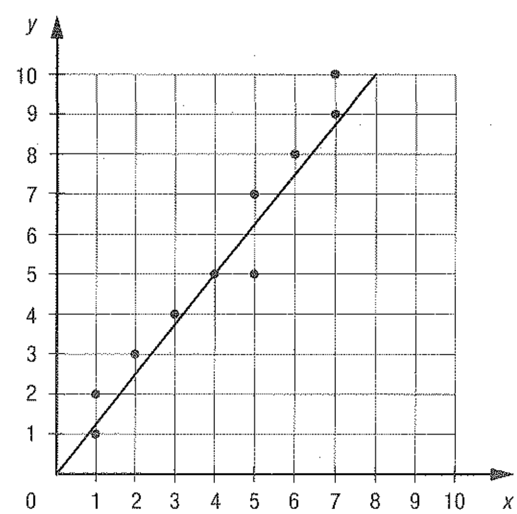
   f) **Function ①:** -3 and -1
   Function ②: none
   Function ③: none

Practice 1.1 (*cont'd*)

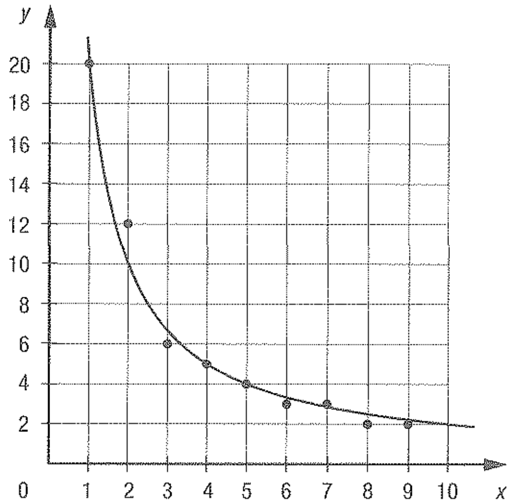
Page 19

4. a) **Graph ①:** First-degree polynomial function.
   Graph ②: Inverse variation function.
   Graph ③: Zero-degree polynomial function.
   Graph ④: First-degree polynomial function.

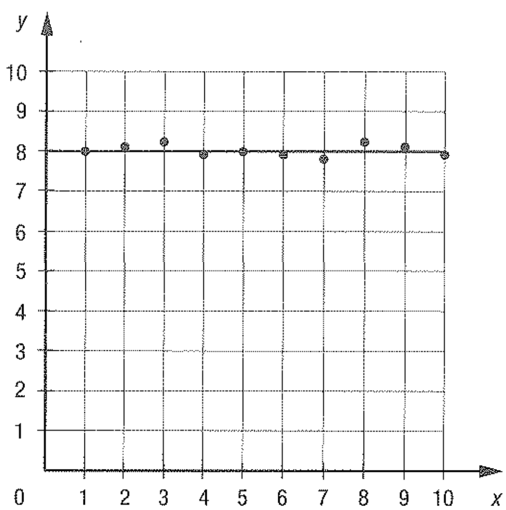
   b) **Graph ①**



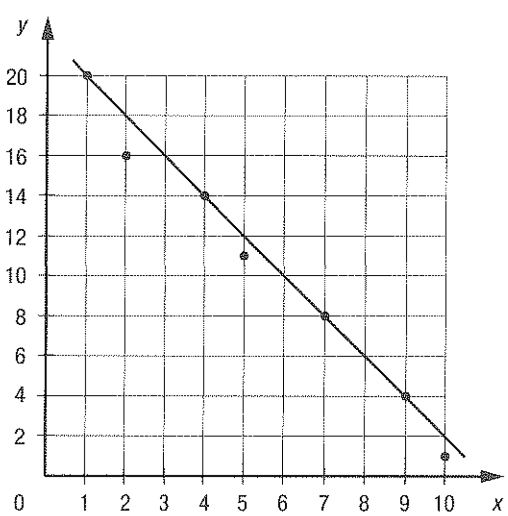
Graph ②



Graph ③



Graph ④



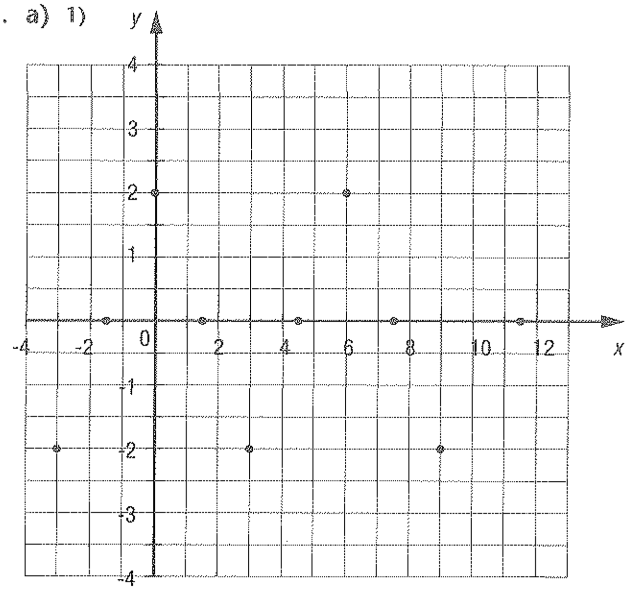
c) **Graph ①**: $y = \frac{5x}{4}$

Graph ②: $y = \frac{10}{x}$

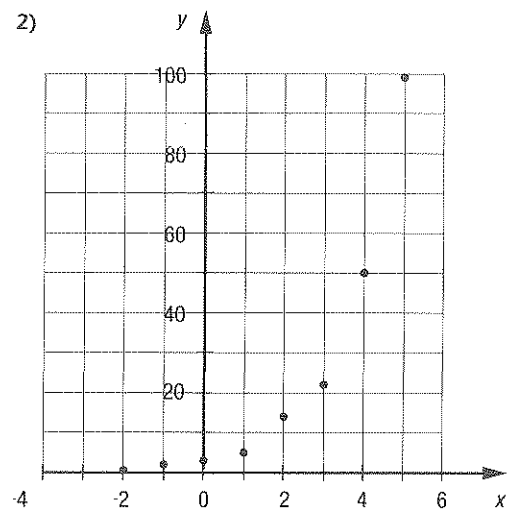
Graph ③: $y = 8$

Graph ④: $y = ^{-}x + 11$

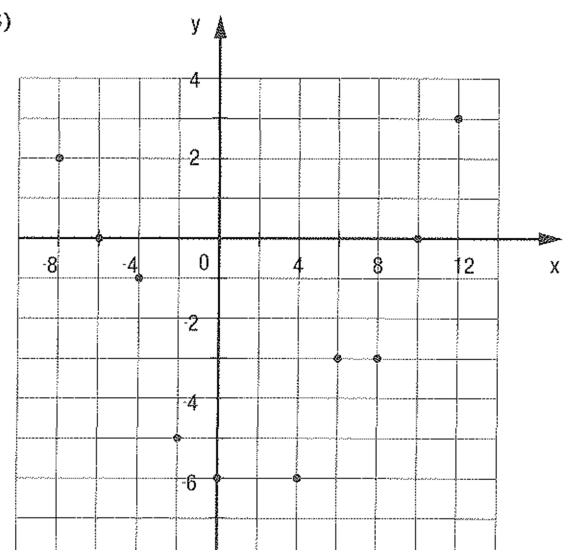
5. a) 1)



2)



3)



b) 1) Periodic function.

2) Exponential function.

3) Absolute value function.

Practice 1.1 (*cont'd*)

Page 20

6. a) The minimum share value was $20.

b) The share value was $40.

c) The share value increased over the interval [30, 90] days.

d) For approximately 40 days.

e) An absolute value function.

f) The share value will be approximately $243.

7. a) $5,250

b) $14,017.23

c) $35,571.74

Practice 1.1 (*cont'd*)

Page 21

8. a) A piecewise function.

b) No, since the graphical representation of the inverse shows that the *x*-coordinate 30 is associated with several data points.

c) The mean drilling speed was 5 m/h.

d) The maximum depth was 80 metres.

e) The drill took 2 hours to rise to the surface.

f) This mechanical failure lasted 2 hours.

9. a) The maximum number of customers was 45.

b) The number of customers increased during the first three hours.

c) At 6:00 p.m., there were 40 customers in this restaurant.

d) There were 25 customers in the restaurant at 5:00 p.m. and 9:00 p.m.

SECTION 1.2 Multiplicative parameters

Problem

Page 22

The aircraft will reach an altitude of 7500 m at the 25th second.

Activity 1

Page 23

a. A periodic function.

b. Signal ①: 24 volts.
Signal ②: 12 volts.

c. 1) The voltage is zero for the same time values.
2) The maximum and minimum voltages are different.

d. 1) The curves have the same zeros.
2) Their extrema are different.

e. Signal ①: 25 Hz.
Signal ③: 12.5 Hz.

f. 1) The maximum and minimum voltages are the same.
2) The frequency of Signal ① is double the frequency of Signal ③.

g. 1) The curves have the same extrema, and some of their zeros are identical.
2) The number of cycles for the curves in a given time is different.

Activity 2

Page 24

a. 1) The expression sin *x* was multiplied by 2.
2) The expression sin *x* was multiplied by 0.5.

b. 1) The curve underwent a vertical stretch.
2) The curve underwent a vertical compression.

c. If the number that multiplies the expression corresponding to the dependent variable of the function $f(x) = \sin x$ is greater than 1, the graph will undergo a vertical stretch. If this number is between 0 and 1, the graph will undergo a vertical compression.

d. 1) The independent variable *x* was multiplied by 2.
2) The independent variable *x* was multiplied by 0.5.

e. 1) The curve underwent a horizontal compression.
2) The curve underwent a horizontal stretch.

f. If the number that multiplies the expression corresponding to the independent variable of the function $f(x) = \cos x$ is greater than 1, the graph will undergo a horizontal compression. If this number is between 0 and 1, the graph will undergo a horizontal stretch.

g. 1) The expression 2^x was multiplied by -1.
2) The independent variable *x* was multiplied by -1.

h. 1) The curve underwent a reflection about the x-axis.
2) The curve underwent a reflection about the y-axis.

i. If the number that multiplies the expression corresponding to the dependent variable of the function $f(x) = 2^x$ is negative, the graph will undergo a reflection about the x-axis. If the number that multiplies the expression corresponding to the independent variable is negative, the graph will undergo a reflection about the y-axis.

Technomath — Page 25

a. 1) Parameter **a**.
2) Parameter **b**.

b. 1) The extrema of Y_1 are -1 and 1, whereas those of Y_2 are -4 and 4.
2) The zeros of Y_1 are -6.28, -3.14, 0, 3.14 and 6.28, whereas those of Y_2 are -6.28, 0 and 6.28.

c. This ratio is always approximately 4.

d. The extrema of Y_2 are four times greater than those of Y_1.

e. The table of values Ⓑ.

Practice 1.2 — Page 28

1. **a)** f_4 **b)** f_1
c) f_2 **d)** f_3

2. **a)** a = 3 b = 5
b) a = -2 b = 1
c) a = -1 b = 2
d) a = 7 b = 3
e) a = 6 b = -1
f) a = -1 b = 1
g) a = 0.5 b = 1
h) a = -7 b = 0.25
i) a = 9 b = 1

3. A 2, B 3, C 1, D 4

Practice 1.2 *(cont'd)* — Page 29

4. 1 D, 2 A, 3 C, 4 B

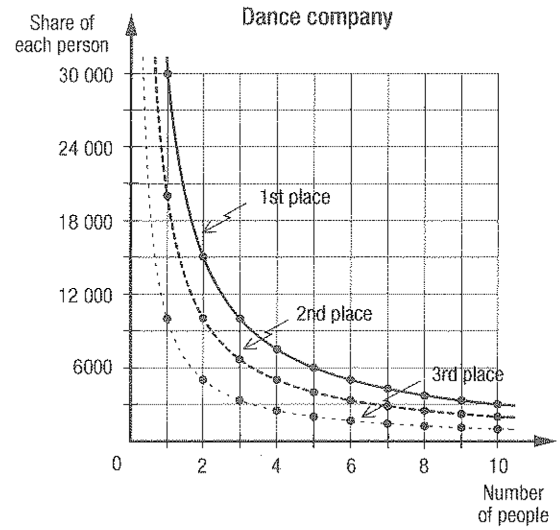
Practice 1.2 *(cont'd)* — Page 30

5. **a)** $g(x) = 4\sin x$ **b)** $g(x) = -0.25|x|$
c) $g(x) = -0.25[x]$ **d)** $g(x) = -2^{-x}$
e) $g(x) = -5x^2$ **f)** $g(x) = -\cos 2x$

Practice 1.2 *(cont'd)* — Page 31

6. **a)** 8 **b)** -0.5
c) -5 **d)** -2

7. **a)**



b) An inverse variation function.

c) *Several answers possible. Example:* A vertical stretch.

d) *Several answers possible. Example:* A vertical compression.

e) With parameter **a**, since the rule is of the form $f(x) = \frac{a}{x}$, where **a** represents the prize amount.

Practice 1.2 *(cont'd)* — Page 32

8. **a)** $g(x) = \cos 2x$ **b)** $g(x) = 0.5 \cos x$
c) $g(x) = -2 \cos x$

9. **a)** A second-degree polynomial function.
b) The curve underwent a horizontal compression since the zeros approached each other.
c) The curve underwent a horizontal stretch since the zeros moved away from each other.
d) A horizontal change in scale affects the length of time that the production is in the air.

Practice 1.2 (*cont'd*)

Page 33

10. a) A first-degree polynomial function.
    b) *Several possible answers. For example:* Parameter a.
    c) The value of the parameter corresponds to the speed of light in a vacuum approximately 299 792 458 m/s.
11. a) A vertical compression or a horizontal stretch.
    b) Yes, because by multiplying the y-coordinate of each point on the curve corresponding to Machine **B** by 2, one obtains the y-coordinate of each point on the curve corresponding to Machine **A**.
    c) The engineer is correct because Machine **B** takes four times longer than Machine **A** to reach the same temperature.
    d) Machine **A** because the temperature increases more quickly with this machine than with Machine **B**.

SECTION 1.3 Graphical modelling

Problem

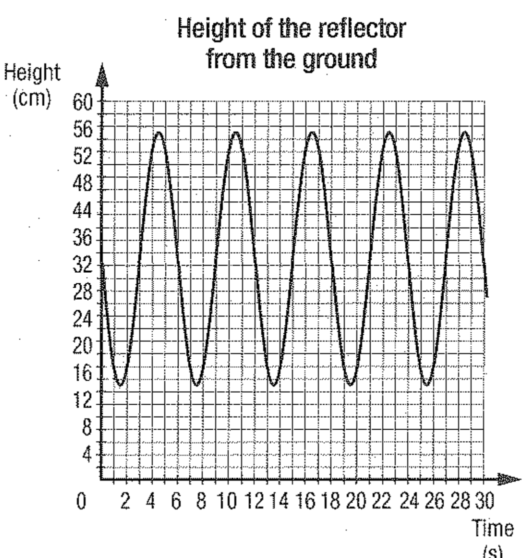
Page 34

During her training, Melissa covered a distance of approximately 1 920 m.

Activity 1

Page 35

a. *Several possible answers. For example:*



b. A periodic function.

c. Domain: [0, ∞[; range: [15, 55].

d. At 6 s, 12 s, 18 s, etc.

e. 1) The range and extrema.
   2) The variation.
   3) The variation.

Activity 2

Page 36

a. A piecewise function.

b. A zero-degree polynomial function for the first part, a first-degree polynomial function for the second and third parts.

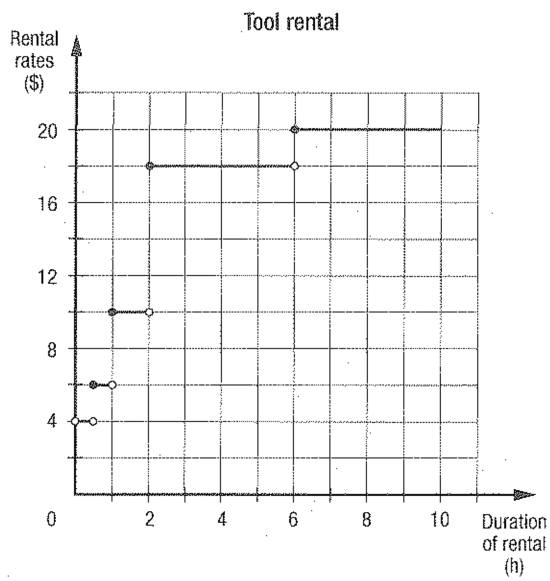
c. *Several answers possible. Example:* For temperatures above or equal to 80 K but less than or equal to 120 K.

d. *Several answers possible. Example:* No, since if the trend is maintained, electric resistance increases for temperatures above 120 K.

Activity 3

Page 37

a.



b. A step function.

c. Domain:]0, ∞[, range: {4, 6, 10, 18, 20, 40, 60, 80, ...}.

d. 1) $10   2) $18
   3) $60

e. Yes, if renting for less than 5 h and 2 min. For a rental of 5 h and 24 min, both companies offer the same rental rate, but for longer rental times the competitor offers a better rate.

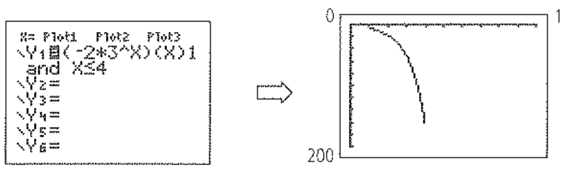
Technomath

Page 38

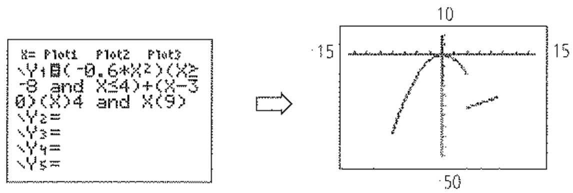
a. 1) Domain: [⁻3, 4[2) Range: [4,5, 8[

b. 1) Domain: [⁻8, 3] 2) Domain: [3, 6]
3) Domain: [⁻8, 6] 4) Range: [74, 64]

c. 1)



2)



b) 1) Domain: [0, 12[, range: [3, 11[.
2) Minimum: 3, maximum: 11.
3) constant over the interval [0, 4[, increasing over the interval [4, 8], decreasing over the interval [8, 12[.
4) 3
5) No zero.
6) Positive over the interval [0, 12[.

c) 1) 3 2) 11
3) 6.5 4) Undefined.

4. No, since a periodic function must return periodically to the same value of the dependent variable. The inverse therefore includes values of the independent variable associated with more than one value of the dependent variable.

5. a) The critical values indicate the times when new files were found.

b) The program found 4 files.

c) Domain:]0, 35] range: {0, 1, 2, 3, 4}.

d) $x \in$]0, 20[, meaning that no file was found during this time interval.

Practice 1.3

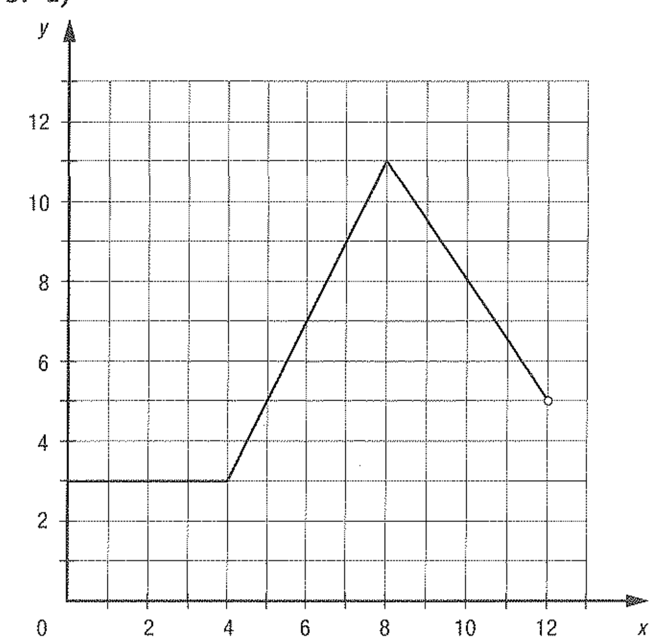
Page 40

1. a) Periodic function.
   b) Step function.
   c) Piecewise function.
   d) Periodic function.
2. a) Step function.
   b) Piecewise function.
   c) Periodic function.

Practice 1.3 (*cont'd*)

Page 41

3. a)



Practice 1.3 (*cont'd*)

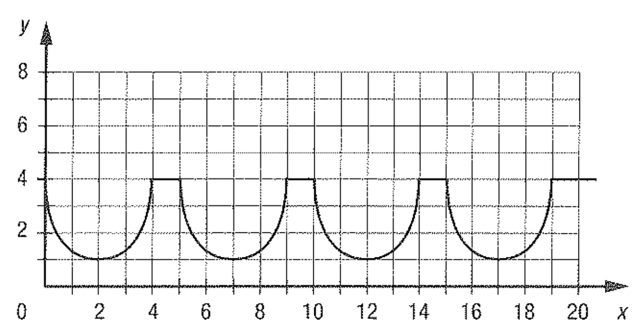
Page 42

6. a) Yes. The period is 2.5.
   b) No.
   c) No.
   d) Yes. The period is 2.
7. a) A periodic function.
   b) At the 5th day, the 10th day, the 15th day, etc.
   c) 1) On January 4.
   2) On May 5, a non-leap year, and on May 4, a leap year.

Practice 1.3 (*cont'd*)

Page 43

8. a) The period is 5.
   b) Domain: ℝ range: [1, 4].
   c)



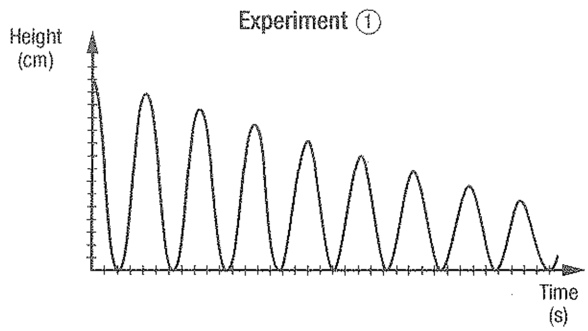
d) 1) No. 2) Yes.
3) No. 4) No.

9. a) Experiment ①: A periodic function.

