



ANSWER KEY

Visions 4 to 7



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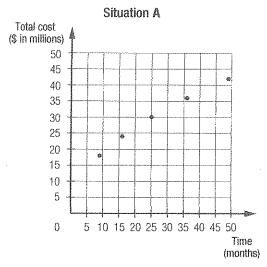


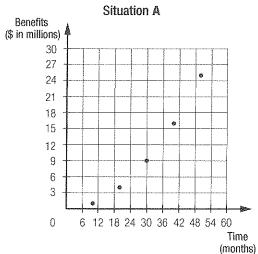


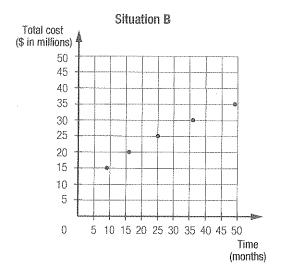
A new drug

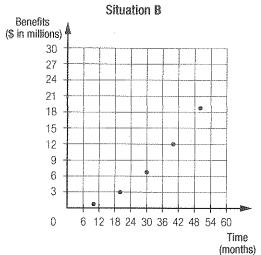
The following procedure can be used to determine the most profitable situation for the marketing of this drug over a period of 100 months.

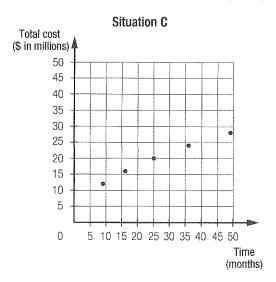
 Model each of the tables of values graphically to determine what type of function they correspond to.

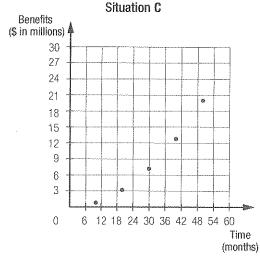












- Associate the scatter plots of the graphs representing the total cost as a function of time with a curve corresponding to a square root function, and associate those of the graphs representing the profits as a function of time with a curve corresponding to a second-degree polynomial function.
- Determine each of the six rules.

Situation A: Total cost: $f(x) = 6\sqrt{x}$

Benefits: $f(x) = 0.01x^2$

Situation B: Total cost: $f(x) = 5\sqrt{x}$

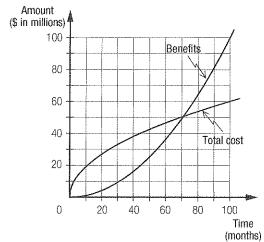
Benefits: $f(x) = 0.0075x^2$

Situation C: Total cost: $f(x) = 4\sqrt{x}$

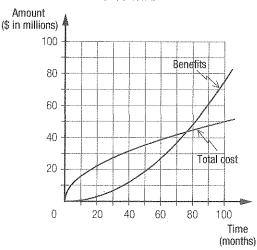
Benefits: $f(x) = 0.008x^2$

 Draw a graphical representation of each situation over a period of 100 months.

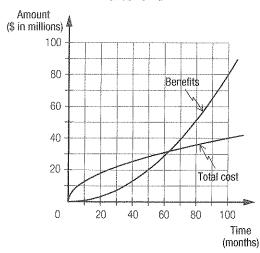




Situation B



Situation C



• For each situation, determine when the benefits and costs are equal.

Note: These points in time can be precisely determined by using the comparison method.

Situation A: \approx 71.14 months Situation B: \approx 76.31 months

Situation C: ≈ 63 months

 Determine the overall costs and benefits after 100 months for each situation.

Situation A: Total cost: \$60 million

Benefits: \$100 million

Situation B: Total cost: \$50 million

Benefits: \$75 million

Situation C: Total cost: \$40 million

Benefits: \$80 million

Determine the most profitable situation.

Situations A and C each show a benefit of \$40 million. However, Situation C appears to be the most profitable since the benefits exceed the costs more rapidly.

- Calculate the number of doses contained in each bottle.
 - Gel A: 250 doses
 - Gel B: 250 doses
 - Gel C: 200 doses
 - Gel D: 187.5 doses
- Calculate the duration of the efficacy of a dose of gel.
 - Gel A: approximately 37 min
 - Gel B: approximately 62 min
 - Gel C: approximately 45 min
 - Gel D: approximately 111 min
- For each of the three gels, determine the number of doses required for each person for one work day.
 - Gel A: 13 doses
 - Gel B: 8 doses
 - Gel C: 11 doses
 - Gel D: 5 doses
- For each of the three gels, calculate the number of doses required for 800 employees during one year of work as well as the number of bottles required to meet this demand.
 - Gel A: 2 496 000 doses, 9984 bottles
 - Gel B: 1 536 000 doses, 6144 bottles
 - Gel C: 2 112 000 doses, 10 560 bottles
 - Gel D: 960 000 doses, 5120 bottles
- For each of the three gels, calculate the purchase price by using the "selling price of the gel" graphs.
 - Gel A: \$59,904
 - Gel B: \$52,224
 - Gel C: \$58,080
 - Gel D: \$53,760
- Gel B represents the most cost effective option with respect to this hospital's needs. Gel B offers a good quality/price ratio.

Prior learning 1

Page 4

- a. 1) 51 infected computers.
 - 2) 2501 infected computers.
 - 3) 125 001 infected computers.
- b. 1) 100 min
 - 2) 300 min or 5 h
- c. 1) 2501 computers would be infected.
 - 2) 6 250 001 computers would be infected.
 - 3) 15 625 000 001 computers would be infected.

2) 28

4) 2^{-3}

6) 2^{-12}

2) 3⁰ 4) 3-5

6) 3²

b) 5

d) 44

f) 5^7

b) b = 4

f) f = -6

d) d = 243

- d. 1) 10 001 computers could be infected.
 - 2) 100 000 001 computers could be infected.
 - 3) $10^{10} + 1$ computers could be infected.

Knowledge in action

Page 6

- 1. a) 1) 2^6
 - 3) 2^{10}
 - $5) 2^{-5}$
 - b) 1) 3⁴
 - 3) 3⁴
 - 5) 3-12
- 2. a) 2¹⁰
 - c) 6^{-3}
 - e) 1
- 3. a) a = 4

 - c) c = 32
 - e) e = 8
- 4. a) $\frac{1}{34}$
 - c) 5^3
 - e) 10^2
 - $g) 6^2$

- b) 8^{2}
- d) $\frac{1}{73}$
- f) $\frac{1}{3}$
- h) $-\frac{6^2}{5^2}$
- 5. a) 1) 4 cells.
 - 2) 32 cells.
 - 3) \approx 181 cells.
 - b) 1) There are 64 cells after 6 h.
 - 2) There are 2048 cells after 11 h.
 - 3) There are 5120 cells after approximately 12.32 h.

Knowledge in action (cont'd)

6. a)
$$\sqrt[3]{8} = \sqrt[3]{2^3} = 2^{\frac{3}{3}} = 2^1 = 2$$

b)
$$\sqrt[3]{\frac{1}{64}} = \sqrt[3]{4^{-3}} = 4^{\frac{-3}{3}} = 4^{-1} = \frac{1}{4}$$

- c) $\sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{3^3}{5^3}} = \frac{3}{5} = \frac{3^1}{5^1} = \frac{3}{5}$
- d) $\sqrt[3]{2^{-6}} = 2^{-\frac{6}{3}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
- e) $\sqrt[3]{216} = \sqrt[3]{6^3} = 6^{\frac{3}{3}} = 6^1 = 6$
- 7. a) The product of any real number multiplied by itself cannot be a negative number.
 - b) The product of a negative number multiplied by itself 3 times is a negative number.
- 8. a) a

b) b6

c) $2^3c^{\frac{2}{3}}$

d) 1

- e) $\frac{\sqrt{3}}{6e}$
- 9. a) The ball reaches a height of 8 m after the 1st bounce
 - b) The ball reaches height of 4.096 m after the 4th bounce.
 - c) The ball reachesa height of $10(\frac{4}{5})^n$ m after the *n*th bounce.
- 10. a) 128 volunteers were recruited.
 - b) There are 254 volunteers at the end of the fundraising campaign.
 - c) \$1,562,500
 - d) \$1,953,100

Knowledge in action (cont'd)

Page 8

- 11. a) 27 bassos.
 - b) 729 sopranos.
 - c) 1093 members.
- 12. a) 40 000 people received a hug in 2005.
 - b) 20 480 000 people will receive a hug in 2014.