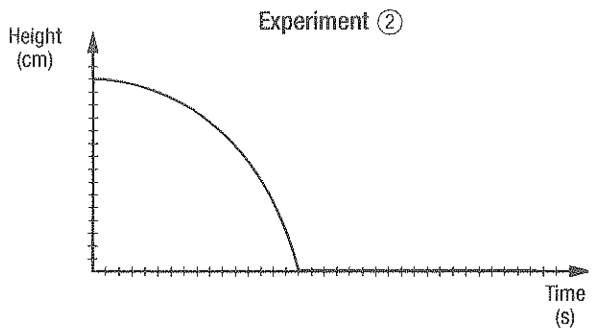
Experiment ②: A piecewise function.

Experiment ③: A piecewise function.

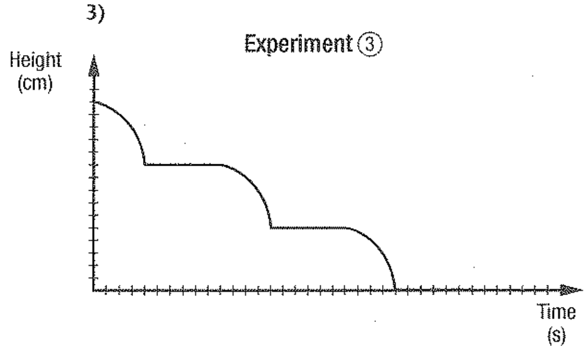
b) 1)



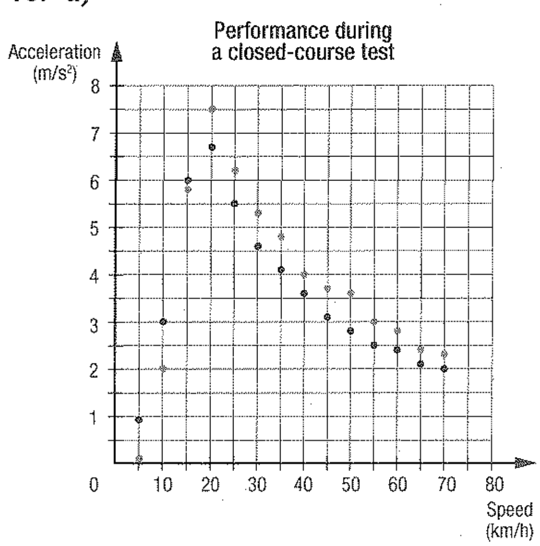
2)



3)



10. a)



b) A piecewise function in both situations.

c) 1) Vehicle **A**: ≈ 7.5 m/s^2
Vehicle **B**: ≈ 6.7 m/s^2

2) Vehicle **A**: 20 km/h
Vehicle **B**: 20 km/h

3) Vehicle **A**: ≈ 1.8 m/s^2
Vehicle **B**: ≈ 1.5 m/s^2

Practice 1.3 (*cont'd*) — Page 44

11. a) ≈ 0.54 V

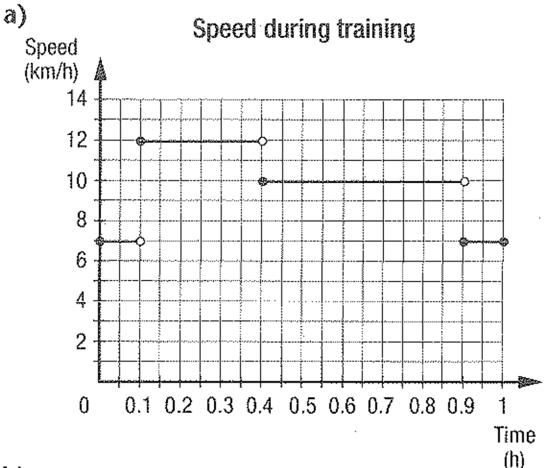
b) No, since this function is not symmetrical.

c) The period is approximately 25 s.

d) Parameter **b**.

Practice 1.3 (*cont'd*) — Page 45

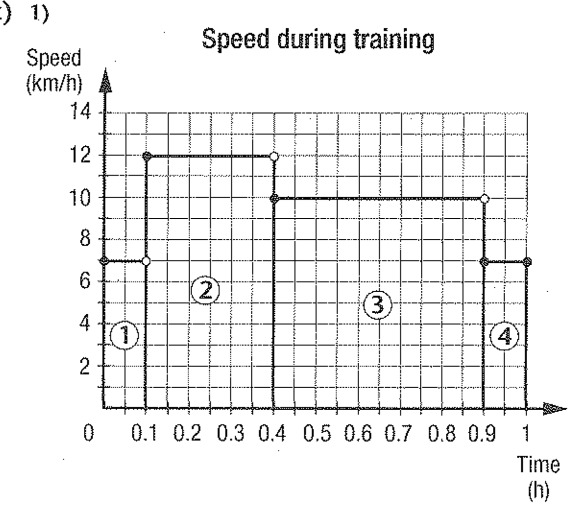
12. a)



b)

Time (h)	Distance covered (km)
[0, 0.1[	0.7
[0.1, 0.4[	3.6
[0.4, 0.9[	5
[0.9, 1]	0.7

c) 1)



2) Rectangle ① = 0.7 u^2 or 0.7 km.
Rectangle ② = 3.6 u^2 or 3.6 km.
Rectangle ③ = 5 u^2 or 5 km.
Rectangle ④ = 0.7 u^2 or 0.7 km.

3) The distance covered over each interval corresponds to the area of the rectangle below each horizontal segments.

d) 1) Jerome will have covered 5.3 km.
2) Jerome will have covered 10 km.

13. a) A piecewise function.

b) 1) 90°C
2) At the boiling point of water at this altitude.

c) The time during which the snow is melting.

d) The vertical distance between the summit and the camp is approximately 1810.9 m.

SPECIAL FEATURES 1

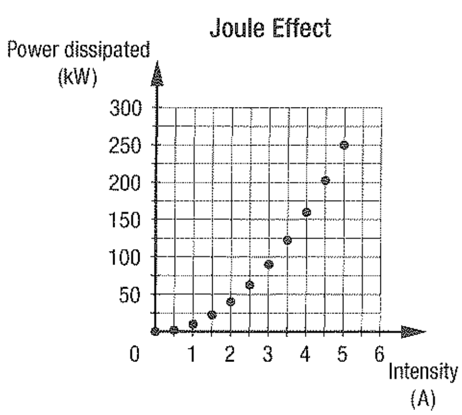
Chronicle of the past
Page 47

1. a) A first-degree polynomial function.

b) The intensity of the current is 8 A.

c) The scatter plot would undergo a horizontal compression meaning that the slope of the scatter plot would be steeper.

2. a)



b) A second-degree polynomial function.

c) The curve will undergo a vertical compression.

In the workplace
Page 49

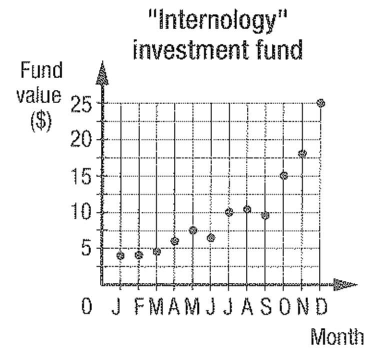
1. a) A step function.

b) A periodic function.

c) A first-degree polynomial function.

2. $0.14

3. a)



b) A second-degree polynomial function.

c) Approximately $35.

Overview
Page 50

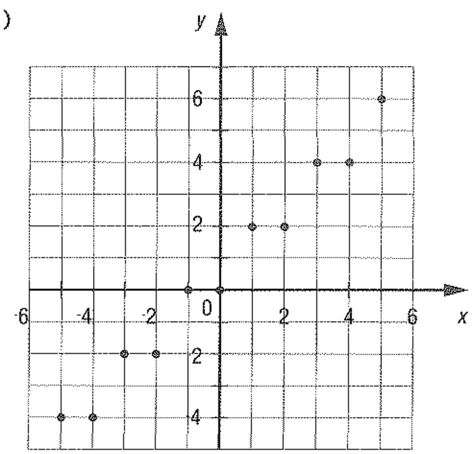
1. a) A periodic function.

b) An exponential function.

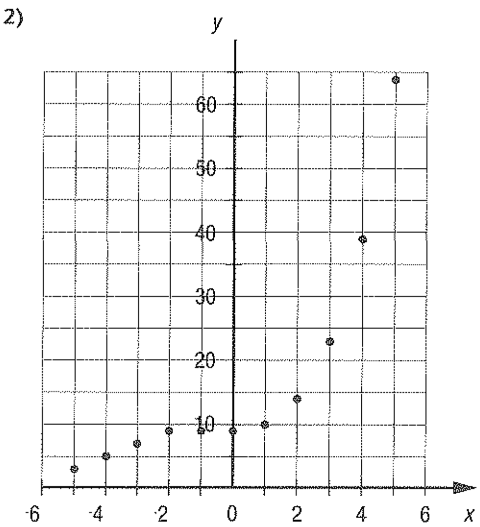
c) A second-degree polynomial function.

d) A first-degree polynomial function.

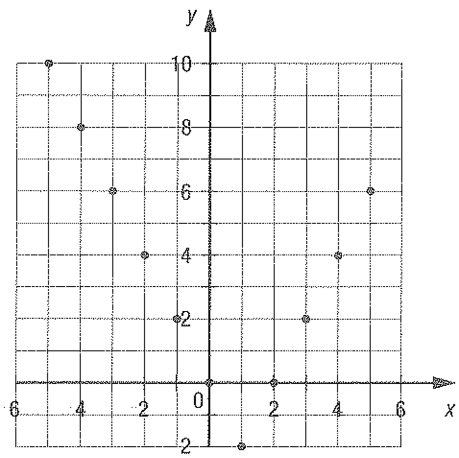
2. a) 1)



2)



3)



b) 1) Step function.
2) Piecewise function.
3) Absolute value function.

Overview (*cont'd*) **Page 51**

3. Exponential function.

4. **a)** 1) Graph ①: domain: ℝ
range: [-3, 3]
Graph ②: domain: ℝ
range:]$-\infty$, 6]
Graph ③: domain: ℝ
range: [-4, ∞[
Graph ④: domain:]$-\infty$, 3[∪]3, ∞[
range:]$-\infty$, 2[∪]2, ∞[

2) Graph ① {..., -8, -4, 0, 4, 8, ...}
Graph ②: -3 and 3
Graph ③: 2
Graph ④: 2

3) Graph ①: minimum: -3, maximum: 3
Graph ②: maximum: 6
Graph ③: no extrema
Graph ④: no extrema

4) Graph ①: 0
Graph ②: 6
Graph ③: -3
Graph ④: ≈ 1.5

b) Graph ①: A periodic function.
Graph ②: A second-degree polynomial function.
Graph ③: An exponential function.
Graph ④: An inverse variation function.

c) Graph ①: The inverse is not a function because several values of the independent variable are associated with more than one value of the dependent variable.

Graph ②: The inverse is not a function because several values of the independent variable are associated with more than one value of the dependent variable.

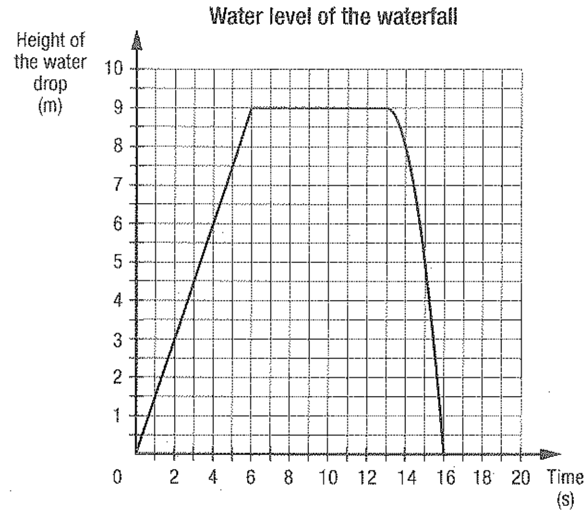
Graph ③: The inverse is a function because for each value of the independent variable only one value is associated with the dependent variable.

Graph ④: The inverse is a function because for each value of the independent variable only one value is associated with the dependent variable.

Overview (*cont'd*) **Page 52**

5. **a)** A piecewise function.

b)

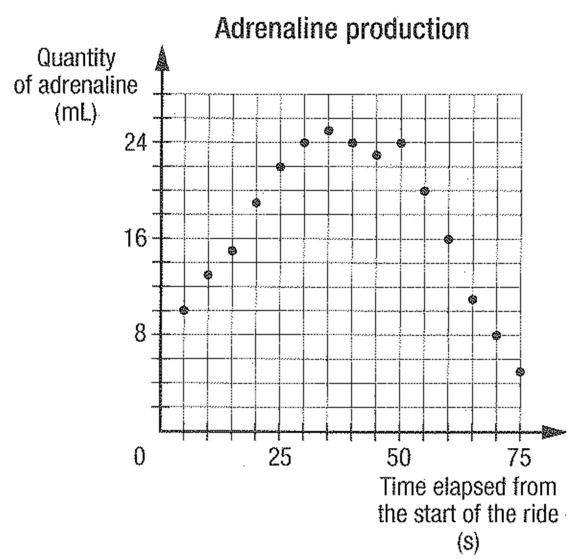


c) Domain: [0, 16] s, range: [0, 9] m.

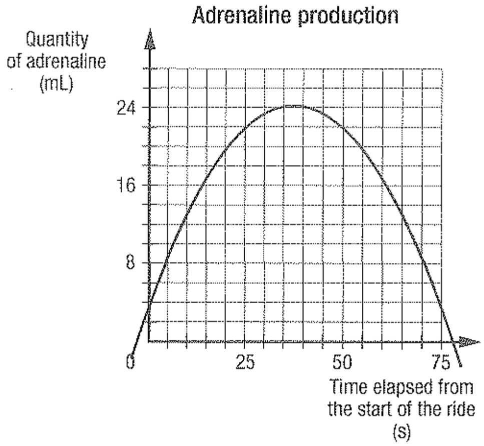
d) The zeros are 0 s and 16 s. They represent the times when the water drop is in the basin.

e) For 7 s.

6. **a)**



b)



c) Approximately 3.5 mg.

Overview (*cont'd*) — Page 53

7. a) 1) 2.5 2) -0.25 3) -2.5

   b) 1) -0.5 2) -1 3) 1.75

8. a) Graph ①: Purple: a: -2.5, b: -0.5
   Green: a: -1.75, b: 0.5
   Pink: a: 1, b: -1

   a) Graph ②: Purple: $a < 0$
   Green: $a > 0$
   Pink: $a < 0$

Overview (*cont'd*) — Page 54

9. ① B, ② D, ③ A, ④ C

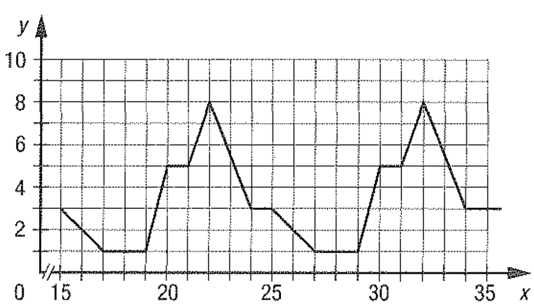
Overview (*cont'd*) — Page 55

10. a) 10

    b) 1) 1 2) 8 3) 2

    c) A maximum of 8.

    d)

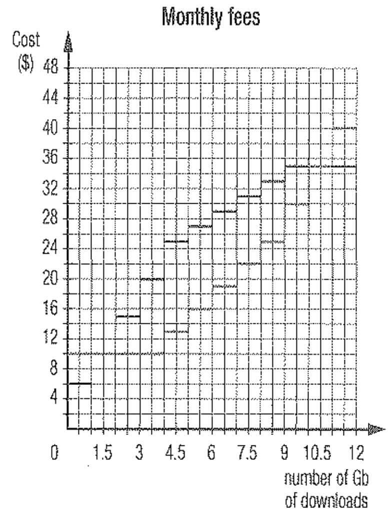


11. a) A second-degree polynomial function.

    b) The maximum height was 9 m.

    c) It will fall 6 s after being struck.

12. a)



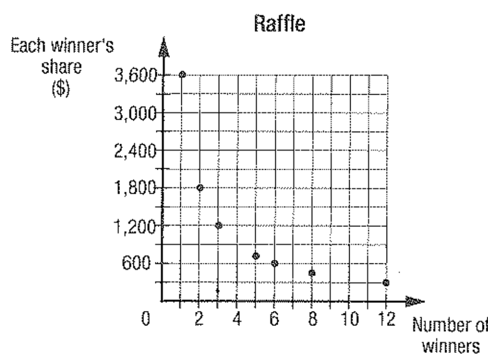
b) The two plans are best represented with a step function.

c) For 1 Gb or less, Monthly Plan **2** is the most advantageous. For downloads greater than 1 Gb and up to 2 Gb, and for downloads greater than 10 Gb without exceeding 11 Gb, the cost is the same for both monthly plans. For downloads greater than 2 Gb and up to 10 Gb, Monthly Plan **1** offers a better price. Finally, for downloads greater than 11 Gb, Monthly Plan **2** offers the best value.

Overview (*cont'd*) — Page 56

13. a) An amount of $3,600.

    b)



c) An inverse variation function.

d) Each person will receive $240.

14. a) The hare population decreased over approximately 180 days.

    b) The maximum population was 150 hares.

    c) There will be approximately 100 hares.

15. a) A step function.

b) 1) This person will pay \$8.
2) This person will pay \$8.

c) The people from 21 (included) to 50 (excluded) years of age.

Overview (*cont'd*) Page 57

16. a)

Braking Distance

Speed (km/h)	10	20	30	40	50	60	70	80	90	100
Distance on dry pavement (m)	0.6	2.4	5.4	9.6	15	21.6	29.4	38.4	48.6	60
Distance on wet pavement (m)	0.9	3.6	8.1	14.4	22.5	32.4	44.1	57.6	72.9	90
Distance on icy pavement (m)	4	16	36	64	100	144	196	256	324	400

b) Distance on dry paved road:
a = 0.006.
Distance on a wet paved road:
a = 0.009.
Distance on icy road:
a = 0.04.

c) An increase in the value of parameter **a** translates into a longer braking distance, and a decrease translates into a shorter braking distance.

17. a) Tempo 1: 9 cm
Tempo 2: 7.5 cm
Tempo 3: 6 cm

b) Tempo 2: *andante*
Tempo 3: *allegro*

Bank of problems Page 58

18.

Training flight

	Take-off phase	Flight phase	Descent phase	Landing phase
Duration (min)	6	10	5	4
Equation where x represents the time (in min) and y represents the height of the aircraft (in m)	$y = 100x$	$y = 600$	$y = ^-80x + 1880$	$y = ^-100\sqrt{x - 21} + 200$

19. Yes, since the two aircraft will be flying at the same altitude after approximately 10 min.

Bank of problems (*cont'd*) Page 59

20.

Cost related to painting

Sculpture	Radius (m)	Paint quantity (dL)	Cost (\$)
Sphere ❶	1	23	27.60
Sphere ❷	2	90	108
Sphere ❸	3	200	240
Sphere ❹	4	360	432
Sphere ❺	5	565	678
Sphere ❻	6	812.19	974.62
Sphere ❼	7	1105.48	1,326.58
Sphere ❽	8	1443.89	1,732.67
Total	–	4599.56	5,519.47

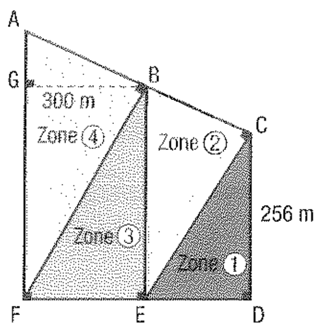
21. a = 2

b = 155

LES 3 | *Reforestation*

Below is an approach that allows you to produce a report where the total project cost is calculated and a diagram of the territory, where each type of tree is planted for each of the four zones, is constructed.

- Prove that the Triangles ①, ②, ③ and ④ are similar by using AA.



- Prove that Triangle ① is similar to Triangle ②:

  m ∠ CDE = m ∠ ECB = 90°
  ∠ ECD ≅ ∠ BEC, because when a transversal intersects two parallel lines, the alternate interior angles are congruent.

  So, Δ ① ~ Δ ②, since two triangles with two congruent corresponding angles are similar.

- Prove that the Triangle ② is similar to Triangle ③:

  m ∠ BCE = m ∠ FEB = 90°
  ∠ EBF ≅ ∠ BEC, because when a transversal intersects two parallel lines, the alternate interior angles are congruent.

  So, Δ ② ~ Δ ③, since two triangles with two congruent corresponding angles are similar.

- Prove that the Triangle ③ is similar to Triangle ④:

  m ∠ BEF = m ∠ FBA = 90°
  ∠ FBE ≅ ∠ AFB, because when a transversal intersects two parallel lines, the alternate interior angles are congruent.

  So, Δ ③ ~ Δ ④, since two triangles with two congruent corresponding angles are similar.

- Triangles ①, ②, ③ and ④ are all similar.

- Determine the ratio of the corresponding sides for triangles ② and ③, to be $\frac{5}{4}$.

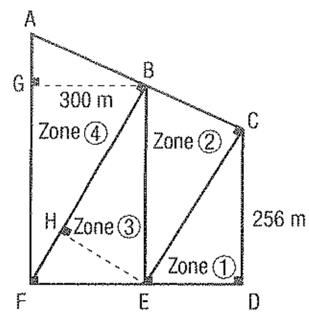
- Calculate m $\overline{BC}$:

  $\frac{5}{4} = \frac{\overline{FE}}{\overline{BC}}$

  $\frac{5}{4} = \frac{300}{\overline{BC}}$

  m $\overline{BC}$ = 240 m

- The length of segment BC can be relayed to the length of segment EH, the altitude from vertex E for Triangle ③.