Greatest-integer functions

Problem

Page 9

The distance covered is approximately 3.07 km.

Activity 1

Page 10

- a. The initial amount of the mortgage loan is \$87,000.
- b. The amount of each payment is \$750.
- c. The time interval between two consecutive payments is 30 days.
- d. The amount of the mortgage loan decreases by \$750 every 30 days.

- e. 1) \$86,250
- 2) \$85,500
- 3) \$84,750
- 4) \$84,750
- f. This loan will be fully reimbursed after 3480 days.

Activity 2

Page 11

- a. 1) The value of parameter a is 2 rather than 1.
 - 2) The value of parameter a is 0.5 rather than 1.
- b. 1) The height between two consecutive segments doubled.
 - 2) The height between two consecutive segments decreased by half.
- c. The vertical distance between two consecutive segments corresponds to the value of parameter a.

Activity 2 (cont'd)

Page 12

- d. 1) The value of parameter b is $\frac{2}{3}$ rather than 1.
 - 2) The value of parameter b is 2 rather than 1.
- e. 1) The length of a segment is 1.5 times greater.
 - 2) The length of a segment is two times smaller.
- f. The length of a segment corresponds to the reciprocal of the value of parameter b.
- g. 1) The value of parameter a is -2 rather than 1.
 - 2) The value of parameter **b** is -2 rather than 1.
- h. 1) The height between two consecutive segments doubled. With the increase in height, each segment became symmetric with respect to the *x*-axis.
 - 2) The length of each segment is two times smaller, and each segment became symmetric with respect to the *y*-axis once its length was decreased.
- i. 1) The vertical distance between two consecutive segments corresponds to the absolute value of parameter a and the segments are located in the 2nd and 4th quadrant.
 - 2) The length of a segment corresponds to the reciprocal of the absolute value of parameter b, the segments are located in the 2nd and 4th quadrant, and the left endpoint of each segment is an open point, whereas the right endpoint is a solid point.

Practice 4.1

Page 14

1. a) 0

b) 1

c) 5

d) 6

e) -10

f) -18

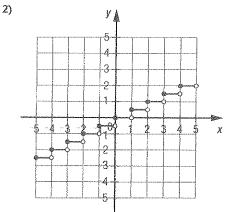
g) 123

h) -8

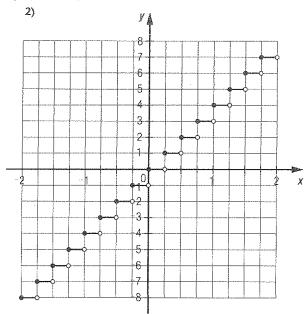
- 2. a) Yes, since an hourly rate is set.
 - b) No, since the carpet production process takes place continuously, not instantaneously once a day.
 - c) Yes, since a rate is set for each 30-min period.
 - d) No, since the number of laps in the pool is covered in a continuous manner.
- 3. A4, 85, C2, B1
- **4.** It is symmetric with respect to the y = x-axis.

Practice 4.1 (cont'd)

5. a) 1)
$$a = 0.5$$
, $b = 1$

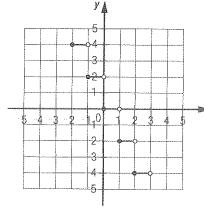


b) 1)
$$a = 1, b = 4$$



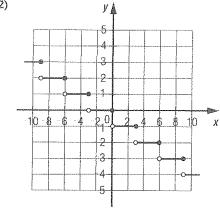
c) 1)
$$a = -2$$
, $b = 1$





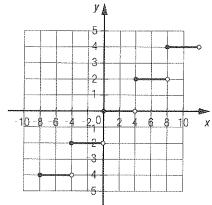
d) 1)
$$a = 1$$
, $b = -\frac{1}{3}$



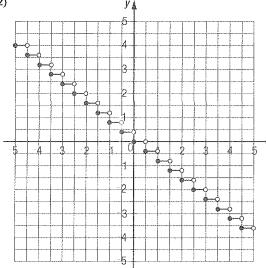


e) 1)
$$a = 2$$
, $b = 0.25$

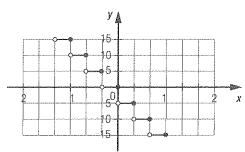




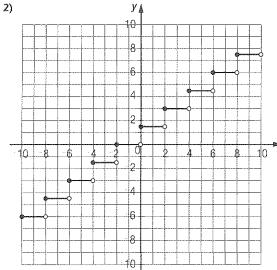
- f) 1) a = -0.4, b = 2
 - 2)