- Calculate m $\overline{AB}$:

  $\frac{\overline{BC}}{\overline{EH}} = \frac{\overline{AB}}{\overline{FE}}$

  $\frac{300}{240} = \frac{\overline{AB}}{300}$

  m $\overline{AB}$ = 375 m

- Calculate m $\overline{AG}$ using Pythagorean theorem:
  m $\overline{AG}$ = 225 m

- Calculate m $\overline{GF}$:

  $\frac{\overline{AG}}{\overline{GB}} = \frac{\overline{GB}}{\overline{GF}}$

  $\frac{225}{300} = \frac{300}{\overline{GF}}$

  m $\overline{GF}$ = 400 m

- Determine measurements $\overline{BF}$, $\overline{BE}$, $\overline{EC}$ and $\overline{ED}$ using the Pythagorean theorem:

  m $\overline{BF}$ = 500 m, m $\overline{BE}$ = 400 m, m $\overline{EC}$ = 320 m, m $\overline{ED}$ = 192 m

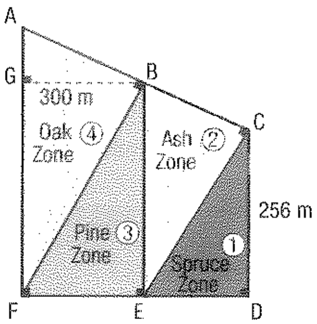
- Calculate the area of the 4 triangles:

  area of Δ ① = 24 576 m^2,
  area of Δ ② = 38 400 m^2,
  area of Δ ③ = 60 000 m^2,
  area of Δ ④ = 93 750 m^2.

- Determine the tree species to be used to reforest each zone.

Zone to be reforested	Tree Species	Number of trees	Cost per plant ($)	Total ($)
Zone ①	Spruce	2457	3	7,371
Zone ②	Ash	9600	2.5	24,000
Zone ③	Pine	7500	2.5	18,750
Zone ④	Oak	7812	4	31,250

The total cost of reforestation will be \$81,369 with a total of 27 369 trees.

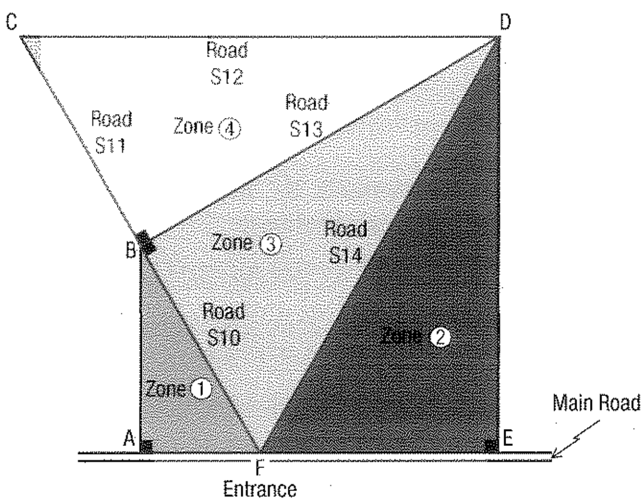


LBS 4 Minimizing travel

*Below is an approach that allows you to demonstrate that the length of Road **S13** is $10\sqrt{3}$ km.*

- Factor the algebraic expressions that represent the area of each zone to determine the possible lengths of the sides of each zone.

  Area of the Zone ①: $xy + 1.5x = \frac{b \times h}{2}$



  $2xy + 3x = b \times h$

  $x(2y + 3) = b \times h$

  Possible lengths for the sides of the right angle of triangle forming Zone ① is therefore x km and $(2y + 3)$ km.

  Area of the Zone ②: $30y + 45 - 2xy - 3x = \frac{b \times h}{2}$

  $$60y + 90 - 4xy - 6x = b \times h$$
  $$15(4y + 6) - x(4y + 6) = b \times h$$
  $$(4y + 6)(15 - x) = b \times h$$

  Possible lengths for the sides of the right angle of the triangle forming Zone ② is therefore $(4y + 6)$ km and $(15 - x)$ km.

- The triangles that form Zones ① and ② are similar by the hypothesis since the sum of the lengths of Roads **S10** and **S14** is minimal. Using the congruent corresponding sides of similar triangles, you can associate with the congruent sides the measurements of x km and $(15 - x)$ km, as well as the measurements $(2y + 3)$ km and $(4y + 6)$ km.
- Associate $(2y + 3)$ km to $\overline{AB}$ and $(4y + 6)$ km to $\overline{ED}$.
- The length of side AB is half the length of side ED. The ratio of the side lengths is 2:1 or 1:2.
- Associate x km to $\overline{AF}$ and $(15 - x)$ km to $\overline{FE}$.
- Calculate the value of x.

  $$\frac{1}{2} = \frac{\overline{AF}}{\overline{FE}}$$
  $$\frac{1}{2} = \frac{x}{15 - x}$$
  $$x = 5$$

  So, $\overline{AF} = 5$ km and $\overline{FE} = 10$ km.

- Prove, by AA, that the triangles forming Zone ① and ④ are similar.

  m ∠ FAB = m ∠ CBD = 90°
  ∠ AFB ≅ ∠ BCD, because when a transversal intersects two parallel lines, then the alternate interior angles are congruent.

  Therefore, Δ ① ~ Δ ④, since two triangles with two congruent angles are similar.

- By transitivity, Triangle ② is similar to the Triangle ④.
- Prove that Triangles ②, ③ and ④ are congruent by ASA.
- Prove that Triangle ③ is congruent to Triangle ④.

  m ∠ CBD = m ∠ FBD = 90°

  Segment BD is a common side to both triangles.

  m ∠ CBD = m ∠ FDB, since the bisector of an angle separates the angle into two congruent angles.

  Therefore, Δ ③ ≅ Δ ④, since two triangles with one congruent side contained between two congruent angles are congruent.

- Prove that Triangle ② is congruent to the Triangle ③.

  Triangle ② is similar to the Triangle ④.
  By transitivity, it is also similar to Triangle ③.

  DF is the side common to both triangles.

  Therefore Δ ② ≅ Δ ③, since two triangles with a congruent side contained between two congruent angles are congruent.
- Angles AFB, BFD and EFD are congruent. They all measure 60°.
- Triangle CDF is equilateral.
- If $\overline{FE}$ = 10 km, then S10 = 10 km and S11 = 10 km.
- The length of Roads **S12** and **S14** are each 20 km.
- Determine the road length **S13**: $10\sqrt{3}$ km or ≈ 17.32 km.

REVISION 2

Prior learning 1 — Page 62

a. The triangle is an isosceles.

b. The angle EGF is 21°.

c. 1) The area of the steel plate is approximately 170.76 cm^2.

2) The area for Section ① is approximately 75.89 cm^2.

3) The area for Section ② is approximately 18.97 cm^2.

4) The area of parallelogram BDFG is approximately 75.9 cm^2.

5) The area of Section ④ is approximately 5.82 cm^2. m $\overline{GE}$ ≈ 5.42 cm, m $\overline{EF}$ ≈ 2.15 cm.

Knowledge in action — Page 66

1. Ⓐ, Ⓔ, Ⓗ
2. a) 5 b) $3n$
   c) $3b$ d) $7p$
   e) 11 f) $6z$
3. a) $8m(m + 3)$ b) $18s^2(4s - 1)$
   c) $^-2t(7t^2 + 1)$ d) $ab(ab^2 + b - 1)$
   e) $2(y - 1)$ f) $3r(2r + 1)$
4. a) All angles of a square are right and its diagonals are congruent.
   b) The parallelogram has two pairs of parallel opposite sides, while a trapezoid only has one pair of parallel opposite sides. Also, the opposite sides of a parallelogram are congruent, while those of the trapezoid are not necessarily.
   c) The four sides of the rhombus are congruent.
   d) The isosceles trapezoid has two congruent diagonals.
5. ①, ④, ③, ②
6. The sum of the lengths of two sides of a triangle is always greater than length of the third side.

   y cm + $(4y + 1)$ cm < $(5y + 3)$ cm

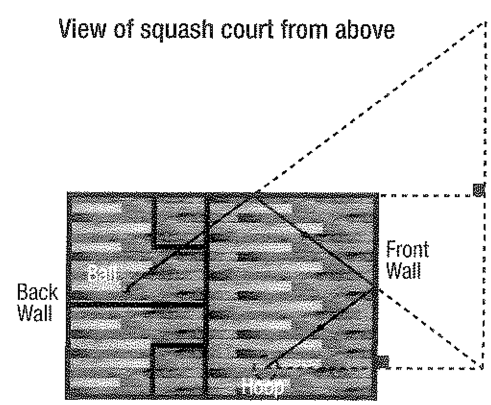
Knowledge in action (*cont'd*) — Page 67

7. a) The sum of the interior angles of a triangle is 180°.
   b) Vertically opposite angles are congruent.
   c) If a transversal intersects two parallel lines, then the corresponding angles are congruent.
   d) Adjacent angles forming a line are supplementary.
8. a) ≈ 1.37 cm^2 b) 8 cm^2
9. a) Isosceles Triangle. b) Isoangular Triangle.
10. ≈ 55.43 cm^2
11. m ∠ A = 91.5°, m ∠ B = 113.5°, m ∠ C = 113.5°, m ∠ D = 86.5°, m ∠ E = 135°

SECTION 2.1 Congruent triangles

Problem — Page 68

Several answers possible. Example:
The ball hits the left wall then the front wall. The angles formed by the trajectory of the ball hitting the left wall and the angle of the ball hitting the front wall are equal. The diagram could be as follows:



Activity 1 Page 69

a. This series of images are representative of a series of rotation.

b. 1) The corresponding side to side AB is side EF.
 2) The corresponding side to side BC is side FG.
 3) The corresponding side to side AC is side EG.

c. 1) The side EF measures 3 cm.
 2) The side FG measures 4 cm.
 3) The side EG measures 5 cm.

d. 1) The heights BD and FH are congruent.
 2) The perimeters are equal.
 3) The areas are equal.

Activity 2 Page 70

a. m ∠ A = 30°, m ∠ B = 120°, m ∠ P = 30°

b. 1) The length of segment CD is 100 m.
 2) m ∠ C = 120°, m ∠ D = 30°, m ∠ P = 30°

c. Triangles ABP and DCP are congruent.

d. You can only construct a single triangle.

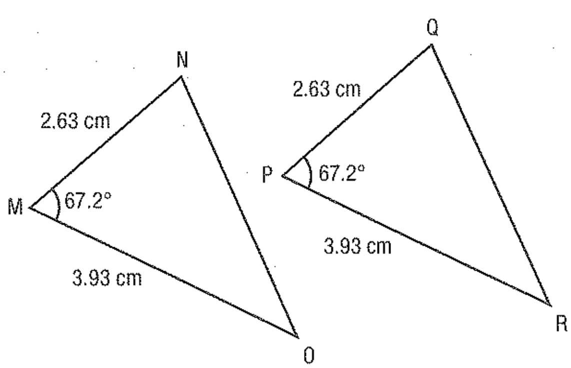
Technomath Page 71

a. 1) One side of triangle ABC is congruent to one side of the triangle DEF, m $\overline{CB}$ = m $\overline{FE}$; no angles of triangle ABC is congruent to any of the angles of triangle DEF.
 2) The three angles of triangle ABC are congruent to the three angles of triangle GHI.
 3) Two sides of the triangle ABC are congruent to two sides of triangle JKL; one of the angles of triangle ABC is congruent to one of the angles of triangle JKL.

b. No.

c. 1) No. 2) No.

d. 1) *Several solutions possible. Example:*

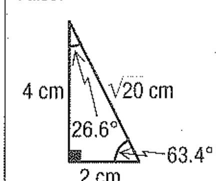
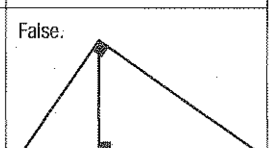


 2) These triangles are congruent only if the pairs of congruent sides are situated on either side of the pair of congruent angles.

Practice 2.1 Page 74

1. a) Two triangles with a congruent angle contained between corresponding congruent sides are congruent (SAS).
 b) Two triangles with corresponding congruent sides are congruent (SSS).
 c) Two triangles that have one congruent side contained between two congruent angles are congruent (ASA).
 d) Two triangles that have one congruent side contained between two congruent angles are congruent (ASA).
 e) Two triangles with corresponding congruent sides are congruent (SSS).
 f) Two triangles with a congruent angle contained between corresponding congruent sides are congruent (SAS).
 g) Two triangles that have one congruent side contained between two congruent angles are congruent (ASA).
 h) Two triangles with a congruent angle contained between corresponding congruent sides are congruent (SAS).

2.

	1) Hypothesis	Conclusion	2) Example or counter-example
a)	Double the length of the legs of a right triangle.	The length of the hypotenuse is doubled.	True. If $a^2 + b^2 = c^2$ then $(2a)^2 + (2b)^2 = (2c)^2$ $\Leftrightarrow 4a^2 + 4b^2 = 4c^2$ $\Leftrightarrow 4(a^2 + b^2) = 4c^2$ $\Leftrightarrow a^2 + b^2 = c^2$
b)	A right triangle where the height is equal to twice the length of its base.	One of the acute angles is twice the size of the other.	False. 
c)	Given a right triangle.	The altitude divides the triangle into two congruent triangles.	False. 

Practice 2.1 (*cont'd*) Page 75

3. a) 8 cm
 b) The angles AED and CEB are congruent since vertically opposite angles are congruent.

 The pair of sides AE and CE, and the pair of sides DE and BE are congruent by hypothesis.

Triangles AED and CEB are congruent because two triangles are congruent if they have a congruent angle contained between two corresponding congruent sides (SAS).

Corresponding elements of congruent figures or solids have the same measurements.

4. a)

STATEMENT	JUSTIFICATION
$\overline{AE} \cong \overline{AB}$	For any regular polygon, all the sides are congruent.
$\overline{ED} \cong \overline{BC}$	For any regular polygon, all the sides are congruent.
$\angle AED \cong \angle ABC$	For any regular polygon all the interior angles are congruent.
$\Delta ADE \cong \Delta ACB$	Two triangles are congruent if they have a congruent angle contained between two corresponding congruent sides (SAS).

b)

STATEMENT	JUSTIFICATION
$m\angle ABC = 180°$	The measurement of an interior angle of a regular polygon equals $\frac{180 \times (n-2)}{n}$, where $n = 5$.
ΔABC is isosceles	$\overline{AB}$ and $\overline{BC}$ are congruent since they are sides of a regular pentagon and for any given regular polygon, all sides are congruent.
$\angle BAC \cong \angle BCA$	In an isosceles triangle, the angles opposite of the two congruent sides are congruent.
$m\angle BAC = 36°$	The sum of the interior angles of any given triangle is 180°.
$m\angle EAB = 108°$	The measurement of an interior angle of a regular polygon equals $\frac{180 \times (n-2)}{n}$, where $n = 5$.
$\angle EAD \cong \angle BCA$	The corresponding angles of congruent triangles are congruent.
$m\angle DAC = m\angle EAB - m\angle EAD - m\angle BAC$	Adjacent angles.
$m\angle DAC = 108° - 36° - 36°$	By substitution.
$m\angle DAC = 36°$	By calculations.

5. a) 1) The opposite sides of a parallelogram are congruent.

2) If a line intersects two parallel lines, then the alternate interior angles are congruent.

3) Two triangles with congruent corresponding sides are congruent (SSS) or two triangles that have an congruent angle contained between corresponding congruent sides are congruent (SAS) or two triangles with one corresponding side contained between congruent angles are congruent (ASA).

b) A rotation.

6. ≈ 6.93 cm

7.

Hypothesis:	Point C is the midpoint of segments AE and BD.
Conclusion:	$\overline{AB} // \overline{DE}$

STATEMENT	JUSTIFICATION
$\overline{AC} \cong \overline{CE}$	Point C is the midpoint of segment AE, by hypothesis.
$\angle ACB \cong \angle DCE$	These angles are vertically opposite, therefore congruent.
$\overline{BC} \cong \overline{CD}$	Point C is the midpoint of segment BD, by hypothesis.
$\Delta ABC \cong \Delta CDE$	Two triangles are congruent if there is a congruent angle contained between two corresponding congruent sides (SAS).
$\angle BAC \cong \angle CED$	In congruent triangles, corresponding angles are congruent.
$\overline{AB} // \overline{DE}$	If alternate interior angles are congruent then the lines intersected by a transversal are parallel.

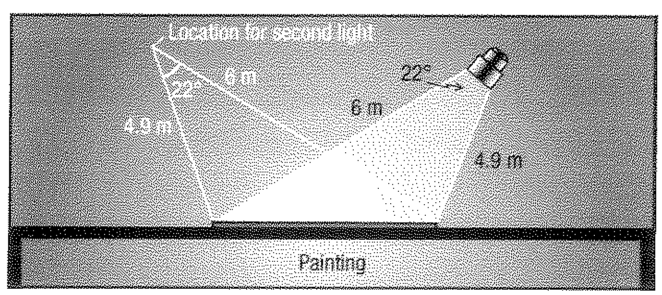
Practice 2.1 (*cont'd*) Page 76

8.

Hypothesis:	Point O is the midpoint of segment $\overline{MN}$, and lines MP and NQ are parallel.
Conclusion:	Point O is the midpoint of segment $\overline{PQ}$.

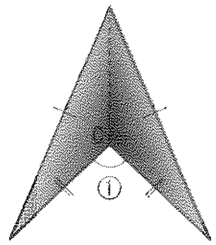
STATEMENT	JUSTIFICATION
$\overline{MO} \cong \overline{NO}$	Definition of "midpoint" and by hypothesis.
$\angle PMO \cong \angle QNO$	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
$\angle MOP \cong \angle NOQ$	These angles are vertically opposite, therefore congruent.
$\Delta MOP \cong \Delta NOQ$	Two triangles are congruent if there is a congruent angle contained between two corresponding congruent sides (SAS).
$\overline{PO} \cong \overline{QO}$	In congruent triangles, corresponding sides are congruent.
Point O is the midpoint of $\overline{PQ}$.	Definition of "midpoint."

9.



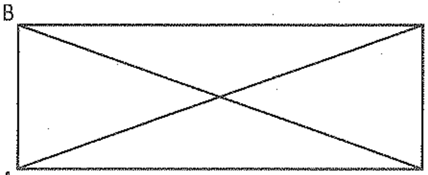
10. a) Two triangles are congruent when their corresponding sides are congruent (SSS).

b)



STATEMENT	JUSTIFICATION
Extend segment AC and create point E.	—
$m\angle BCE = m\angle BAC + m\angle ABC$	The measure of an exterior angle of a triangle is equal to the sum of the interior angles that are not adjacent to it.
$m\angle BCE = 180° - m\angle BCA$	Adjacent angles where the exterior sides form a straight line, are supplementary.
$m\angle DCE = 180° - m\angle DCA$	Adjacent angles where the exterior sides form a straight line, are supplementary.
$m\angle BCA = m\angle DCA$	Corresponding elements of plane figures or solid congruent figures have the same measurements.
$m\angle BCE = \frac{m\angle BCD}{2}$	By substitution.
$m\angle BAC = \frac{m\angle BAD}{2}$	Corresponding elements of plane figures or solid congruent figures have the same measurements.
$m\angle BAC = \frac{m\angle BCD}{4}$	By substitution and by hypothesis.
$\frac{m\angle BCD}{2} = \frac{m\angle BCD}{4} + m\angle ABC$	By substitution in the first equation.
$\frac{m\angle BCD}{4} = m\angle ABC$	By calculations.
$m\angle ABC = m\angle BAC$	By transitivity.
Δ ABC is an isosceles triangle	In any isosceles triangle, the angles opposite to the corresponding congruent sides are congruent.
Δ ACD is an isosceles triangle	Corresponding elements of plane figures or solid congruent figures have the same measurements.

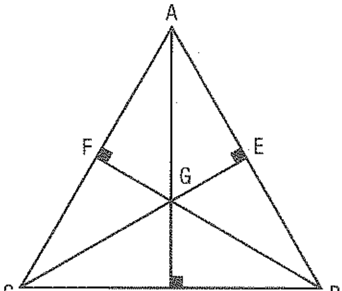
11. a) *Several answers possible. Example:*



Hypothesis:	The quadrilateral ABCD is a rectangle and segments BD and CA are diagonals.
Conclusion:	Triangles ACD and ACB are congruent as are triangles BCD and DAB.

STATEMENT	JUSTIFICATION
$\overline{BC} \cong \overline{DA}$	For any given rectangle, the pairs of opposite sides are congruent.
$\overline{AB} \cong \overline{CD}$	The opposite sides of a rectangle are congruent.
$\overline{AC} \cong \overline{AC}$	Any side is congruent to itself.
Δ ACD ≅ Δ ACB	Two triangles with corresponding congruent sides are congruent (SSS).
$\overline{BD} \cong \overline{BD}$	Any side is congruent to itself.
Δ BCD ≅ Δ DAB	Two triangles with corresponding congruent sides are congruent (SSS).

b)



Hypothesis:	ABC is an equilateral triangle and segments AD, CE and BF are the three altitudes of this triangle.
Conclusion:	The six triangles are congruent, two by two.

STATEMENT	JUSTIFICATION
$\overline{CF} \cong \overline{AF}$	The axis of symmetry of an isosceles triangle is a median, a perpendicular bisector, a bisector and an altitude of this triangle.
∠ CFG ≅ ∠ AFG	The supplementary angle to a right angle is also a right angle.
$\overline{FG} \cong \overline{FG}$	Reflexivity.
Δ CFG ≅ Δ AFG	Two triangles with a congruent angle contained between two corresponding congruent sides are congruent (SAS).
Similarly, you can demonstrate that Δ AEG ≅ Δ BEG and Δ BDG ≅ Δ CDG.	
$\overline{AE} \cong \overline{AF}$	The axis of symmetry of an isosceles triangle is a median, a perpendicular bisector, a bisector and altitude of this triangle. In an equilateral triangle, there are three axes of symmetry considering that there are three congruent sides.
∠ AEG ≅ ∠ AFG	The altitudes of a triangle.
$\overline{AG} \cong \overline{AG}$	Any side is congruent to itself.
Δ AEG ≅ Δ AFG	Two triangles with a congruent angle contained between two corresponding congruent sides are congruent (SAS).
Similarly, you can demonstrate that Δ BDG ≅ Δ BEG and Δ CFG ≅ Δ CDG.	
The six triangles are congruent, by transitivity.	

12.

Hypothesis:	• Segment BD is the bisector of angle ADC. • $\overline{AD} \cong \overline{CD}$
Conclusion:	$\overline{AB} \cong \overline{BC}$

STATEMENT	JUSTIFICATION
$\overline{AD} \cong \overline{CD}$	By hypothesis.
$\angle ADB \cong \angle BDC$	Since, by hypothesis, segment BD is the bisector of angle ADC it separates the angle ADC into two congruent angles.
$\overline{BD} \cong \overline{BD}$	This side is congruent to itself (reflexivity).
$\Delta ABD \cong \Delta BCD$	Two triangles with a congruent angle contained between two corresponding congruent sides are congruent (SAS).
$\overline{AB} \cong \overline{BC}$	In congruent triangles, the corresponding sides are congruent.