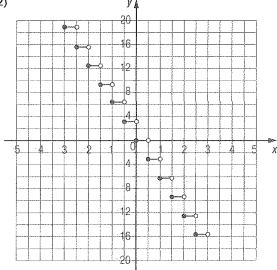
- g) 1) a = 5, b = -3
 - 2)



- **h)** 1) a = -1.5, $b = -\frac{1}{2}$



- i) 1) $a = -\pi, b = 2$
 - 2)

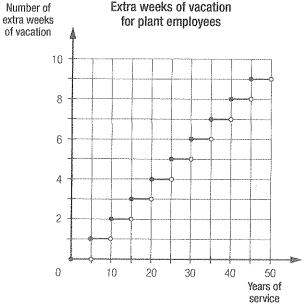


- 6. A 2, B 1, C 3, D 4
- 7. No. If the value of parameter a in the rule of the function is not an integer, then the values of the range will not be integers.

Practice 4.1 (cont'd)

- 8. a) 1) f(x) = 2[x]
 - 2) Domain: ℝ; range: $\{y \in \mathbb{Z} \mid y = 2x \text{ where } x \in \mathbb{Z}\}.$
 - 3) [0, 1]
 - 4) Increasing over \mathbb{R} .
 - **b)** 1) $f(x) = -3\left[\frac{-1}{3}x\right]$
 - 2) Domain: R; range: $\{y \in \mathbb{Z} \mid y = 3x \text{ where } x \in \mathbb{Z}\}.$
 - 3)]-3, 0]
 - 4) Increasing over \mathbb{R} .
 - c) 1) $f(x) = 2 \frac{x}{2}$
 - 2) Domain: R; range: $\{y \in \mathbb{Z} \mid y = 2x \text{ where } x \in \mathbb{Z}\}.$
 - 3) [0, 2[
 - 4) Increasing over \mathbb{R} .
 - **d)** 1) $f(x) = 2 \frac{x}{2}$
 - 2) Domain: R; range: $\{y \in \mathbb{Z} \mid y = 2x \text{ where } x \in \mathbb{Z}\}.$
 - 3) [-2, 0]
 - 4) Decreasing over \mathbb{R} .
- 9. a) An employee receives a retirement allowance after 8 years.
 - b) An employee receives a retirement allowance after 24 years.
 - c) The employee would receive \$1,000.

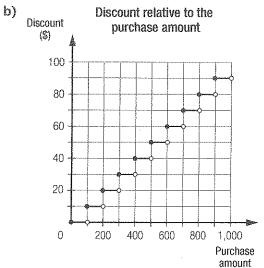
10. a)



b)
$$f(x) = \left[\frac{x}{5}\right]$$

c) An employee has six extra weeks of vacation after 30 years of service.

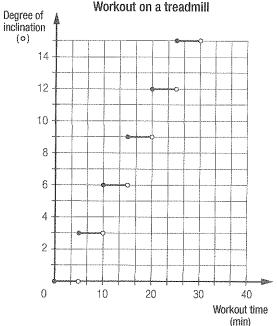
11. a)
$$f(x) = 10 \left[\frac{x}{100} \right]$$



- c) This person receives a \$50 discount.
- d) No, since the discount is offered only for every \$100 worth of purchases. For example, the discount is the same whether a person buys \$100 worth of products or \$199 worth of products.
- 12. a) The amount of the loan is \$1,500.
 - b) The amount of each payment is \$300.
 - c) The time interval between two consecutive payments is 3 months.

Practice 4.1 (cont'd)

13. a)



b)
$$f(x) = 3 \left| \frac{x}{5} \right|$$

- c) No, since the left endpoints of the intervals are open.
- **14.** a) Domain:]0, +∞[; range: {2.5, 3.5, 4.5, 5.5, 6.5, 7.5}.
 - b) The rate for each kilogram is the lowest for masses greater than 7500 kg.