Practice 2.1 (*cont'd*)

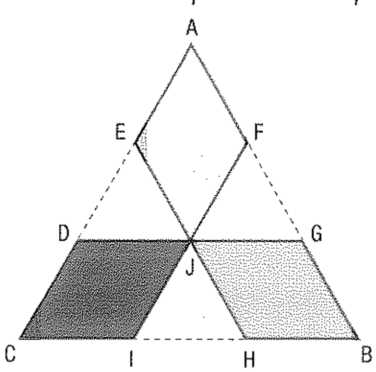
Page 77

13. a)

STATEMENT	JUSTIFICATION
$\overline{AD} \cong \overline{CD}$	By hypothesis.
$\angle ADB \cong \angle CDB$	The supplementary angle to a right angle is also a right angle.
$\overline{BD} \cong \overline{BD}$	Any side is congruent to itself.
$\Delta ABD \cong \Delta BCD$	Two triangles with a congruent angle contained between two corresponding congruent sides are congruent (SAS).

b) The observer must place the hammer at a distance of $\sqrt{4332}$ cm or ≈ 65.82 cm.

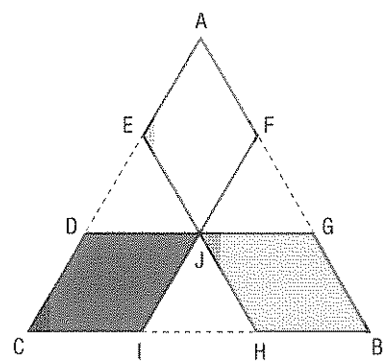
14. a) *Several answers possible. Example:*



Hypothesis:	AEJF, BGJH and CIJD are congruent rhombuses.
Conclusion:	Triangles EDJ, FJG and IJH are congruent equilateral triangles.

STATEMENT	JUSTIFICATION
$\angle EJF \cong \angle IJH$	Vertically opposite angles are congruent.
$\angle EJD \cong \angle GJH$	Vertically opposite angles are congruent.
$\angle FJG \cong \angle DJI$	Vertically opposite angles are congruent.
$\angle EJF \cong \angle GJH \cong \angle DJI$	Corresponding sides of congruent rhombuses are congruent.
$\overline{EJ} \cong \overline{FJ} \cong \overline{GJ} \cong$ $\overline{HJ} \cong \overline{IJ} \cong \overline{DJ}$	Congruent sides of a rhombus.
$\Delta EDJ \cong \Delta FGJ \cong \Delta HIJ$	Two triangles with a congruent angle contained between two corresponding congruent sides are congruent (SAS).
Triangles EDJ, FJG and JIH are isosceles.	Definition of an "isosceles triangle."
$m\angle EJD = m\angle GJF =$ $m\angle HJI = 60°$	Six congruent angles of 60° form an angle of of 360°.
$m\angle EDJ = m\angle DEJ =$ $m\angle FGJ = m\angle GFJ =$ $m\angle HIJ = m\angle IHJ = 60°$	In any given isosceles triangle, the angles opposing the congruent sides are congruent and the sum of the measures of the interior angles of a triangle is 180°.
Triangles EDJ, FJG and IJH are equilateral	A triangle with three congruent sides is called an equilateral triangle.

b) *Several possible answers. Example:*



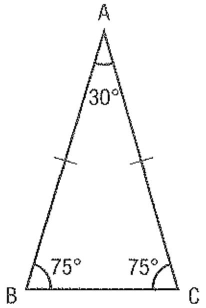
Hypothesis:	Quadrilaterals AEJF, BGJH and CIJD are congruent rhombuses.
Conclusion:	Triangle ABC is an equilateral triangle.

STATEMENT	JUSTIFICATION
$\overline{AE} \cong \overline{DC} \cong \overline{CI} \cong \overline{HB} \cong$ $\overline{BG} \cong \overline{FA}$	Corresponding sides of a congruent rhombuses are congruent.
$\overline{ED} \cong \overline{IH} \cong \overline{GF}$	Corresponding elements of plane figures or congruent solids have the same measurements.
$\overline{AC} \cong \overline{CB} \cong \overline{AB}$	The segments are formed from congruent segments.
Δ ABC is equilateral	A triangle with three congruent sides is called an equilateral triangle.

Practice 2.1 (*cont'd*)

15. a) The conjecture is false, an appropriate counter-example would be:

Several answers possible. Example:

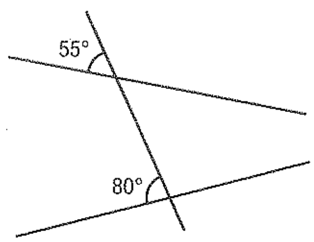


b) The conjecture is false, an appropriate counter-example would be:

Several answers possible. Example:
The diagonals of a parallelogram are congruent.

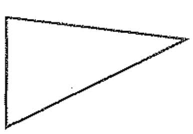
c) The conjecture is false, an appropriate counter-example would be:

Several answers possible. Example:



d) The conjecture is false, an appropriate counter-example would be:

Several answers possible. Example:

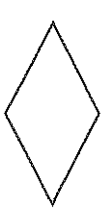


A scalene triangle without an obtuse angle.

e) The conjecture is true.

f) The conjecture is false, an appropriate counter-example would be:

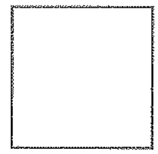
Several answers possible. Example:



A rhombus.

g) Conjecture is false, an appropriate counter-example would be:

Several answers possible. Example:



A square.

16.

STATEMENT	JUSTIFICATION
$\overline{AC} \cong \overline{AB}$	The triangle is isosceles because the angles formed at the base are congruent.
$\overline{DB} \cong \overline{EC}$	By hypothesis and $\overline{AC} \cong \overline{AB}$.
$\overline{BC} \cong \overline{CB}$	This side is congruent to itself.
$\angle ABC \cong \angle ACB$	By hypothesis.
$\Delta DBC \cong \Delta ECB$	Two triangles are congruent if they have a congruent angle contained between two corresponding congruent sides (SAS).
$\overline{CD} \cong \overline{BE}$	Corresponding congruent sides of congruent triangles are congruent.

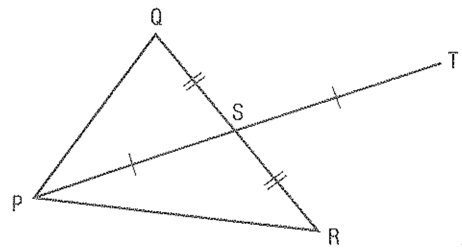
17. The angles formed by the hands are not congruent on the two clocks. On the clock showing 12:15, the hour hand has turned a quarter of the distance between 12 and 1, while on the clock reading 3:30, the hour hand has moved half the distance between 3:00 and 4:00.

18. No, triangles EFG and LMN are not congruent.

Triangle EFG	Triangle LMN
If the volume of the pyramid equals $\frac{20\ 000}{3}$ cm³ and if the height (i.e the sement EG) equals 10 cm, then: Volume $= \frac{20\ 000}{3}$ $\frac{\text{area of the base} \times \text{height}}{3} = \frac{20\ 000}{3}$ Area of the base $\times$ height $= 20\ 000$ Area of the base $\times$ 10 $= 20\ 000$ Area of the base $= 2000$ m $\overline{DC} = \sqrt{2000}$ m $\overline{DC} = 20\sqrt{5}$ m $\overline{GF} = 20\sqrt{5}$ m $\overline{GF} = 10\sqrt{5}$ m $\overline{EF}^2 =$ m $\overline{EG}^2 +$ m $\overline{GF}^2$ m $\overline{EF}^2 = 10^2 + (10\sqrt{5})^2$ m $\overline{EF}^2 = 600$ m $\overline{EF} = 10\sqrt{6}$ The measurements of the sides of triangle EFG are 10 cm, $10\sqrt{5}$ cm and $10\sqrt{6}$ cm.	If m $\angle NML = 30°$ and m $\angle LNM = 90°$, therefore the triangle LMN is a triangle 30° – 60° – 90°. Because segements LM measures 20 cm segment LN measures 10 cm, since in a 30° – 60° – 90°, triangle the measurement of the side opposite 30° angle is equal to half the length of the hypotenuse. m $\overline{LM}^2 =$ m $\overline{LN}^2 +$ m $\overline{NM}^2$ $20^2 = 10^2 +$ m $\overline{NM}^2$ $400 = 100 +$ m $\overline{NM}^2$ $300 =$ m $\overline{NM}^2$ $10\sqrt{3} =$ m $\overline{NM}$ The sides of the triangle therefore measure $10\sqrt{3}$ cm, 10 cm and 20 cm.

Practice 2.1 (*cont'd*) Page 79

19.

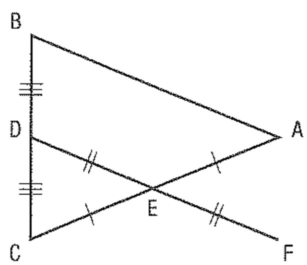


Hypothesis:	• $\overline{PS} \cong \overline{ST}$ • $\overline{QS} \cong \overline{SR}$
Conclusion:	The quadrilateral PQTR is a parallelogram.

STATEMENT	JUSTIFICATION
Parallelogram PQTR is a quadrilateral.	A quadrilateral is a plane figure formed by broken lines.
$\overline{QS} \cong \overline{SR}$	By hypothesis.
$\overline{PS} \cong \overline{ST}$	By hypothesis.
Point S is the midpoint of segment $\overline{QR}$ and the midpoint of segment $\overline{PT}$.	By hypothesis.
Quadrilateral PQTR is a parallelogram.	The diagonals of a parallelogram intersect at their midpoints.

20. The minimum width of the case is approximately 40.08 cm.

21. a)



Hypothesis:	$\overline{AE} \cong \overline{EC}$, $\overline{BD} \cong \overline{DC}$, $\overline{DE} \cong \overline{EF}$
Conclusion:	$\overline{AF} \cong \overline{CD}$

STATEMENT	JUSTIFICATION
$\overline{DE} \cong \overline{EF}$	By hypothesis.
$\overline{AE} \cong \overline{EC}$	By hypothesis.
$\angle AEF \cong \angle CED$	Vertically opposite angles are congruent.
$\Delta AEF \cong \Delta DBC$	Two triangles are congruent if they have a congruent angle contained between two corresponding congruent sides (SAS).
$\overline{AF} \cong \overline{CD}$	Corresponding sides of congruent triangles are congruent.

c)

Hypothesis:	• ABC is a triangle • m $\overline{CD}$ = m $\overline{BD}$ • m $\overline{CE}$ = m $\overline{AE}$
Conclusion:	$\overline{DE}$ is parallel to $\overline{BA}$ and m $\overline{DE} = \frac{1}{2} \times$ m $\overline{BA}$.

STATEMENT	JUSTIFICATION
m $\overline{CB}$ = m $\overline{CD}$ + m $\overline{BD}$ = 2 × m $\overline{CD}$ $\frac{m\overline{CD}}{m\overline{CB}} = \frac{1}{2}$	By hypothesis.
m $\overline{CA}$ = m $\overline{CE}$ + m $\overline{AE}$ = 2 × m $\overline{CE}$ $\frac{m\overline{CE}}{m\overline{CA}} = \frac{1}{2}$	By hypothesis.
$\angle DCE \cong \angle BCA$	Superimposed angles.
$\Delta CED \sim \Delta CAB$	Two triangles are similar if the corresponding sides are proportional.
$\angle EDC \cong \angle ABC$	When two triangles are similar, corresponding angles are congruent (SAS).
$\overline{DE}$ is parallel to $\overline{BA}$	If two corresponding angles are congruent, they are formed by a transversal intersecting two parallel lines.
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{1}{2}$ m $\overline{DE} = \frac{1}{2} \times$ m $\overline{BA}$	Since $\Delta CED \cong \Delta CAB$ by SAS, the ratio of all corresponding sides is $\frac{1}{2}$.

22. a) 1 triangle. **b)** 1 triangle.

SECTION 2.2 Similar triangles

Problem Page 80

Several answers possible. Example:
The maximum dimensions of the sound box is 175 cm, 175 cm and 210 cm.

Activity 1 Page 81

a. Segments AE and BD are parallel.

b. If a transversal cuts two parallel lines then the corresponding angles are congruent.

c. Angle ACE and angle BCD are congruent, because the angle is common to both triangles ACE and BCD.

d. The ratio of similarity is 8.

e. The spotlight is set up at 4 m from the wall.

Activity 2

Page 82

a. 1) Line l_3 was moved downwards.

2) The angle between the transversals was enlarged.

3) The slope of lines l_1, l_2 and l_3 were modified.

b.

Screen	1) $\frac{m\overline{DE}}{m\overline{AB}}$	2) $\frac{m\overline{EF}}{m\overline{BC}}$
1	$\frac{15}{14}$	$\frac{15}{14}$
2	$\frac{15}{14}$	$\frac{15}{14}$
3	$\frac{10}{7}$	$\frac{10}{7}$
4	$\frac{3}{2}$	$\frac{3}{2}$

c. *Several possible answers. For example:*
When three parallel lines are intersected by two transversals, the segments formed on the transversals are proportional.

Technomath

Page 83

a. 1) 3 pairs of angles. 2) None.

3) 1 pair of angles.

b. 1) $\frac{10.0}{5.0} = \frac{12.0}{6.0} = \frac{6.0}{3.0} = 2$

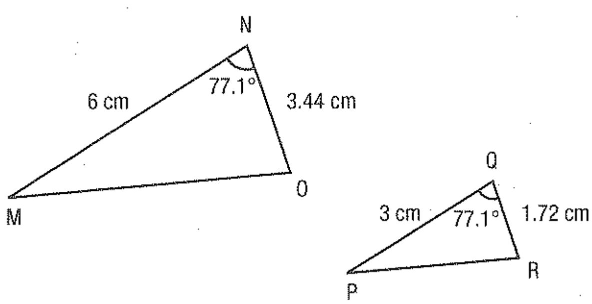
2) $\frac{7.5}{5.0} = \frac{4.5}{3.0} = 1.5$

3) $\frac{6.0}{3.0} = \frac{12.0}{6.0} = 2$

c. Triangles ABC and DEF are similar.

d. 1) No. 2) No.

e. 1) *Several possible answers. For example:*



2) Only if the pairs of proportional sides are located on either side of the pair of congruent angles.

Practice 2.2

Page 85

1. a) Two triangles whose corresponding sides are proportional are similar (SSS).

b) Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar (SAS).

c) Two triangles that have two congruent corresponding angles are similar (AA).

2. a) Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar (SAS).

b) Two triangles that have two congruent corresponding angles are similar (AA).

c) Two triangles whose corresponding sides are proportional are similar (SSS).

d) Two triangles that have two congruent corresponding angles are similar (AA).

e) Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar (SAS).

f) Two triangles whose corresponding sides are proportional are similar (SSS).

3. Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar (SAS).

Practice 2.2 *(cont'd)*

Page 86

4. ≈ 6.64 m

5. The perimeter is 27.6 cm.

6. a) $x = 3$ cm and $y = 2.8$ cm.

b) $x = 2.8$ cm and $y = 0.71$ cm.

c) $x = 1$ cm and $y = 3.51$ cm.

d) $x = 3.1$ cm and $y = 4.9$ cm.

e) $x = 5.76$ cm and $y = 1.78$ cm.

f) $x = 1.77$ cm and $y = 2.97$ cm.

7. a) No, because the 50° angle can be, in the case of one triangle, the angle between two congruent sides and in the other triangle, one of two congruent angles.

b) Yes, with the exception of an angle of 60°.

Practice 2.2 *(cont'd)*

Page 87

8. a) A single triangle.

Two triangles that have two congruent corresponding angles are similar (AA).

Vertically opposite angles are congruent.
If a transversal intersects two parallel lines, then the alternate interior angles formed are congruent.

b) 4 triangles.

Two triangles that have two congruent corresponding angles are similar (AA).

Any angle is congruent to itself. If a transversal line intersects two parallel lines, then the corresponding angles are congruent.

9. The light source must be at 1.05 m from the object.

10. a) 4 m b) 11.4 m

11. a)

STATEMENT	JUSTIFICATION
∠ ADB ≅ ∠ BDC	Two right angles.
∠ ABD ≅ ∠ CBD	Two complementary angles to the same angle.
Δ ABD ~ Δ BCD	Two triangles with two corresponding congruent angles are similar (AA).

b) 1) $\frac{16}{3}$ mm 2) $\frac{20}{3}$ mm

Practice 2.2 (*cont'd*)

Page 88

12.

Hypothesis:	• The quadrilateral ABCD is a trapezoid. • The diagonals BD and AC intersect at point E.
Conclusion:	$\frac{m\overline{BE}}{m\overline{ED}} = \frac{m\overline{AE}}{m\overline{CE}}$

STATEMENT	JUSTIFICATION
∠ ABE ≅ ∠ EDC	These angles are alternate interior, therefore congruent.
∠ BEA ≅ ∠ CED	Vertically opposite angles are congruent.
Δ ABE ~ Δ CDE	By AA.
$\frac{m\overline{BE}}{m\overline{ED}} = \frac{m\overline{AE}}{m\overline{CE}}$	The ratio of the lengths of the corresponding sides of the two triangles is constant.

13. a) The height of the pyramid is 137 m.

b) The length of the shadow would have been 106.86 m.

14.

Hypothesis:	$\overline{AE}$, $\overline{CD}$ and $\overline{FB}$ are three altitudes of the isosceles triangle ABC.
Conclusion:	$\frac{m\overline{AE}}{m\overline{CD}} = \frac{m\overline{BE}}{m\overline{BD}}$

STATEMENT	JUSTIFICATION
∠ AEB ≅ ∠ CDB	The altitude of a triangle forms a right angle to the base.
∠ ABE ≅ ∠ DBC	Common angle.
Δ ABE ~ Δ CDB	By AA.
$\frac{m\overline{AE}}{m\overline{CD}} = \frac{m\overline{BE}}{m\overline{BD}}$	The ratio of the lengths of the corresponding sides of the two triangles is constant.

15. 13.6 m

SECTION 2.3

Manipulating algebraic expressions

Problem

Page 89

The possible dimensions of the panel are 27 dm by 10 dm and 31 dm by 11 dm.

Activity 1

Page 90

a. 1) $12xy + 9x + 8y + 6$

2) $3x(4y + 3) + 2(4y + 3)$

3) $(4y + 3)(3x + 2)$

b. Yes. When multiplying the two binomials you obtain the original polynomial.

Activity 1 (*cont'd*)

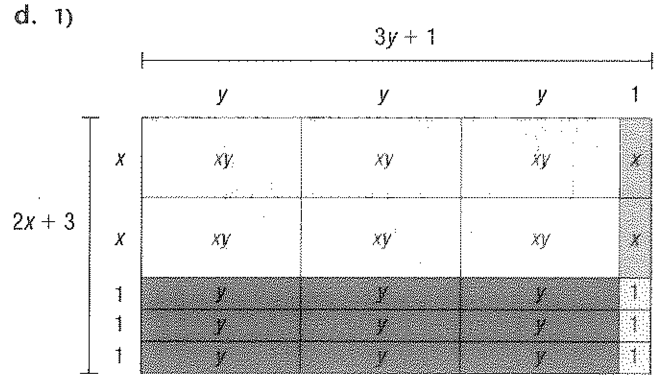
Page 91

c. 1) $12xy - 3x + 16y - 4$

2) $3x(4y - 1) + 4(4y - 1)$

3) $(4y - 1)(3x + 4)$

d. 1)



2) $(3y + 2)(2x + 3)$

3) The product of these two algebraic expressions is initial polynomial.

4) $2x + 3$

e. 1) $(3x + 2)(y + 3)$ 2) $(2x + 3)(y + 2)$

Activity 2

Page 92

a. $x^2 - y^2$

b. No. There is no common factor to both terms.

c. 1) $x(x - y)$ 2) $y(x - y)$

3) $x(x - y) + y(x - y)$ 4) $(x - y)(x + y)$

d. $(x - y)(x + y) = x^2 - xy + yx - y^2 = x^2 - y^2$

e. 1) The green sections creates a square.
2) The red sections creates a square.
3) Figure ABCD is a square.

f. $(2x + 3)(2x + 3)$

g. $(2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Activity 3 — Page 93

a. 1) $t = \frac{6}{s}$ 2) $t = \frac{3}{s}$

b. $t = \frac{9}{s}$

c. No, $s \neq 0$ because you cannot divide by 0.

d. $\frac{4n + 12}{4n} = \frac{4(n + 3)}{4n} = \frac{n + 3}{n}$

e. $d = st \Rightarrow \left(\frac{3n}{n + 3}\right)\left(\frac{4n + 12}{4n}\right) = \left(\frac{3n}{n + 3}\right)\left(\frac{4(n + 3)}{4n}\right) = 3$ m

f. No, because you cannot divide by 0, $n \neq 0$ and $n \neq -3$.

g. Yes, by factoring the expression in the denominator and then by simplifying, you get: $\frac{5}{3(2t + 1)}$.

h. $s = \frac{d}{t} \Rightarrow \left(\frac{5t^2}{2t + 1}\right) \div \left(\frac{5t}{6t^2 + 3t}\right) = \left(\frac{5t^2}{2t + 1}\right)\left(\frac{3t(2t + 1)}{5t}\right) =$ $3t^2$ km/h

i. No, because you cannot divide by 0, $t \neq -\frac{1}{2}$.

Practice 2.3 — Page 98

1. a) Yes. $1 > 0$, $4 > 0$ and $4 = 2\sqrt{1}\,\sqrt{4}$.

b) No. $-144 < 0$

c) No. $-9 < 0$

d) Yes. $64 > 0$, $81 > 0$ and $144 = 2\sqrt{64}\,\sqrt{81}$.

e) No. $15 \neq 2\sqrt{3}\,\sqrt{5}$

f) Yes. $\frac{9}{2} > 0$, $4 > 0$ and $\frac{4}{3} = 2\sqrt{1}\,\sqrt{\frac{4}{9}}$.