56CTION **4.**2

Second-degree polynomial functions

Problem

Page 19

The difference between the kinetic energy of Ball B and that of Ball A is 11 858 J.

Activity 1

- **a.** 1) The maximum power output of this electrical appliance is 3000 W.
 - 2) The current flowing through the appliance when it is operated at its maximum power output is approximately 22.36 A.
- b. The resistance of this appliance is 6 ohms.
- c. 1) The appliance consumes 486 W of power.
 - 2) The appliance consumes 2646 W of power.
 - 3) The appliance consumes 4213.5 W of power.
 - 4) The appliance consumes 2999.82 W of power.

d. Variations Power Current (W) (A)Column A Column B 0 0 +150 +300 5 150 +450 +300 10 600 +750 +300 1.5 1350 +1050 20 2400

- e. 1) An increasing arithmetic sequence.
 - 2) A constant arithmetic sequence.

Activity 2

Page 21

- a. 1) i) $\sqrt[5]{3^2}$
- ii) $\sqrt[3]{7^3}$
- iii) √5
- iv) [√]√*a*^m
- 2) The square root of a negative number does not exist in the set of real numbers.
- **b.** 1) i) $\sqrt{6}$ iii) $\sqrt{38.5}$
- ii) $\sqrt{255}$
- iv) \sqrt{ab}
- 2) The degree of the root of each term of the product is not the same.
- c. 1) i) $\sqrt{6}$
- ii) $\sqrt{2}$
- iii) $\sqrt{10}$ iv) $\sqrt{}$
- 2) The degree of the root of the dividend and that of the root of the divisor are not the same.

Activity 3

Page 22

- a. 1) The independent variable is the time (in s).
 - 2) The dependent variable is the speed (in km/h).
- **b.** The car accelerates for 45 s, after which it reaches its maximum speed.
- c. $v = 62\sqrt{t}$
- d. 1) 186 km/h
- 2) $\approx 345.20 \text{ km/h}$
- 3) $\approx 240.12 \text{ km/h}$
- 4) $\approx 415.91 \text{ km/h}$

Technomath

Page 23

- a. The three curves pass through the origin and are all symmetric with respect to the *y*-axis.
- **b.** The coefficients of x^2 are -2.5, 0.6 and 4.
- c. The coefficient determines the sign of the function, as well as whether the curve will be compressed or stretched vertically.

- **d.** 1) The function is positive meaning the curve is located in the first and second quadrant.
 - 2) The function is negative meaning the curve is located in the third and fourth quadrant.
 - 3) The curve underwent a vertical compression.
 - 4) The curve underwent a vertical stretch.

Practice 4.2

Page 27

1. 42,84,C4,D3,E6,E2,G5,H5

Practice 4.2 (cont'd)

Page 28

2. a) f(x)

- b) g(x)
- c) h(x) and i(x).
- d) f(x) and g(x).

3. a) k(x)

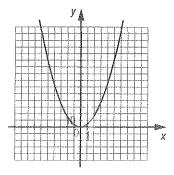
b) I(x)

 ϵ) j(x)

- d) m(x)
- 4. a) A first-degree polynomial function since, if the variations in the independent variable are constant, the variations in the dependent variable are also constant.
 - b) A second-degree polynomial function since, if the variations in the independent variable are constant, the variations in the dependent variable yield an arithmetic sequence and the difference between each of the variations is constant.
 - c) A zero-degree polynomial function since, if the variations in the independent variable are constant, the variations in the dependent variable are nil.
 - d) A second-degree polynomial function since, if the variations in the independent variable are constant, the variations in the dependent variable yield an arithmetic sequence and the difference between each of the variations is constant.
- 5. a) 1) Γ

Х	у
-6	144
-4	64
2	16
0	0
2	16
4	64
6	144

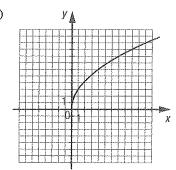
2)



b) 1)