2. a) $(x + 2)(y + 4)$ **b)** $(13x - 1)^2$

c) $(x + 4)(y - 2)$ **d)** $-2(y - 2)(x + 1)$

e) $4(x - 3y)(x + 3y)$ **f)** $(3x + 2)(7y - 6)$

g) $(5x + 3y)(x - y)$ **h)** $(3x + 1)(y - 1)$

i) $(7 - x)^2$ **j)** $(x - y)(x - 1)$

k) $\left(\frac{x}{3} + \frac{1}{2}\right)(y + 8)$

l) $\left(\left(\sqrt{2} - 2\right)x - 3\right)\left(\left(\sqrt{2} + 2\right)x + 3\right)$

3. a) $x + 1$ **b)** $3x - 4$

c) $3x - 7 + \frac{17}{x + 3}$ **d)** $5x - 4$

e) $x + 2$ **f)** $x^2 - 2x - 1$

4. a) $\frac{8x^2}{3}$ **b)** $\frac{xy}{x - 3}$

c) $\frac{1}{x - 3}$ **d)** $\frac{x + y}{3}$

e) $\frac{4(x - 1)}{3}$ **f)** $-\frac{2}{1 - 4x}$

5. a) $\frac{2}{15x}$, $x \neq 0$, $y \neq 0$

b) $\frac{3}{2x}$, $x \neq 0$

c) $-\frac{21}{4}$, $x \neq -3$

d) $-\frac{x + 1}{y}$, $x \neq 0$, $y \neq 0$

e) $\frac{5x - 1}{yy}$, $x \neq 0$, $y \neq 0$

f) $\frac{(x - 7)(x - 3)}{2}$, $x \neq 3$, $x \neq -7$

g) $\frac{8y}{9x}$, $x \neq 0$, $y \neq 0$

h) $\frac{2(4x + 1)}{x(x + 1)}$, $x \neq 0$, $x \neq -1$

i) $\frac{-x^2 - x + 8}{(x + 4)^2}$, $x \neq -4$

Practice 2.3 (*cont'd*) — Page 99

6. a) 1) $(x^2 + 2x)$ cm²

2) $(5x + 10)$ cm²

3) $(x^2 + 7x + 10)$ cm²

b) 1) $(x + 5)(x + 2)$

2) $(x + 7)(x + 2) - 2(x + 2)$

c) You will get the same factors as **b)** 1).

7. a) $m = 5$, $n = 1$ **b)** $m = -3$, $n = -2$

c) $m = -\frac{1}{8}$, $n = -\frac{1}{3}$ **d)** $m = -\frac{1}{3}$, $n = 3$

e) $m = \frac{1}{2}$, $n = 2$ **f)** $m = \frac{3}{2}$, $n = -1$

8. The radius of the dial is 4 cm.

9. a) The length of the red line measurement is $(4x - 4y + 40)$ m.

b) The area if the combat surface measures $(20x - 20y + 100)$ m².

Practice 2.3 (*cont'd*) — Page 100

10. a) $x - 4 + \frac{56}{x + 7}$ **b)** $2x - \frac{26}{3} + \frac{28}{3(3x - 2)}$

c) $2x + 9$ **d)** $7x + 5$

11. a) Two triangles that have two congruent angles are similar (AA).

b) $\left(\frac{9x^2}{2} + \frac{27x}{2} + 9\right)$ cm²

12. a) The area of the habitat is $(x + 10)(y + 12)$ m².

b) The width of the trench to the West and East of the dry area is 5 m. The width of the trench to the North and South of the dry area is 6 m.

Practice 2.3 (cont'd) — Page 101

13. $\frac{3}{2(5b-1)}$

14. $\frac{7x^2-8x+9}{2(x+1)^2}$

15. a) 1) Matty's cycling time is represented by the expression $t_1 = \frac{10}{s}$, where s corresponds to the running speed.

   2) Matty's running time is represented by the term $t_2 = \frac{10}{s}$, where s corresponds to the running speed.

   3) The total time is represented by the expression $t_{total} = \frac{1}{3} + \frac{20}{s}$.

   b) His cycling speed was 40 km/h.

16. $7x + 19 + \frac{39}{x-2}$

SECTION 2.4 Optimizing a distance

Problem — Page 102

The distance from Person **A** to the reflection of Person **B** is 15.2 m.

Activity 1 — Page 103

a. AP_2B route is the shortest.

b. 1) $75 - x$ 2) $\sqrt{(20^2 + x^2)}$

3) $\sqrt{(10^2 + (75-x)^2)}$

Activity 1 (cont'd) — Page 104

c.

Distance from D to P (m)	Distance from C to P (m)	Distance from A to P (m)	Distance from B to P (m)	Length of trajectory (m)
10	65	≈ 22.36	≈ 65.76	≈ 88.13
25	50	≈ 32.02	≈ 50.99	≈ 83.01
40	35	≈ 44.72	≈ 36.40	≈ 81.12
45	30	≈ 49.24	≈ 31.62	≈ 80.87
55	20	≈ 58.52	≈ 22.36	≈ 80.88
60	15	≈ 63.25	≈ 18.03	≈ 81.27
70	5	≈ 72.80	≈ 11.18	≈ 83.98

d. Scientists should locate the probe's Recharge Station 45 m from point D.

e. 1) Two triangles with two corresponding congruent angles are similar (AA).

2) Point B' is a reflection of point B over axis CD.

3) Two triangles with two corresponding congruent angles are similar (AA).

f. Corresponding angles of similar plane figures or similar solids are congruent and the measures of the corresponding sides are proportional. So in this situation, this proportion is related to corresponding sides.

g. The minimum distance that the probe will have to travel is approximately 80.77 m.

Technomath — Page 105

a) 1) $\sqrt{(12-x)^2 + 5^2}$ 2) $\sqrt{(x^2 + 2^2)}$

b) *Several possible answers. For example:* Approximately 3.5 km from point D.

c) 150 km

Practice 2.4 — Page 107

1. a) The triangles are not similar.

   b) The triangles are similar.

   Two triangles that have one congruent angle contained between corresponding sides of proportional lengths are similar (SAS).

   c) The triangles are not similar.

   d) The triangles are not similar.

2. a) 1) 17.28 km 2) ≈ 36.80 km

   b) 1) ≈ 4.59 km 2) ≈ 21.4 km

3. a) $\left(\frac{62}{9}, 0\right)$ b) $\left(0, \frac{56}{11}\right)$

Practice 2.4 (cont'd) — Page 108

4. ≈ 30.92 cm

5. a) The height of Part Ⓑ is approximately 10.71 cm.

   b) The lateral area of Parts Ⓐ and Ⓑ of the hourglass is approximately 215.46 cm².

6. The oil company should build a platform at approximately 1,272.73 m from point A.

Practice 2.4 (cont'd) — Page 109

7. The surface to repaint is 381.3 m².

8. Each purple section has an area of 27 cm².

9. ≈ 44.72 cm

Practice 2.4 (*cont'd*) — Page 110

10. The amount of lumber required is approximately 25.19 m.

11. Leg ①: ≈ 116.62 km

    Leg ②: ≈ 102.59 km

12. Building A: ≈ 117.81 m

    Building B: ≈ 235.61 m

    Building C: ≈ 153.22 m

SECTION 2.5 Metric relations

Problem — Page 111

The height of the Dubai Tower is approximately 705.18 m.

Activity 1 — Page 112

a. 1) Two triangles with two corresponding congruent angles are similar (AA).

2) Two triangles with two corresponding congruent angles are similar (AA).

3) Two triangles with two corresponding congruent angles are similar (AA).

b. 1) $\frac{m\,\overline{CD}}{m\,\overline{AC}} = \frac{m\,\overline{AC}}{\mathbf{m\,\overline{CB}}} = \frac{\mathbf{m\,\overline{DA}}}{m\,\overline{AB}}$

2) $\frac{m\,\overline{AC}}{m\,\overline{AD}} = \frac{m\,\overline{BC}}{\mathbf{m\,\overline{AB}}} = \frac{\mathbf{m\,\overline{AB}}}{m\,\overline{DB}}$

3) $\frac{m\,\mathbf{\overline{AC}}}{m\,\overline{AB}} = \frac{m\,\overline{AD}}{\mathbf{m\,\overline{BD}}} = \frac{\mathbf{m\,\overline{CD}}}{m\,\overline{AD}}$

c. 1) 4.8 mm

2) 6 m

3) 6.25 cm

Technomath — Page 113

a. 1) Screen 3: $\frac{m\,\overline{CH}}{m\,\overline{BH}} = \frac{m\,\overline{BH}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{AB}} = \frac{2}{3}$

Screen 4: $\frac{m\,\overline{CH}}{m\,\overline{BH}} = \frac{m\,\overline{BH}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{AB}} = 1.6$

Screen 5: $\frac{m\,\overline{CH}}{m\,\overline{BH}} = \frac{m\,\overline{BH}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{AB}} = 1.4$

Screen 6: $\frac{m\,\overline{CH}}{m\,\overline{BH}} = \frac{m\,\overline{BH}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{AB}} = 0.5$

Two triangles whose corresponding sides are proportional are similar.

2) Screen 3: $\frac{m\,\overline{AB}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{BH}} = \frac{m\,\overline{AC}}{m\,\overline{AB}} \approx 1.20$

Screen 4: $\frac{m\,\overline{AB}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{BH}} = \frac{m\,\overline{AC}}{m\,\overline{AB}} \approx 1.89$

Screen 5: $\frac{m\,\overline{AB}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{BH}} = \frac{m\,\overline{AC}}{m\,\overline{AB}} \approx 1.72$

Screen 6: $\frac{m\,\overline{AB}}{m\,\overline{AH}} = \frac{m\,\overline{BC}}{m\,\overline{BH}} = \frac{m\,\overline{AC}}{m\,\overline{AB}} \approx 1.12$

Two triangles whose corresponding sides are proportional are similar.

3) Screen 3: $\frac{m\,\overline{AB}}{m\,\overline{BH}} = \frac{m\,\overline{BC}}{m\,\overline{CH}} = \frac{m\,\overline{AC}}{m\,\overline{BC}} \approx 1.80$

Screen 4: $\frac{m\,\overline{AB}}{m\,\overline{BH}} = \frac{m\,\overline{BC}}{m\,\overline{CH}} = \frac{m\,\overline{AC}}{m\,\overline{BC}} \approx 1.18$

Screen 5: $\frac{m\,\overline{AB}}{m\,\overline{BH}} = \frac{m\,\overline{BC}}{m\,\overline{CH}} = \frac{m\,\overline{AC}}{m\,\overline{BC}} \approx 1.23$

Screen 6: $\frac{m\,\overline{AB}}{m\,\overline{BH}} = \frac{m\,\overline{BC}}{m\,\overline{CH}} = \frac{m\,\overline{AC}}{m\,\overline{BC}} \approx 2.24$

Two triangles whose corresponding sides are proportional are similar.

b. 1) Screen 3: 32.955 cm^2

Screen 4: 35.6 cm^2

Screen 5: 83.916 cm^2

Screen 6: 22.5 cm^2

2) Screen 3: 32.955 cm^2

Screen 4: 35.6 cm^2

Screen 5: 83.916 cm^2

Screen 6: 22.5 cm^2

c. In a right triangle, the product of the length of the hypotenuse and the corresponding altitude equals the product of the length of the sides of the right angle.

d. 1) No.

2) No.

Practice 2.5 — Page 115

1. a) Δ ABC, Δ ADB and Δ BDC.

   b) Δ ABC ~ Δ ADB: $\frac{m\,\overline{AC}}{m\,\overline{AB}} = \frac{m\,\overline{AB}}{m\,\overline{AD}} = \frac{m\,\overline{BC}}{m\,\overline{BD}}$

   Δ ABC ~ Δ BDC: $\frac{m\,\overline{AC}}{m\,\overline{BC}} = \frac{m\,\overline{AB}}{m\,\overline{BD}} = \frac{m\,\overline{BC}}{m\,\overline{DC}}$

   Δ ABD ~ Δ BCD: $\frac{m\,\overline{AB}}{m\,\overline{BC}} = \frac{m\,\overline{AD}}{m\,\overline{BD}} = \frac{m\,\overline{BD}}{m\,\overline{DC}}$

   c) Δ ABC ~ Δ ADB × k = 1.25

   Δ ABC ~ Δ ADB × k = 1.67

   Δ ABC ~ Δ BDC × k = 1.33

   d) Two triangles are similar if their corresponding sides are proportional.

2. a) 2 cm

   In a right triangle, the length of the altitude drawn from the right angle is the geometric mean of the length of the two segments that determine the hypotenuse.

b) 3.2 cm

In a right triangle, the product of the length of the hypotenuse and its corresponding altitude is equal to the product of the length of the legs.

3. 5.82 cm

4. 12.99 cm

5. 3079.49 cm^2

Practice 2.5 (*cont'd*) Page 116

6. A computer-controlled saw must start cut at 9.67 cm from point D.

7.

Length of segments

	a	*b*	*c*	*m*	*n*	*h*
a)	9	12	15	5.4	9.6	7.2
b)	$4\sqrt{5}$	$8\sqrt{5}$	20	4	16	8
c)	10	7.5	12.5	8	4.5	6

8. 23.31 cm

9.

	1) Length associated with *x*	2) Geometric statement
a)	Approximately 1.98 cm	$(m\ \overline{AB})^2 = m\ \overline{AD} \times m\ \overline{BC}$
b)	40.5 cm	$(m\ \overline{CD})^2 = m\ \overline{AD} \times m\ \overline{BD}$
c)	12 cm	$(m\ \overline{BC})^2 = m\ \overline{CD} \times m\ \overline{AC}$
d)	40 cm	$(m\ \overline{AB})^2 = (m\ \overline{AC})^2 + (m\ \overline{BC})^2$

Practice 2.5 (*cont'd*) Page 117

10. a) The distance is 4 km.

b) The distance is 6.93 km.

c) The height of the balloon is 3.46 km.

11. ≈ 837.08 cm^2

12. a) The height of the dam is 61.47 m.

b) The minimum water level to maintain in the basin is approximately 36.29 m.

13. The console is 2.83 m tall.

Practice 2.5 (*cont'd*) Page 118

14. a) 5 m from the foot of the tree.

b) ≈ 8.33 m

c) 5 m above the ground.

d) ≈ 2.08 m

e) ≈ 5.42 m

15. The maximum headroom is 2.08 m.

16. The volume of this recycling container is approximately 58.14 m^3.

Practice 2.5 (*cont'd*) Page 119

17. a) The length of one side of this coin is 5.4 mm.

b) The length of segment CD is 5.22 mm.

18. The distance is approximately 44.32 m.

19. ≈ 74.88 m

20. The loudspeaker are approximately 14.74 m off the ground.

21. The length of the wooden spreader BD, is 76.8 cm.

SPECIAL FEATURES 2

Chronicle of the past Page 121

1. ≈ 4.71 cm

2. a)

Hypothesis:	• The lines AB and DC are parallel. • Lines AD and BC meet at point O.
Conclusion:	Δ ABO ~ Δ DCO

STATEMENT	JUSTIFICATION
$\overline{AB}$ // $\overline{CD}$	By hypothesis.
∠ BAO ≅ ∠ CDO	When a transversal intersects two parallel lines, the corresponding angles are congruent.
∠ ABO ≅ ∠ DCO	When a transversal intersects two parallel lines, the corresponding angles are congruent.
Δ ABO ~ Δ DCO	Two triangles are congruent if they have two corresponding congruent angles (AA).

b) Yes. Since the pairs, sides AO and DO and sides AB and DC, are two pairs of corresponding sides of similar triangles the ratio of their lengths is the same.

3.

Hypothesis:	• m $\overline{AB}$ = 2 m • m $\overline{AC}$ = 3.2 m • m $\overline{AD}$ = 1.25 m
Conclusion:	Δ ABC ~ Δ ADB

STATEMENT	JUSTIFICATION
$\frac{m\ \overline{AC}}{m\ \overline{AB}} = \frac{m\ \overline{AB}}{m\ \overline{AD}}$	Because $\frac{m\ \overline{AC}}{m\ \overline{AB}} = \frac{3.2}{2} = 1.6$ and $\frac{m\ \overline{AB}}{m\ \overline{AD}} = \frac{2}{1.25} = 1.6$.
∠ BAD ≅ ∠ BAC	Common angle.
Δ ABC ~ Δ ADB	Two triangles that have a congruent angle contained between two corresponding sides of proportional length are similar (SAS).

In the workplace

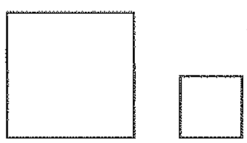
Page 123

1. The lenght steel rod AD is approximately 80.78 cm.
2. Triangles EFI and FGH are congruent if segments EF and FG are congruent by ASA, otherwise they are similar by AA, since there are two pairs of corresponding congruent angles formed by parallel segments intersected by a transversal.
3. The length of the beam is approximately 1.67 m.
4. The depth of the Catch basin ① is 3.83 m and the depth of Catch basin ② is 3.53 m.

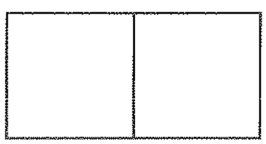
Overview

Page 124

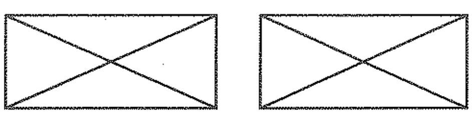
1. a) No.



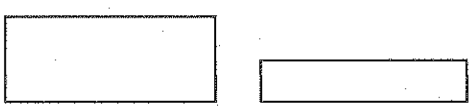
b) Yes.



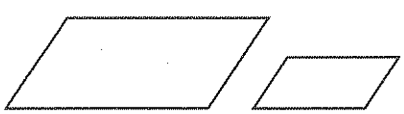
c) Yes.



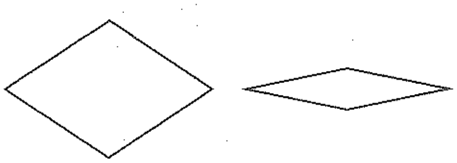
d) No.



e) No.



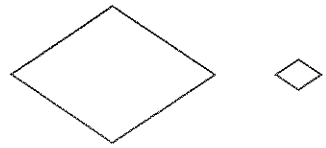
2. a) False.



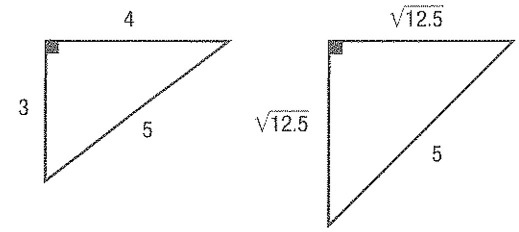
b) True.

c) True.

d) False.



e) False.



3. a) 1) $\frac{12}{17}$ or ≈ 0.71 cm. 2) 1.62 cm

3) $\frac{6\sqrt{15}}{5}$ or ≈ 1.47 cm. 4) $\frac{196}{75}$ or ≈ 2.61 cm.

b) No, because the lines EF and GH are not parallel.

Overview *(cont'd)*

Page 125

4. a) $m = 2$ b) $m = 40$ c) $m = 625$

d) $m = -\frac{16}{3}$ e) $m = 3$ f) $m = 20\ 000$

5. $\frac{x + 5}{3}$

6. a) Neither.

b) Congruent triangles. Two triangles are congruent if a congruent corresponding side is contained between congruent angles (ASA).

c) Similar triangles. Two triangles with two congruent corresponding angles are similar (AA).

d) Neither.

e) Similar triangles. Two triangles whose corresponding sides are proportional, are similar (SSS).

f) Similar triangles. Two triangles whose corresponding side sare proportional, are similar (SSS).

Overview *(cont'd)*

Page 126

7. a) Two triangles that have corresponding congruent sides are congruent (SSS).

b) Two triangles that have corresponding congruent sides are congruent (SSS).

c) Two triangles are congruent if the angles between the two corresponding sides are congruent (SAS).

d) Two triangles that have a corresponding congruent side contained between two congruent angles are congruent (ASA).

8. a) $(a + b)(x + y)$
   b) $3x^3(x - 2)$
   c) $(x + 3)(4y + 5)$
   d) $(5a^4 - 8b^2)(5a^4 + 8b^2)$
   e) $(b + 3)(c - 2)$
   f) $2(z - 2)^2$
   g) $(2a - 3)(2x + y)$
   h) $(7a^2x + y)(x - 2ay)$
   i) $(x - 1)(x + 2)$
   j) $(3b - a)(4b - x^2)$
   k) $2(m - 1)$
   l) $(z - 2)(3z^4 - 5z^2)$
   m) $(x - y - 13)(x - y + 13)$
   n) $(m - n - 1)(m - n + 1)$

9. a) $(5x + 2)$ dm and $(y - 4)$ dm.
   b) 12 dm and 6 dm.
   c) $x \geq -\frac{2}{5}$ and $x \geq 4$.

10. This person would cover approximately 35.36 m.

Overview (*cont'd*) — Page 127

11. The volume of the semi-trailer is 88.44 m^3.

12. a) The ratio between the perimeter of Rectangle **A** and the Rectangle **B** is: $\frac{3x + 5}{2x + 3}$.
    b) The ratio between the area of the Rectangle **B** and the area of Rectangle **A** is: $\frac{x + 3}{2(x + 5)}$.

13. a) $x = 2.88$ cm   b) $x = 1.2$ cm
    c) $x = \frac{\sqrt{19}}{5}$ or ≈ 0.87 cm.   d) $x = 3.6$ cm

Overview (*cont'd*) — Page 128

14. A distance of 3.18 m.
15. The perimeter is $6(m + 4n + 2)$ m.
16. The school is approximately 3.21 m tall.
17. The area of the green triangle is 4 dm^2.

Overview (*cont'd*) — Page 129

18. m $\overline{AB}$ = 5 hm, m $\overline{BE}$ = 3 hm, m $\overline{CD}$ = 4.2 hm
19. The height of the image of the person is 10.8 cm.
20. $\approx$ 12.99 cm
21. The area of the orange triangle is 32 dm^2.

Overview (*cont'd*) — Page 130

22. The area of the recordable section of the compact disc is $\pi(R - r)(R + r)$.

23. a) The expression that corresponds to the speed of the moving object is $(x^2 + 3)$ m/s.
    b) The expression that corresponds to the time of movement is $(x - y)$ s.

24. a) 1) *Several possible answers. For example:* Triangles ABD, BCD, ADF and BDF are all similar to triangle ABC.
       2) Triangle ECG is congruent, therefore similar to triangle EBG.
       3) No triangle is similar to triangle BED.
    b) 1) The triangle EBG is congruent to triangle ECG.
       2) No triangle is congruent to triangle FDB.

25. The shortest length of wire to purchase is at least 29.73 m.

Overview (*cont'd*) — Page 131

26. a) The length of segment BC is 0.65 m.
    b) The length of segment AJ is 0.81 m.

27. a)

STATEMENT	JUSTIFICATION
∠ BRA ≅ ∠ DOP	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
∠ BAR ≅ ∠ PDO	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
$\overline{DO}$ ≅ $\overline{AS}$	By hypothesis.
Δ BRA ≅ Δ DOP	Two triangles with a corresponding congruent side contained between two corresponding congruent angles are congruent (ASA).

b)

STATEMENT	JUSTIFICATION
∠ FGP ≅ ∠ ODP	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
∠ FPG ≅ ∠ OPD	Vertically opposite angles are congruent.
Δ FPG ~ Δ OPD	Two triangles are similar if they have two corresponding congruent angles (AA).
∠ PDO ≅ ∠ OSC	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
∠ DOP ≅ ∠ SOC	Vertically opposite angles are congruent.
Δ OPD ~ Δ OSC	Two triangles are similar if they have two corresponding congruent angles (AA).
∠ SOC ≅ ∠ SRQ	If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
∠ QSR ≅ ∠ CSO	Vertically opposite angles are congruent.
Δ QSR ~ Δ CSO	Two triangles are similar if they have two corresponding congruent angles (AA).
Δ SRQ ~ Δ GFP	By transitivity.

c) 1) ≈ 5.66 cm   2) ≈ 8 cm

3) ≈ 11.31 cm   4) ≈ 14.78 cm

28. a) Two triangles that have corresponding congruent sides are congruent (SSS).

b) Two triangles that have two corresponding congruent angles are similar (AA).

c) 1) 3.6 cm   2) 6.4 cm

3) 4.8 cm

29. $10x + 6y + 6$ cm

Bank of problems

Page 132

30. The distance between the two antennas is approximately 63.25 m.

31. m $\overline{CI} \approx 1.38$ m

32. The height of lamppost is 21.8 m.

33. The area of the blue section is $63(x + 1)(y + 9)$ cm^2.

Bank of problems (*cont'd*)

Page 133

34.

STATEMENT	JUSTIFICATION
$\overline{AE} \cong \overline{AB}$	Two sides of a regular pentagon.
$\overline{AB} \cong \overline{BC}$	Two sides of a regular pentagon.
∠ EAB ≅ ∠ ABC	Two interior angles of the same regular pentagon.
Δ AEB ≅ Δ ABC	Two triangles with corresponding congruent angle between two corresponding congruent sides, are congruent (SAS).
$\overline{AC} \cong \overline{BE}$	The corresponding sides of congruent triangles are congruent.

35. The length of the base of Building **A** is approximately 23.38 m and the base of the Building **B**, approximately 12.83 m.

36. The volume of right circular cone is approximately 138.24 cm^3.

LES 5 | Air traffic control

The following is an approach used to determine the lines that form the boundaries of each air corridor, the coordinates of the vertices of each of the three polygons that correspond to the common flight zones as well as the area of these zones.

- Determine the coordinates of point D by using the point of division formula: $\left(400 + \frac{1}{6} \times 480, 100 + \frac{1}{6} \times 360\right) = (480, 160)$.
- Determine the coordinates of point I by using the point of division formula.
- Since point J shares segment DI in ratio 4:1, point J is situated $\frac{4}{5}$ of length segment DI: $\left(480 + \frac{4}{5} \times \Delta x, 160 + \frac{4}{5} \times \Delta y\right) = (240, 480)$ $\Rightarrow \Delta x = -300$ and $\Delta y = 400$
- Therefore I: $(480 + \Delta x, 160 + \Delta y) = (180, 560)$.
- Determine the equation of lines AF, DI, AL, FL and GK.

  $\overline{AF}: y = \frac{3}{4}x - 200$, $\overline{DI}: y = -\frac{4}{3}x + 800$,

  $\overline{AL}: y = -\frac{4}{3}x + \frac{1900}{3}$, $\overline{FL}: y = -\frac{1}{7}x + \frac{4100}{7}$

  and $\overline{GK}: y = -\frac{1}{7}x + \frac{3600}{7}$.
- Determine the coordinates of point B. Between point B and point D, the variation is (80, 60). With lines AL and AF being perpendicular and the air corridors having the same width, between point A and point B, the variation must be (-60, 80). Therefore B = (340, 180).
- Determine the equation of line BE: $y = \frac{3}{4}x - 75$.
- By using the comparison method, determine all the coordinates of points A to L, which are the intersection points between two lines.
- The coordinates of points A, F and J are provided in the graph.
- The coordinates of points B, D and I were determined in the procedure above.
- The coordinates of the other points are: C(420, 240), E(740, 480), G(800, 400), H(660, 420), K(100, 500) and L(40, 580).
- Calculate the area of the zone formed by square ABCD.
- By using the formula of the distance between two points, calculate the length of side AB.

  $\overline{AB} = \sqrt{(400 - 340)^2 + (100 - 180)^2}$

  $\Rightarrow \overline{AB} = 100$ m
- The area of this zone is 10 000 m^2.
- Calculate the area of quadrilateral EFGH. Since it consists of a rhombus, the lengths of the diagonals must be calculated.
- Determine the distance between points E and G.

  $\overline{EG} = \sqrt{(740 - 800)^2 + (480 - 400)^2}$

  $\Rightarrow \overline{EG} = 100$ m
- Determine the distance between points H and F.

  $\overline{FH} = \sqrt{(880 - 660)^2 + (460 - 420)^2}$

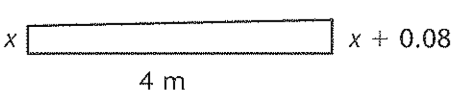
  $\Rightarrow \overline{FH} \approx 223.6068$ m
- The area of rhombus EFGH is approximately 11 180.34 m^2.
- Calculate the area of quadrilateral IJKL. The two diagonals have the same length as those obtained for the previous rhombus. The area of this rhombus is therefore 11 180.34 m^2.
- Present the final results in a clear manner.

Equations of the lines defining the boundary of Air corridor ①:	
$y = -\frac{4}{3}x + \frac{1900}{3}$	$y = -\frac{4}{3}x + 800$
Coordinates of the points defining the boundary of the common fly zone ABCD:	
A(400, 100), B(340, 180);	C(420, 240), D(480, 160)
Area of zone ABCD:	
10 000 m^2	
Equations of the lines forming the boundary of Air corridor ②:	
$y = \frac{3}{4}x - 200$	$y = -\frac{3}{4}x - 75$
Coordinates of the points forming the boundary of the common fly zone EFGH:	
E(740, 480), F(880, 460)	G(800, 400), H(660, 420)
Area of zone EFGH:	
11,180.34 m^2	
Equations of the lines defining the boundary of Air corridor ③:	
$y = -\frac{1}{7}x + \frac{4100}{7}$	$y = -\frac{1}{7}x + \frac{3600}{7}$
Coordinates of the points defining the boundary of the common fly zone IJKL:	
I(180, 560), J(240, 480);	K(100, 500), L(40, 580)
Area of zone IJKL:	
11 180.34 m^2	

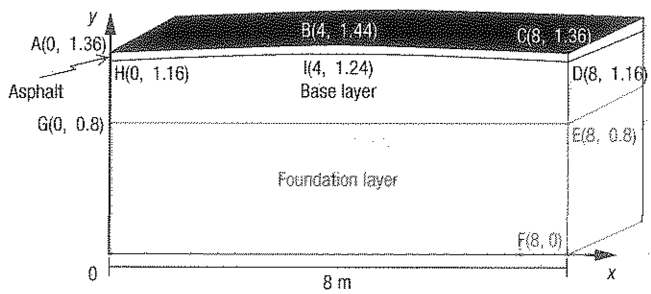
LES 6 | *A precision tool*

The following is an approach on how to write up a proposal for the construction of the 1-km of road.

- Establish that the thickness of the foundation layer is 80 cm and the thickness of the asphalt layer is 20 cm.
- Calculate the area of the foundation layer. $0.8 \text{ m} \times 8 \text{ m} = 6.4 \text{ m}^2$
- Calculate the volume of the foundation layer. $6.4 \text{ m}^2 \times 1{,}000 \text{ m} = 6400 \text{ m}^3$
- Calculate the cost of this layer. $6400 \text{ m}^3 \times \$5/\text{m}^3 = \$32{,}000$
- Determine the difference in level of the road starting from the slope: $\frac{2}{100} = \frac{x}{4} \Rightarrow x = 0.08$ m. Therefore, from the centre of the road to the shoulder of the road, the difference in level is 8 cm.
- Calculate the area of the asphalt: imagine a large rectangle measuring 8 m in width by 28 cm in height separated into four triangles measuring 4 m in base by 8 cm in height.
  Area $= 8 \times 0.28 - 4\left(\frac{4 \times 0.08}{2}\right) = 1.6 \text{ m}^2$
- Calculate the volume of the asphalt. $1.6 \text{ m}^2 \times 1000 \text{ m} = 1600 \text{ m}^3$
- Calculate the cost of the layer of asphalt. $1600 \text{ m}^3 \times \$80/\text{m}^3 = \$128{,}000$
- The volume of the foundation layer and the volume of the asphalt layer must correspond to $\frac{5}{7}$ of the total volume; the volume is 8000 m³.
- Calculate the volume of the base layer by proportion: 3200 m³.
- Calculate the cost of the base layer. $3200 \text{ m}^3 \times \$10/\text{m}^3 = \$32{,}000$
- The area of the base layer is therefore 3.2 m².
- The base layer is formed by two trapezoids.
- Calculate the measure of both vertical sides of these trapezoids:
  $\frac{(x + x + 0.08) \times 4}{2} = 1.6$
  $x = 0.36$ m



- Determine the coordinates of points A, G and H on the y-axis: A(0, 1.36), G(0, 0.8) and H(0, 1.16).
- Determine the coordinates of points B and I: B(4, 1.44) and I(4, 1.24).
- Determine the coordinates of points C, D, E and F: C(8, 1.36), D(8, 1.16), E(8, 0.8) and F(8, 0).
- Determine the equation of lines AB, HI, BC and ID.
  $\overline{AB}$: $y = 0.02x + 1.36$, $\overline{HI}$: $y = 0.02x + 1.16$,
  $\overline{BC}$: $y = {}^-0.02x + 1.52$ $\overline{ID}$: $y = {}^-0.02x + 1.32$.
- Determine the inequalities that border each layer.
  - Foundation layer: $x \geq 0$, $x \leq 8$, $y \geq 0$ and $y \leq 0.8$.
  - Base layer: $x \geq 0$, $x \leq 8$, $y \geq 0.8$, $y \leq 0.02x + 1.16$ and $y \leq {}^-0.02x + 1.32$.
  - Asphalt layer: $x \geq 0$, $x \leq 8$, $y \geq 0.02x + 1.16$, $y \geq {}^-0.02x + 1.32$, $y \leq 0.02x + 1.36$ and $y \leq {}^-0.02x + 1.52$.
- Present the results in a clear manner.



Cost of materials		
Asphalt $128,000	**Foundation layer** $32,000	**Base layer** $32,000
The total cost for the construction of 1 km of road would be $192,000.		
Inequalities defining the foundation layer: $x \geq 0, x \leq 8,$ $y \geq 0, y \leq 0.8$	Inequalities defining the base layer: $x \geq 0, x \leq 8, y \geq 0{,}8,$ $y \leq 0.02x + 1.16,$ $y \leq {}^-0.02x + 1.32$	Inequalities defining the asphalt layer: $x \geq 0, x \leq 8,$ $y \geq 0.02x + 1.16,$ $y \geq {}^-0.02x + 1.32,$ $y \leq 0.02x + 1.36,$ $y \leq {}^-0.02x + 1.52$

REVISION 3

Prior learning 1

Page 136

a. 1) $y = 15\,000x$

2) $y = 14\,900x + 6000$

b. The company must build 60 vehicles to make a profit.

Prior learning 2

Page 137

a. For February, the expression is $2a - 550$.

For March, the expression is $a + 100$.

For April, the expression is $2a - 550$.

For May, the expression is a.

For June, the expression is $a + 50$.

b. The employee received $500 in January, $450 in February, $600 in March, $450 in April, $500 in May and $550 in June.

Knowledge in action

Page 140

1. a) $(14, 16)$ b) $\left(\frac{5}{2}, -\frac{15}{2}\right)$
   c) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ d) $(10, -13)$
   e) $\left(\frac{8}{7}, \frac{4}{7}\right)$ f) $(21, 1)$
   g) $\left(2, \frac{54}{5}\right)$ h) $\left(1, -\frac{93}{10}\right)$
   i) $\left(6, \frac{76}{5}\right)$
2. a) $\approx (15.09, 1.45)$ b) $\left(\frac{-16}{3}, \frac{-8}{3}\right)$
   c) $(-15, 130)$ d) $(-32, 72)$
   e) $(1, 35)$ f) $\left(\frac{2}{11}, -\frac{29}{11}\right)$

Knowledge in action (*cont'd*)

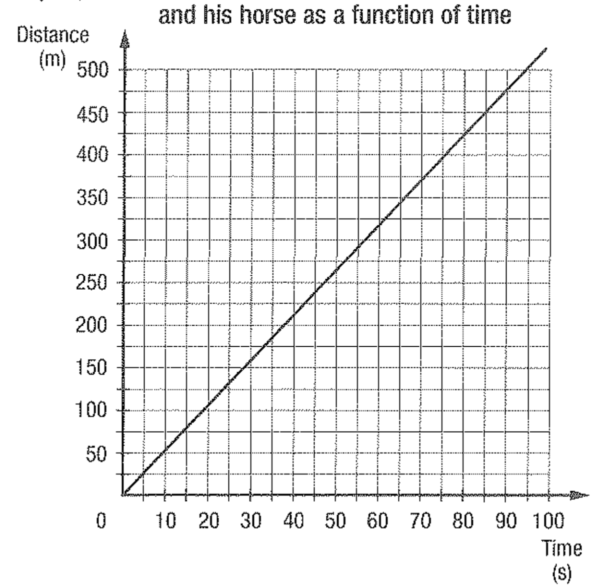
Page 141

3. a) $d \geq 2$ b) $p < 19$
   c) $a \leq 3$ d) $g \geq 8$
   e) $t < v$ f) $r \leq b$
   g) $2c > -6$ h) $s \geq m + 5$
4. A 1, B 5, C 4, D 6, E 2, F 3
5. a) $x > -2$ b) $a \leq 8$
   c) $t \leq 19$ d) $b < 110$
   e) $m \geq -4.2$ f) $c \geq -5$
   g) $n > -\frac{19}{5}$ h) $x > 2$
   i) $x \geq \frac{11}{4}$ j) $x \geq 1$
   k) $x < -\frac{2}{3}$ l) $x > -\frac{16}{3}$
6. a) x: time in h
   y: quantity of fuel left in the plane
   b) $y = -500x + 1850$; $y = -100x + 1000$
   c) Both planes hold the same quantity of fuel approximately 2.02 h or 2 h 1 min 15 s after the first plane takes off.

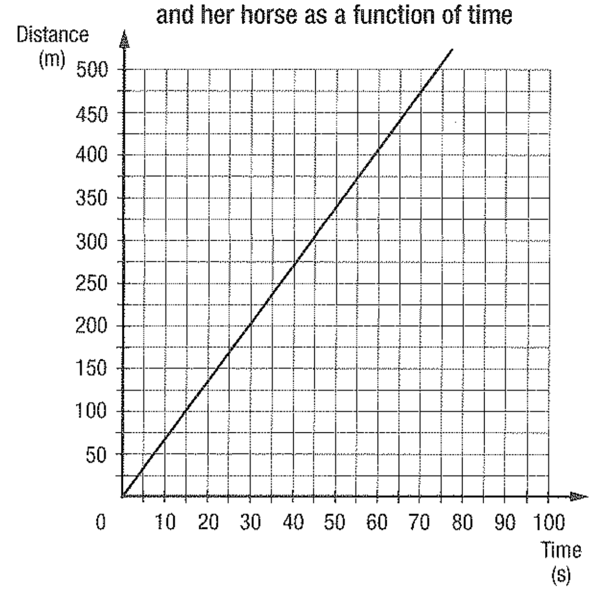
Knowledge in action (*cont'd*)

Page 142

7. a) 1) $x > 0$ b) 1) $x > 0$
   2) $x > 13.16$ 2) $x > 9.6$
   3) $x \in]0, 8.45]$ 3) $x \in]0, 7.07]$
   c) 1) $x > \frac{4}{3}$ d) 1) $x > 0$
   2) $x > 13.87$ 2) $x > 4.17$
   3) $x \in \left]\frac{4}{3}, 10.69\right]$ 3) $x \in]0, 2.69]$
8. a) 1) Distance covered by Ian and his horse as a function of time



2) Distance covered by Malika and her horse as a function of time



b) Both horse riders would be side by side 160 m from the finishing line.

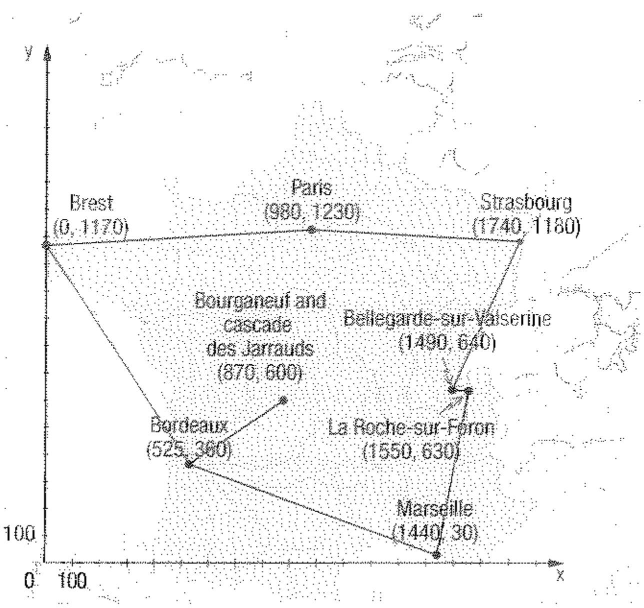
c) 1) Ian and his horse take approximately 80.19 s to complete the race.
   2) Malika and her horse take approximately 73.53 s to complete the race.

SECTION 3.1

Points and segments in the Cartesian plane

Problem

Page 143



The shortest length of cable for this project is approximately 5367.58 km.

Activity 1

Page 144

a. The vertical change is −2.

b. The horizontal change is 10.

c. The coordinates are (2, 0.25).

d. It is a right triangle.

e. 1) The distance is 2 m.

2) The distance is 10 m.

3) The distance is approximately 10.2 m.

Activity 2

Page 145

a. The coordinates are (10, −30).

b. 1) The coordinates are (−40, −30).

2) The coordinates are (10, 30).

3) The coordinates are (−40, 30).

c. The Survey Marker, P is situated closer to point C since if the segment were to be divided into three, point P is situated two parts from point D and one part from point C.

d. The Survey Marker shares segment DC in the ratio 2:1.

e) 1) The coordinates are (40, 30).

2) The coordinates are (60, 90).

3) The coordinates are (60, 30).

Activity 3

Page 146

a. It must be shown that m $\overline{AM}$ = m $\overline{BM}$ = m $\overline{CM}$.

b. A(0, b), B(a, 0), C(0, 0)

c. $\left(\frac{a}{2}, \frac{b}{2}\right)$

d. 1) $\frac{\sqrt{a^2 + b^2}}{2}$ 2) $\frac{\sqrt{a^2 + b^2}}{2}$ 3) $\frac{\sqrt{a^2 + b^2}}{2}$

e. m $\overline{AM}$ = m $\overline{BM}$ = m $\overline{CM}$

f. This representation facilitates working with the coordinates of the vertices of the triangle, because they consist of a minimum number of variables.

Technomath

Page 147

a. 1) The right triangle has been moved.

2) The dimensions of the right triangle have been changed.

3) The dimensions of the right triangle and slope of segment AB have been changed.

b. 1) For example, for Screen 3: $\frac{1.7 + 6.7}{2} = 4.2$.

2) For example, for Screen 3: $\frac{1.5 + 4.7}{2} = 3.1$.

c. 1) The quotient and the slope of segment AB are identical or almost identical.

2) Screen 4: ≈ 5.94 cm

Screen 5: ≈ 10.57 cm

Screen 6: ≈ 10.8 cm

d. 1) No, there is no relation between the slope of a segment and its length.

2) The slope is 0.

3) The slope is undefined.

Practice 3.1

Page 150

1. *Several answers possible. Example:*

Because $(3 - 6)^2 = (6 - 3)^2$ since $(-3)^2 = 3^2$ and because $(4 - 8)^2 = (8 - 4)^2$ since $(-4)^2 = 4^2$.

2. a) $\sqrt{58} \approx 7.62$ u

b) $\sqrt{349} \approx 18.68$ u

c) $10\sqrt{145} \approx 120.42$ u

d) $\sqrt{5785} \approx 76.06$ u

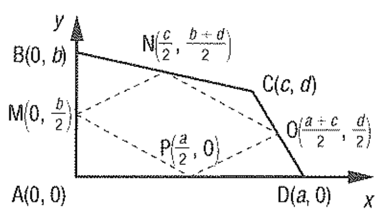
e) $\sqrt{154.25} \approx 12.42$ u

3. a) $\frac{3}{5}$ b) $-\frac{1}{7}$
   c) -6 d) 1
4. a) A right triangle.
   b) An equilateral triangle.
   c) An isosceles triangle.
   d) A scalene triangle.
   e) A parallelogram.
   f) A parallelogram.
   g) A rectangle.
   h) An isosceles trapezoid.
   i) A square.
   j) A rhombus.
5. a) (9, 7) b) (10, 14)
   c) (0, 8) d) (1, 0)

Practice 3.1 *(cont'd)* **Page 151**

6. a) $\left(7, \frac{25}{3}\right)$ b) (6, 10)
   c) (7, 9.25) d) (1, −4)
   e) (6, 12) f) $\left(\frac{79}{12}, \frac{158}{15}\right)$

7.



Given quadrilateral ABCD and points M, N, O and P as the respective midpoints of sides AB, BC, CD and AD. Calculate the slope of segments MN, OP, NO and MP.

Slope of $\overline{MN}$	Slope of $\overline{OP}$	Slope of $\overline{NO}$	Slope of $\overline{MP}$
$\frac{\frac{b+d}{2} - \frac{b}{2}}{\frac{c}{2}} = \frac{d}{c}$	$\frac{\frac{d}{2}}{\frac{a+c}{2} - \frac{a}{2}} = \frac{d}{c}$	$\frac{\frac{d}{2} - \frac{b+d}{2}}{\frac{a+c}{2} - \frac{c}{2}} = \frac{-b}{a}$	$\frac{\frac{-b}{2}}{\frac{a}{2}} = \frac{-b}{a}$

Quadrilateral MNOP is a parallelogram.

8. a) The coordinates are $\left(\frac{-1}{2}, \frac{7}{2}\right)$.
   b) The length of the new road is $\sqrt{74.5}$ km, that is, approximately 8.63 km.

9. a) 90°
   b) 0
   c) −10
   d) This ratio causes a division by 0.
   e) The slope is undefined.

Practice 3.1 *(cont'd)* **Page 152**

10. a) $M_1(0, a)$ $M_2(b + c, a)$
    b) 1) 0 2) 0 3) 0
    c) 1) $2b$ 2) $b + c$ 3) $2c$
    d) This segment is parallel to the bases since their slopes are equal and $b + c$ is equal to half the sum of $2b$ and $2c$.
11. a) The 2nd outlet divides segment AB in the ratio 1:3.
    b) The 4th outlet divides segment BA in the ratio 1:3.
    c) The 3rd outlet is located at half the length $\overline{AB}$.

Practice 3.1 *(cont'd)* **Page 153**

12. The agent can cover 212π u², that is approximately 666.02 u².
13. a) The object must cover a distance of approximately 34.98 m.
    b) 1) The object moves at 0.6 m/s.
       2) The object moves at approximately 1.21 m/s.
       3) The object moves at approximately 0.72 m/s.
    c) 122 objects can be processed.
14. 164 882 students were enrolled.

Practice 3.1 *(cont'd)* **Page 154**

15. a) The coordinates are $\left(\frac{11}{2}, 0\right)$.
    b) The distance is 1.5 u.
    c) The distance is approximately 6.18 u.
16. a) The distance is approximately 92.2 m.
    b) The distance is approximately 26.34 km.
17. a) 1) The rope is attached at a height of $\frac{1}{30}$ m.
       2) The rope is attached at a height of $\frac{14}{45}$ m.
    b) 1) The length of the rope is approximately 1.54 m.
       2) The length of the rope is approximately 1.34 m.

Practice 3.1 (*cont'd*) **Page 155**

18. a) The length of the humerus is approximately 37 cm.
    b) Andrea's height is approximately 174 cm.
    c) The difference is approximately 5.5 cm.
19. a) Both species must cover approximately 3400 km.
    b) The coordinates are ($^-1444.2$, $^-1440.6$).
    c) The hummingbird would catch up with the monarch butterfly approximately 3.04 days after the monarch's departure.

SECTION 3.2 Lines in the Cartesian plane

Problem **Page 156**

The irrigation system would use approximately 53 207.47 L of water.

Activity 1 **Page 157**

a. $y = \frac{-12}{5}x + 118$

b. The slope is $-\frac{12}{5}$ and the y-intercept is 118.

c. 1) Replace y by 0, subtract 118 from each side of the equation, multiply each side of the equation by 5, then divide them by $^-12$.
   2) Multiply each side of the equation by 5, then subtract $5y$ from each side of the equation.

d. 1) 1 2) $^-2$ 3) 120

e. Subtract x from each side of the equation, subtract 120 from each side, then divide each side of the equation by $^-2$.

f. 1) $-\frac{A}{B}$ 2) $-\frac{C}{B}$ 3) $-\frac{C}{A}$

Activity 2 **Page 158**

a. m $\overline{AC} = \sqrt{117}$, m $\overline{AB} = \sqrt{234}$, m $\overline{BC} = \sqrt{117}$ and so (m $\overline{AC})^2$ + (m $\overline{BC})^2$ = (m $\overline{AB})^2$, 117 + 117 = 234.

b. 1) $-\frac{3}{2}$ 2) 5 3) $\frac{2}{3}$

c. $-\frac{3}{2} \times \frac{2}{3} = -\frac{6}{6} = {}^-1$

d. 1) $\left(\frac{3}{2}, -\frac{1}{2}\right)$
   2) $y = -\frac{1}{5}x - \frac{1}{5}$
   3) $5 \times -\frac{1}{5} = -\frac{5}{5} = {}^-1$

e. The product of the slopes of two perpendicular lines is $^-1$.

f. 1) The slope of l_4 is 1 and the slope of l_5 is 1: both lines have the same slope.
   2) Lines l_4 and l_5 are parallel.

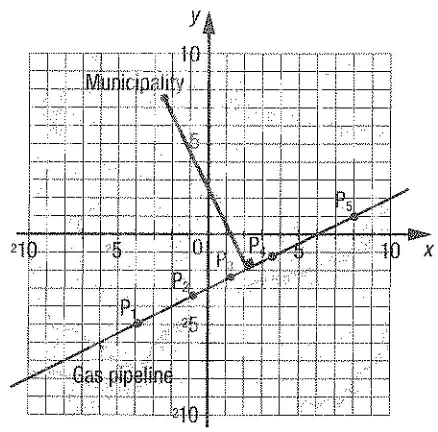
g) $1 \times {}^-1 = {}^-1$

h) Quadrilateral DEFG is a right trapezoid.

Activity 3 **Page 159**

a. 1) Point P_1 is the furthest.
   2) Point P_4 is the closest.
   3) It would have to connect between points P_3 and P_4.

b. 1)



   2) The line is perpendicular to the gas pipeline.

c. 1. $^-2$
   2. $y = {}^-2x + \frac{5}{2}$
   3. $\left(\frac{11}{5}, \frac{-19}{10}\right)$
   4. ≈ 12.34 km

   It would cost the city at least $382,540 to connect it to the gas pipeline.

Technomath **Page 160**

a. *Several answers possible. Example:* (0, 3.43), (4, 4.43), (8, 5.43), (12, 6.43)

b. Lines l_1 and l_2 have the same slope, that is 0.25.

c. $0.25 \times {}^-4 = {}^-1$

d. $y = -\frac{4}{5}x - \frac{11}{30}$

e. The y-intercept of line l_1 = 3.47.
   The y-intercept of line l_2 = $^-1.5$.
   The y-intercept of line $l_3 \approx 2.23$.

f. Line l_2 is parallel to line l_1 and it is perpendicular to line l_3.

g. 1) The slope becomes 0 and the equation is $y = b$ or $By + C = 0$.

2) The slope would no longer be defined and the equation would be $Ax + C = 0$.

Practice 3.2 **Page 163**

1. a) $y = 2x - 4$ b) $y = 3$
   c) $y = {}^{-}3x$ d) $y = \frac{3}{2}x + 4$
   e) $y = {}^{-}x + 2$ f) $x = -5$

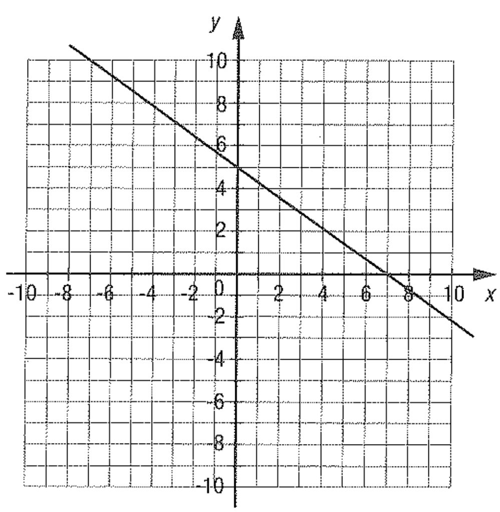
2.

	Slope	*y*-intercept	*x*-intercept
a)	$^{-}1$	23	23
b)	$^{-}12$	5	$\frac{5}{12}$
c)	$^{-}1$	15	15
d)	3	$-\frac{19}{2}$	$\frac{19}{6}$
e)	$^{-}1$	7	7
f)	$\frac{3}{4}$	$\frac{5}{4}$	$-\frac{5}{3}$
g)	1	0	0
h)	0	5.7	None
i)	$\frac{147}{2}$	$^{-}168$	$\frac{16}{7}$

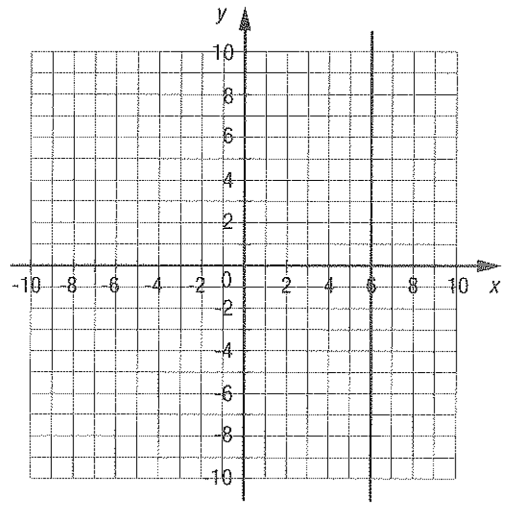
3. a) 1) $y = 8x - 3$ 2) $8x - y - 3 = 0$
   b) 1) $y = {}^{-}2x + 29$ 2) $2x + y - 29 = 0$
   c) 1) $y = \frac{4}{5}x + \frac{26}{5}$ 2) $4x - 5y + 26 = 0$
   d) 1) $y = {}^{-}8x + 17$ 2) $8x + y - 17 = 0$
   e) 1) $y = 2x + 20$ 2) $2x - y + 20 = 0$
   f) 1) $y = {}^{-}1.3x$ 2) $13x + 10y = 0$

Practice 3.2 (*cont'd*) **Page 164**

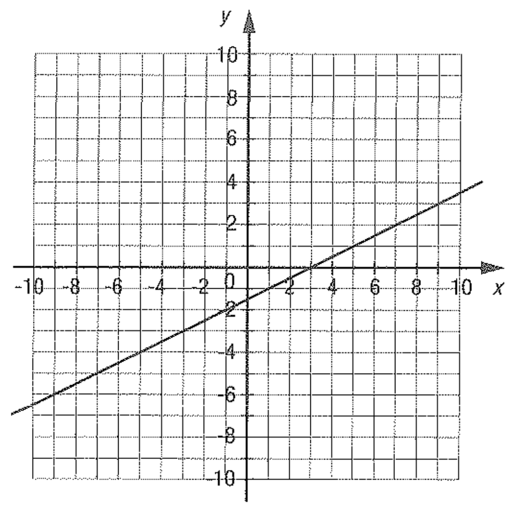
4. a)



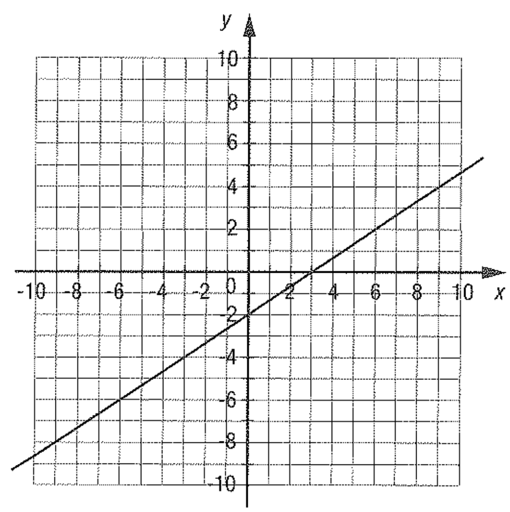
b)



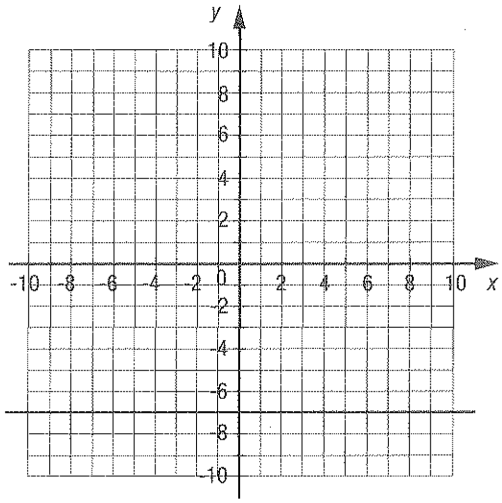
c)



d)



e)



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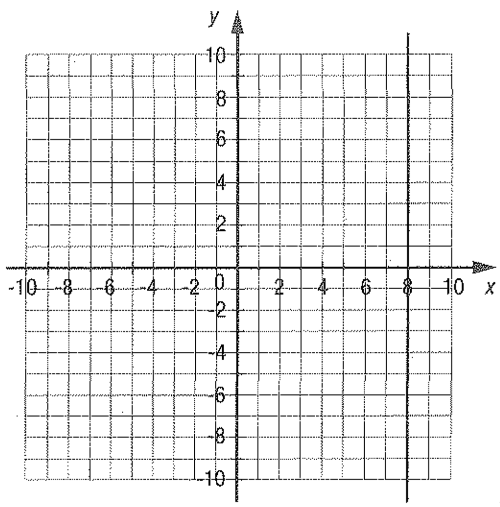


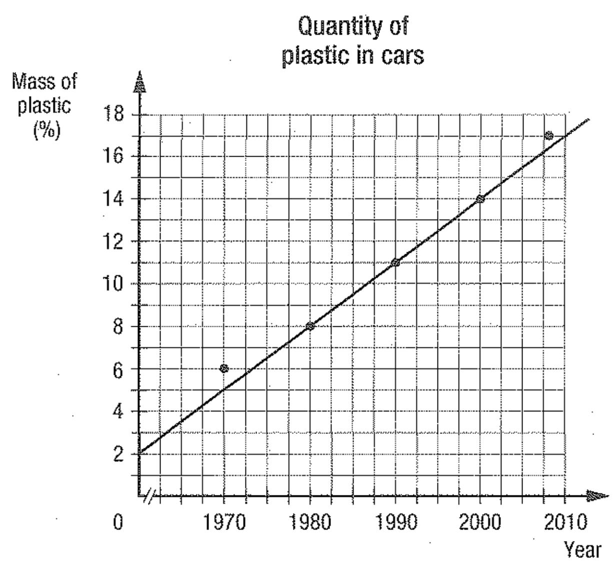
5. a) $y = 2x + 3$ b) $y = 4$
   c) $x = ^-2$ d) $y = -\frac{1}{2}x + 4$
   e) $y = -\frac{5}{4}x + 17$ f) $y = \frac{2}{5}x - \frac{28}{5}$
6. a) ≈ 9.22 u b) ≈ 12.52 u
7. a) $\frac{-1}{2a}$ b) $\frac{2}{ab}$
   c) $\frac{-1}{7a^2}$ d) $\frac{-5}{2a + 2b}$

Practice 3.2 *(cont'd)* **Page 165**

8. a) 30
   b) $5x - 2y + 60 = 0$
   c) *Several possible answers. For example:* $y = -\frac{2}{5}x + 8$
9. a) 60 u^2 b) ≈ 74 u^2
10. a) $2x + y + 1 = 0$
    b) $x - 7y + 14 = 0$
    c) $4x + 5y + 165 = 0$
11. a) $y = x + 15$
    b) $y = ^-x + 112$
    c) $y = 5x + \frac{5}{2}$
12. ① and ②.
13. a) $y = 0$ b) $x = 0$

Practice 3.2 *(cont'd)* **Page 166**

14. a) $5x - 3y - 14 = 0$
    b) $y = \frac{-3}{5}x + 1$
    c) Since the slope of $\overline{AB}$ is 0.25 and the slope of $\overline{AC}$ is $^-4$, their product is -1.
15. a) 1) Two lines. 2) $y = 9$ and $y = 21$.
    b) 1) Two lines. 2) $x = 12$ and $x = 24$.
    c) An infinite number of lines.
    d) A circle.
16. a) and b)



    c) The equation of this line is $y = 0.3x - 586$.
    d) The mass of the plastic would be 1840 kg.

Practice 3.2 *(cont'd)* **Page 167**

17. The area is 3750 cm^2.
18. $k = 4$
19. a) ①, ② and ④ as well as ③ and ⑤.
    b) None.

20. a) Determine the slope of the parallel lines, calculate the slope of a perpendicular line, determine the equation of the parallel lines and perpendicular line, determine the intersection points of the parallel lines with the perpendicular line and finally, calculate the distance between these points.

b) ≈ 2.45 u

21. a) The slope is $^{-}\frac{1}{4}$.

b) The equation of the line is $x + 4y - 2200 = 0$.

Practice 3.2 (*cont'd*) Page 168

22. a) The coordinates are (19.8, 8.61).

b) The required time is approximately 4.15 min or 4 min 9 s.

c) The distance is approximately 16.06 km.

23. a) 1) The equation is $= \frac{-1}{2}x + 45$.

2) The equation is $y = 2x - 30$.

3) The equation is $y = 0$.

4) The equation is $y = 2x$.

b) The coordinates are (15, 0).

c) 1) The perimeter of the seigneury is approximately 102.21 km.

2) The area of the seigneury is approximately 495 km^2.

24. a) The equation of the line is $y = 2.5$.

b) The distance between the top of his head and the highest point of the roof is 4.73 m.

SECTION 3.3 System of equations

Problem Page 169

Planes **A** and **B** would carry the same amount of fuel in their tank after having covered 10 000 km.

Activity 1 Page 170

a. $\frac{a + b}{2} = 80$ and $a = 2b - 116$.

b. 1) The variables are situated on the same side of the equation.

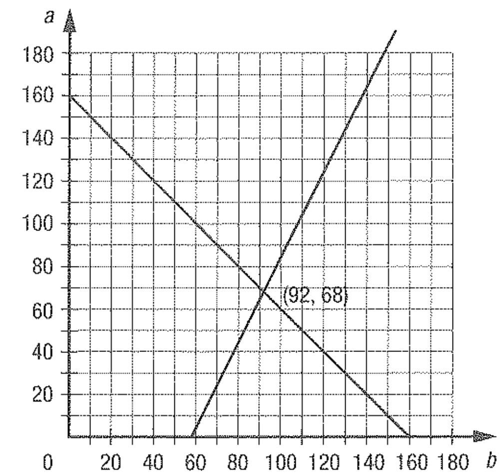
2) The variables are situated on both sides of the equation.

c. $\frac{3b - 116}{2} = 80$

d. $b = 92$, which indicates that at the second try, the speed of the vehicle was 92 km/h.

e. The speed of the vehicle was 68 km/h.

f.



The intersection point corresponds to the solution that was obtained in the previous examples.

Activity 2 Page 171

a. $30x + 60y = 1275$ and $45x + 15y = 975$.

b. 1) The variables are situated on the same side of the equation.

2) The variables are situated on the same side of the equation.

c. No, since the duration of the production is only prolonged; the machines continue to work at the same rhythm.

d. It has the same value, that is 90.

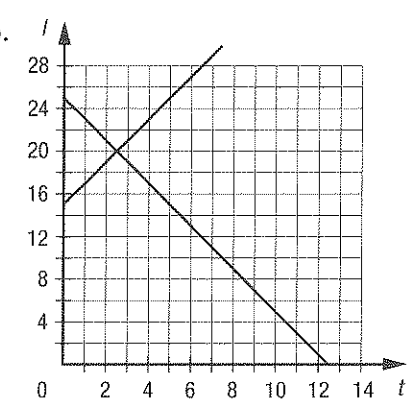
e. The following equation is obtained: $150y = 1875$.

f. 12.5 bottles of 1 L and 17.5 bottles of 500 mL are produced per minute.

Activity 3 Page 172

a. $l = {}^{-}2t + 25$ and $l = 2t + 15$.

b.

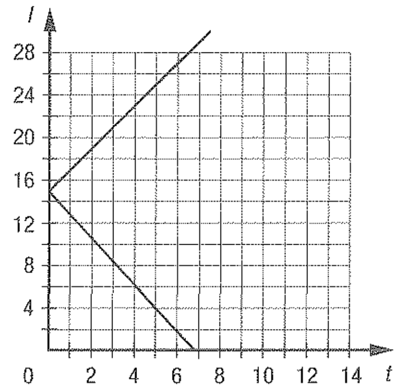


c. These lines intersect.

d. Based on the diagram drawn in **b.**, one can establish that after 2.5 min, the level of water of both basins is 20 m.

e. $l = {}^-2t + 15$ and $l = 2t + 15$.

f.



g. These lines intersect.

h. Based on the diagram drawn in f., one can establish that the level of water of both basins is 15 m at the time of opening.

Technomath

Page 173

a. (⁻4.8, 3.2)

b. The scale. The *x*-axis is scaled by 1. The *y*-axis is scaled by 1.

c. (1, 3), (⁻5, ⁻15), (7, ⁻27)

Practice 3.3

Page 176

1. a) (2, 3) b) (1, 4)
   c) (2, ⁻1) d) (1, 1)
   e) $\left(-\frac{8}{7}, -\frac{27}{7}\right)$ f) (2, 0)
   g) $\left(\frac{1}{5}, \frac{1}{10}\right)$ h) $\left(\frac{1525}{2}, \frac{3205}{2}\right)$
   i) (4, 4)

2.

	1) Unknowns	2) System of equations	3) Solution
a)	*x*: number of hours of work *y*: amount charged ($)	$y = 25x + 50$ $y = 35x + 20$	(3, 125): For 3 hours of work, the amount charged is the same, that is $125.
b)	*x*: cost of resistor ($) *y*: cost of condenser ($)	$50x + 75y = 90$ $125x + 90y = 135$	$\left(\frac{27}{65}, \frac{12}{13}\right)$: A resistor costs approximately $0.42 and condenser costs approximately $0.92.
c)	*x*: number of skiers *y*: number of snowboarders	$x + y = 54$ $x = 2y$	(36, 18): There are 36 skiers and 18 snowboarders.
d)	*x*: mass of a bottle (g) *y*: mass of a glass (g)	$2x + 5y = 440$ $3x + 3y = 534$	(150, 28): The mass of a bottle is 150 g and the mass of a glass is 28 g.

3. The coordinates of the rooftop are (7, 12).

Practice 3.3 (*cont'd*)

Page 177

4.

	1) System of equations	2) Mass of the object
a)	*x*: mass of a ball (g) *y*: mass of a brick (g) $4x + 3y = 4000$ $y = 250 + 3x$	(250, 1000): The mass of a ball is 250 g and the mass of a brick is 1000 g.
b)	*x*: mass of a ball (g) *y*: mass of a cube (g) $x = 41 + 7y$ $x = 77 + 4y$	(125, 12): The mass of a ball is 125 g and the mass of a cube is 12 g.
c)	*x*: mass of a ball (g) *y*: mass of a cube (g) $5x + 4y = 92$ $2x + 2y = 40$	(12, 8): The mass of a ball is 12 g and the mass of a cube is 8 g.

5. a) *Several answers possible. Example:*
   $^-5x + 2y + 9 = 0$
   b) *Several answers possible. Example:*
   $2y = 24x + 10$

6. The diameter of Mercury is 4880 km.

7. (⁻9, 5.25)

Practice 3.3 (*cont'd*)

Page 178

8. a) (⁻1.25, ⁻32.5) b) no solution
   c) $\left(\frac{-4}{3}, \frac{23}{3}\right)$ d) $\left(\frac{-9}{4}, \frac{-7}{4}\right)$

9. 3865 adults and 4500 children have visited the exhibition within the last month.

10. The first machine produces 1000 screws/h whereas the second machine produces 1300 screws/h.

11. Company ③ is in the best financial situation because after 8 months, its assets would exceed those of the other companies.

12. ⁻40°

Practice 3.3 (*cont'd*)

Page 179

13. For sales of over $1,000,000, Employer B's offer is more appealing.

14. a) k = 27
    b) For all values of k other than 27.

15. a) The coordinates of point B are (−4, 8) and the coordinates of point C are (6, 3).
    b) He must buy approximately 15.63 m^3 of soil.

16. After 25 min, the temperature would be the same.

17. Both planes would fly at the same altitude after 7.5, 25 and 35 s.

SECTION 3.4

Half-planes in the Cartesian plane

Problem — Page 180

No, since the total cost is \$50 and the amount budgeted is less than \$50.

Activity 1 — Page 181

a. No, because by replacing the values in the inequality, one obtains $15 > 3 \times 5$, which is false.

b. There would be at least 30 green spotlights.

c. The degree of precision of the GPS is greater than 60%.

d. No, because by replacing the values in the inequality, one obtains $45 - 30 \leq 10$, which is false.

e. There is an infinite number of solutions.

Activity 2 — Page 182

a. $y > \frac{10}{3}x$

b. 1) No. 2) No. 3) No. 4) Yes.

c. 1) If the inequality sign is $>$, the coloured half-plane would be situated above the boundary line.

2) If the inequality sign is $>$, the boundary line would be dotted.

d. 1) No, none of the points situated below the dotted line satisfy the inequality $y > \frac{10}{3}x$.

2) Yes, all the points situated above the dotted line satisfy the inequality $y > \frac{10}{3}x$.

3) No, none of the points situated on the dotted line satisfy the inequality $y > \frac{10}{3}x$.

e. 1) The inequality becomes $y \geq \frac{10}{3}x$.

2) The boundary line of the equation $y = \frac{10}{3}x$ is represented by a solid line rather than a dotted line.

3) The coordinates of the points situated on the boundary line are part of the solution set.

Technomath — Page 183

a. 1) $y \geq x + 14$ 2) $y \leq -0.5x - 13$

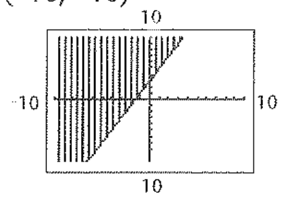
b. 1) $y \geq x + 14 \Rightarrow -10 \geq 16 + 14 \Rightarrow -10 \geq 30$ is false, therefore the ordered pair (16, −10) does not belong to the solution set of the inequality.

2) $y \leq -0.5x - 13 \Rightarrow -15 \leq 0.5 \times -23 - 13 \Rightarrow -15 \leq -1.5$ is true, therefore the ordered pair (−23, −15) belongs to the solution set of the inequality.

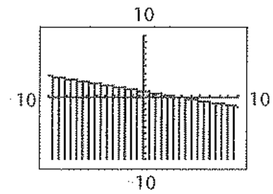
c. *Several answers possible. Example:*

1) (−10, −10) 2) (−40, 1)

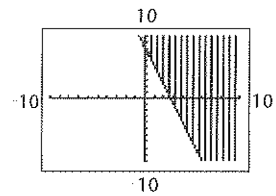
d. 1)



2)



3)



Practice 3.4 — Page 186

1. a) $5x + 10y \leq 300$ b) $x > 3y$

c) $x \geq 2y$ d) $x \geq 5y$

2. a) $y \leq \frac{-2x + 4}{3}$ b) $y \geq \frac{-5x}{2} + 11$

c) $y \geq -3x - 5$ d) $y \geq \frac{-5x}{4} + 5$

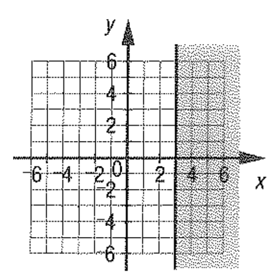
e) $y \leq \frac{x}{2} - 10$ f) $y \geq -x + \frac{13}{3}$

g) $y \geq -\frac{10}{9}x + 5$ h) $y \geq \frac{3x}{4} + \frac{17}{2}$

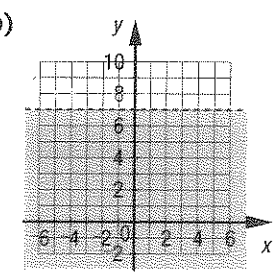
i) $y > -x + \frac{1}{2}$

3. A 3, B 1, C 4, D 2

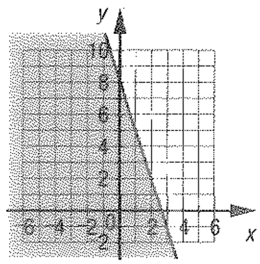
4. a)



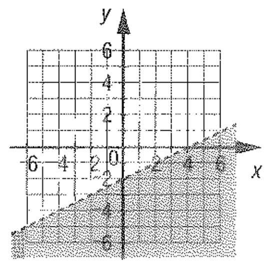
b)



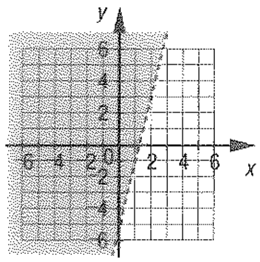
c)



d)



e)



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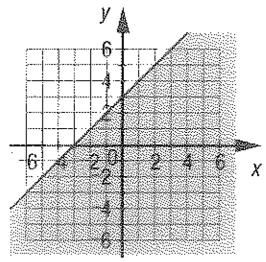
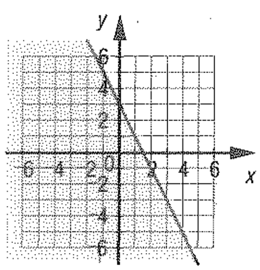
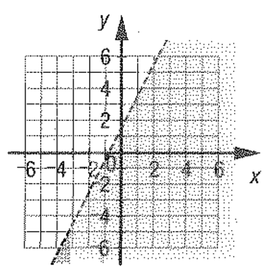


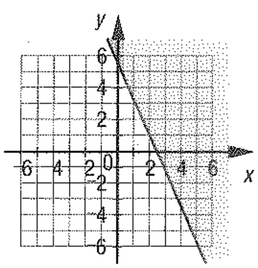
g)



h)



i)



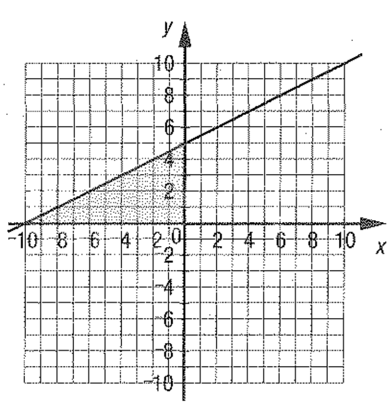
Practice 3.4 (*cont'd*)

Page 187

5. a) $y \geq \frac{1}{2}x - 5$ b) $y > 5x - 25$

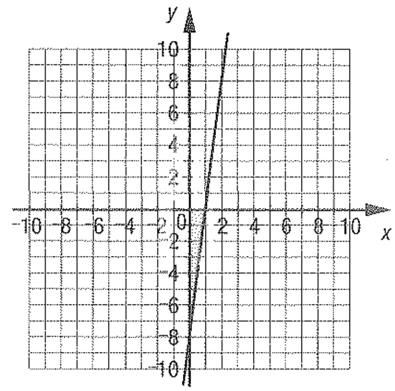
   c) $y < -\frac{1}{5}x + 40$ d) $y \leq -\frac{3}{2}x + 1$

6. a) 1)



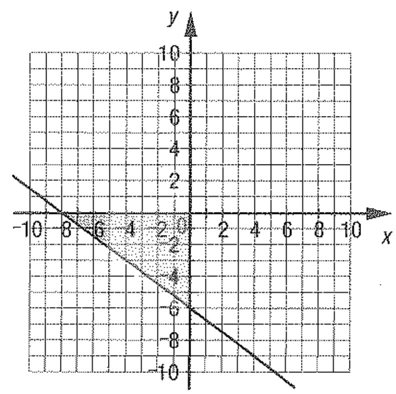
2) The area of the region is 25 u^2.

b) 1)



2) The area of the region is 4 u^2.

c) 1)



2) The area of the region is 24 u^2.

7. a) *Several answers possible. Example:* $y \geq 0$

   b) *Several answers possible. Example:* $y \geq 0$

   c) *Several answers possible. Example:* $y \geq 2$

   d) *Several answers possible. Example:* $y \geq -2$

8. There are two solutions: (1, 1) and (1, 2).

Practice 3.4 (*cont'd*)

Page 188

9. a) A, B, C b) B, D, E

   c) A, B, D d) A, B, C, F, G

10. a) $x + y \geq 24$ and $x + y \leq 45$.

    b) The blue region represents the solutions for both inequalities.

11. a) $y \geq -0.8x + 16$

    b) The number of hours of training per week.

    c) 1) No.

       2) Yes.

       3) Yes.

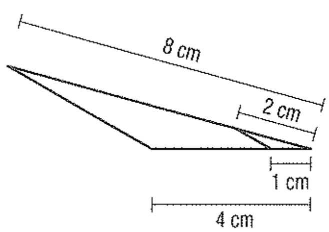
Practice 3.4 (*cont'd*) **Page 189**

12. a) $0.05a + 0.06b \leq 207$
    b) $0.05a + 0.06b = 207$
    c) The points situated on the boundary line are part of the solution set, since they correspond to the points where the value of the combined annual interests is equal to \$207.
    d) Region **A** represents the solution set.
13. a) x: width of the field (m)
    y: length of the field (m)
    b) 1) $x \geq 40$ 2) $y > 500 - x$
    c) 1) $2x + 2y > 1000$ 2) $2x + 2y \leq 2000$
14. The area of this industrial park is 3 km^2.

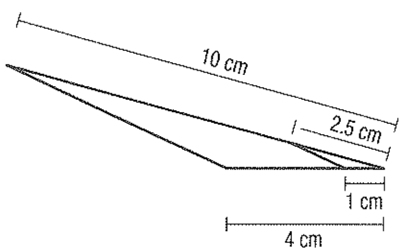
SPECIAL FEATURES 3

Chronicle of the past **Page 191**

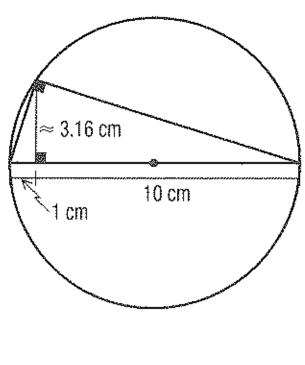
1. $2x^2 + 2x + 4 = 0$
2. *Several answers possible. Example:* 9cc + 5c.5
3. a)



b)

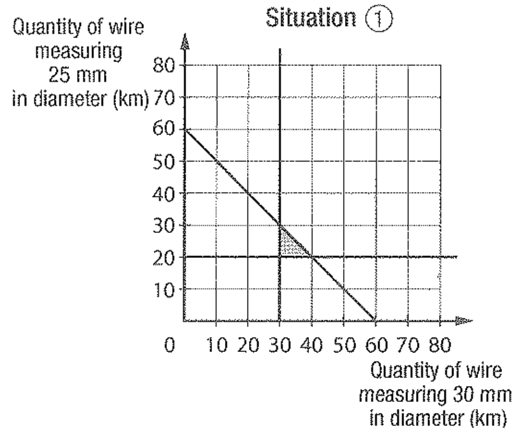


c)



In the workplace **Page 193**

1. a) x: the quantity of wire measuring 30 mm in diameter
   y: the quantity of wire measuring 25 mm in diameter
   $x \geq 30$, $y \geq 20$, $x + y \leq 60$
   b)



   c) This zone represents all the solutions that satisfy to all three demands.
2. a) The equation that corresponds to the high-tension line is $y = -4x + 119$.
   b) The equation of the line passing through City **A** and City **B** is $y = \frac{x + 85}{4}$.
   c) The distance between both cities is approximately 20.62 km.
   d) The coordinates of the dam are (20, 39).

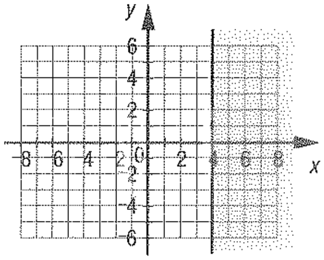
Overview **Page 194**

1. a) $\left(\frac{2}{3}, \frac{7}{3}\right)$ b) $(-2, -1)$
   c) $\left(\frac{9}{2}, -\frac{9}{2}\right)$ d) $\left(-\frac{3}{8}, \frac{9}{4}\right)$
   e) $(4, -9)$ f) $\left(-\frac{127}{50}, \frac{13}{5}\right)$
   g) $(-12, -4)$ h) $\left(\frac{8}{3}, 8\right)$
   i) $\left(\frac{33}{8}, \frac{15}{8}\right)$
2. a) 1) $\frac{5}{6}$ 2) ≈ 31.24 u 3) Square
   b) 1) $-\frac{2}{5}$ 2) ≈ 44.1 u 3) Triangle
   c) 1) 4 2) ≈ 32.98 u 3) Rectangle
   d) 1) $-\frac{8}{9}$ 2) ≈ 38.08 u 3) Parallelogram

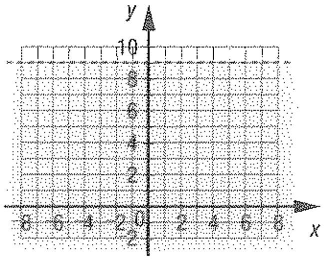
Overview (*cont'd*) **Page 195**

3. a) The inequality that corresponds to this situation is $x + 2y < 120$.
   b) 1) No. 2) No. 3) No.
   4) Yes. 5) Yes. 6) No.

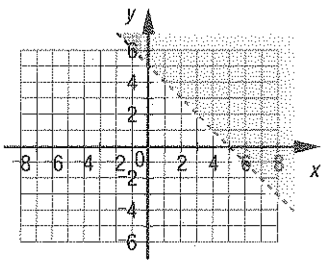
4. a)



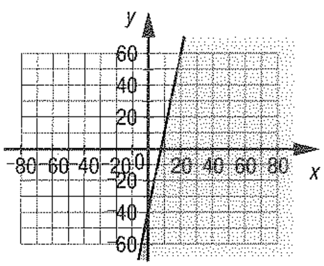
b)



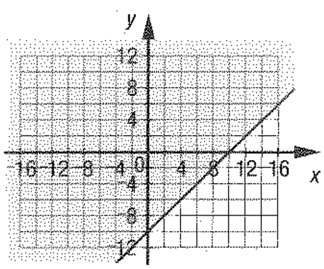
c)



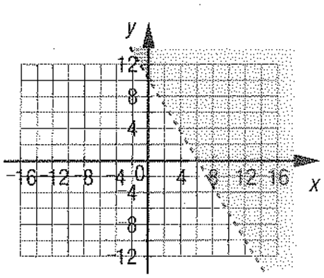
d)



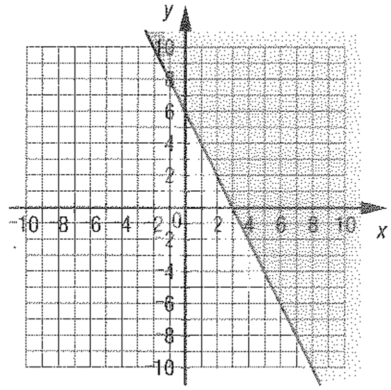
e)



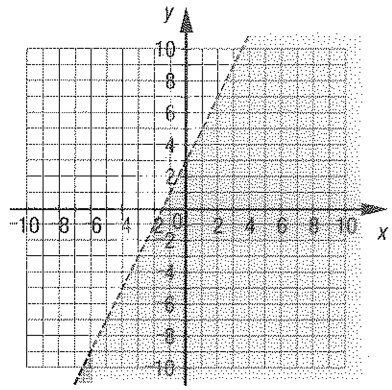
f)



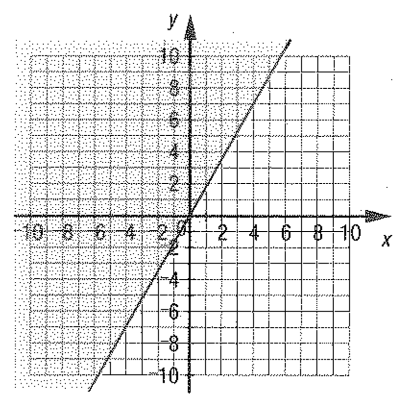
g)



h)



i)



5. a) $y < \frac{^-x}{4} - 1$ b) $y \leq 12x + 2$
   c) $y > {^-}\frac{4}{5}x + 1$ d) $y \geq \frac{5}{4}x$

Overview (*cont'd*) **Page 196**

6. a) (12, 16) b) (3, 12)
   c) $\left(^-4, \frac{9}{2}\right)$ d) (59, 40)
7. a) The lines are non-coinciding parallel lines because they have the same slope but a different *y*-intercept.
   b) The lines are coinciding parallel lines because they have the same slope and the same *y*-intercept.
   c) The lines are perpendicular because the product of their slope is equal to -1.

d) The lines are non perpendicular intersecting lines because the product of their slope is not equal to -1.

8. a) $l_1: y = 2x - 12$, $l_2: y = \frac{-1}{2}x + 8$

b) $l_3: y = \frac{9x + 15}{8}$, $l_4: y = \frac{8x + 108}{15}$

c) $l_5: y = -x + \frac{\sqrt{3} + 1}{2}$; $l_6: y = \frac{\sqrt{3}}{3}x$

9. a) *Several answers possible. Example:* A(2, 10)

b) *Several answers possible. Example:* A(36, $^-$7)

c) *Several answers possible. Example:* $A(\sqrt{13}, 8)$

d) *Several answers possible. Example:* $A(6 + \sqrt{242}, ^-3)$

10. a) $y = 3x - 5$ b) $y = x + 8$

Overview *(cont'd)* **Page 197**

11. a) 1) *x*: rhythm of a metronome set to *vivace*
y: rhythm of a metronome set to *andante*

2) $x \geq 2y + 6$

b) 1) *x*: quantity of solar energy captured by a heliostat
y: quantity of solar energy captured by a system of fixed solar panels

2) $x \leq 1.7y$

c) 1) *x*: time required to take measurements with a carpenter's compass
y: time required to take measurements with a regular ruler

2) $x \leq \frac{y}{2}$

d) 1) *x*: size of images enlarged by the overhead projector
y: size of images of the original document

2) $x \leq 15y$

12. a) ($^-$1, 4) b) ($^-$1, $^-$6)

c) ($^-$2.8, $^-$2.4) d) ($^-$2.8, 1.6)

Overview *(cont'd)* **Page 198**

13. a) Helicopter ① must cover approximately 11 390.63 m.

b) The coordinates of this point are approximately (10 057.59, 4468.92).

c) 1) Helicopter ① covered approximately 0.53 of the initial distance.

2) The equation associated to this new route is $y = -\frac{3661}{3860}x + \frac{71\ 379\ 699}{7720}$.

d) The equation associated to the route of this helicopter is $y = \frac{10\ 775x - 44\ 465\ 590}{20\ 072}$.

14. 5 ml of a 5% hydrochloric acid solution must be added to 5 ml of a 20% hydrochloric acid solution.

15. The fire hydrant would be situated at point $\left(\frac{5}{8}, \frac{27}{4}\right)$.

Overview *(cont'd)* **Page 199**

16. a) The maximum area is 784 m^2.

b) The maximum area is 1500 m^2.

c) The maximum area is 3000 m^2.

17. a) The coordinates of the centre of gravity are $\left(\frac{4}{3}, 0\right)$.

b) The coordinates of the centre of gravity are $\left(\frac{251}{226}, \frac{56}{113}\right)$.

c) The coordinates of the centre of gravity are $\left(-\frac{7}{2}, -\frac{3}{2}\right)$.

d) The coordinates of the centre of gravity are $\left(1, -\frac{4}{3}\right)$.

18. The production would be 50 kL of oil and 125 kL of natural gas.

Overview *(cont'd)* **Page 200**

19. The cost to restore the floor is approximately \$249.41/m^2.

20. a) 1) The distance is approximately 340.27 km.

2) The distance is approximately 749.09 km.

3) The distance is approximately 309.23 km.

4) The distance is approximately 708.89 km.

b) No, the slope of Johannesburg-Umtata is $-\frac{353}{32}$ and the slope of Pietersburg-Pietermariztburg is $-\frac{373}{34}$.

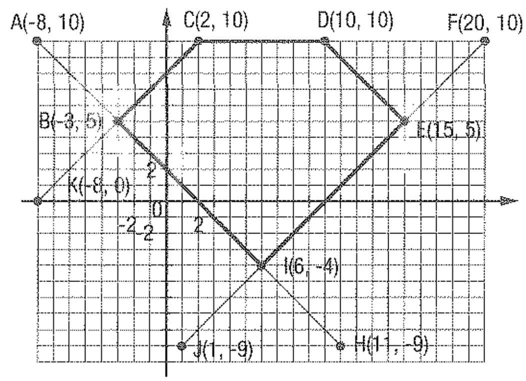
c) This route would have been longer, that is approximately 2978.6 km instead of approximately 2107.48 km.

Bank of problems

Page 201

21. The coordinates of the trap door are (8, 6).

22.



23. 1) Determine the unknowns:
   x: quantity (in ml) of the 1st solution
   y: quantity (in ml) of the 2nd solution

   2) Establish the equations needed to produce 550 mL of Cecoluse for \$60.
   $50x + 350y = 550$
   $12.5x + 11y = 60$

   3) Solve the system of equations
   $x \approx 3.91$, $y \approx 1.01$

   4) Determine the quantity of each solution needed to produce 20 L.
   $x \approx 7109.09$ mL
   $y \approx 12\ 854.55$ mL

Bank of problems (*cont'd*)

Page 202

24. The distance travelled is approximately 1623.39 m.

25. The archaeologists would explore 36 locations.