Х	У
0	0
2	≈ 4.24
4	6
6	≈ 7.35
8	≈ 8.49
10	≈ 9.49
12	≈ 10.39

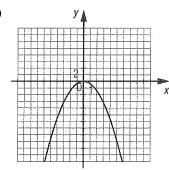
2)



c) 1)

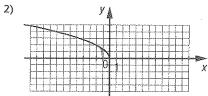
. Х	у
-6	-25.2
-4	-11.2
-2	2.8
0	0
2	2.8
4	-11.2
6	-25.2

2)



d) 1)

х	y
-12	≈ 4.90
-10	≈ 4.47
-8	4
-6	≈ 3.46
-4	≈ 2.82
-2	2
0	0



Practice 4.2 (cont'd)

Page 29

6. a)
$$x^2$$

c)
$$-9x^2$$

b)
$$-675x^2$$
 d) $\frac{875}{16}x^2$

7. a)
$$250\sqrt{x}$$

b)
$$-30\sqrt{-x}$$

c)
$$-3\sqrt{x}$$

d)
$$25.92\sqrt{-x}$$

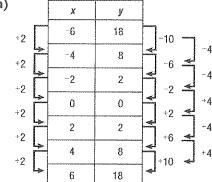
8. a)
$$y = -1.5x^2$$

b)
$$y = -0.5\sqrt{x}$$

c)
$$y = 0.4\sqrt{-x}$$

d)
$$y = 0.75x^2$$

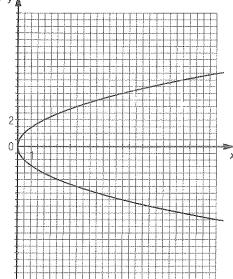
9. a)



b)

X	У
18	-6
8	-4
2	-2
0	0
2	2
8	4
18	6

c) y



d) The inverse of function *f* is not a function since at least one element of the domain corresponds to more than one element in the range.

Practice 4.2 (cont'd)

Page 30

10. a) 1) Domain: \mathbb{R} ; range: $[0, \infty[$.

- 2) Increasing over]0, ∞[and decreasing over $]-\infty$, 0[.
- 3) Positive.
- 4) Minimum: (0, 0).
- 5) 0
- 6) 0

b) 1) Domain: [0, ∞[; range: [0, ∞[.

- 2) Increasing.
- 3) Positive.
- 4) Minimum: (0, 0).
- 5) 0
- 6) 0

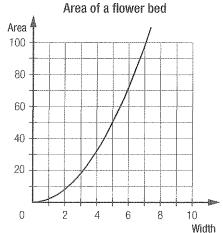
c) 1) Domain: \mathbb{R} ; range: $]-\infty$, 0].

- 2) Increasing over $]-\infty$, 0[and decreasing over $]0, \infty[$.
- 3) Negative.
- 4) Maximum: (0, 0).
- 5) 0
- 6) 0

d) 1) Domain: $]-\infty$, 0]; range: $]-\infty$, 0].

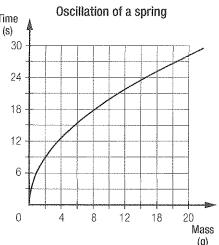
- 2) Increasing over $]-\infty$, 0[.
- 3) Negative.
- 4) Maximum: (0, 0).
- 5) 0
- 6) 0

- 11. a) 1) $y = 3x^2$
 - 2) $y = -5x^2$
 - 3) $y = 2.4x^2$
 - b) 1) $v = 2\sqrt{-x}$
 - 2) $y = 250\sqrt{x}$
 - 3) $y = 3.2 \sqrt{x}$
- 12. a) $A = 2x^2$
 - b)



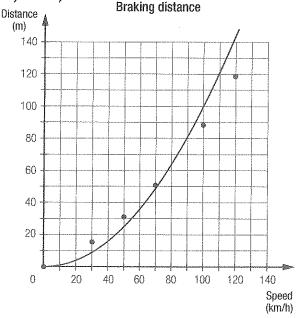
- c) 1) The curve would undergo a vertical stretch.
 - 2) The curve would undergo a vertical compression.

13. a) _{Time}



- **b)** 1) $\approx 34.41 \text{ s}$
 - 2) $\approx 39.74 \text{ s}$
 - 3) $\approx 48.67 \text{ s}$

14. a) and b)



c) Several answers possible. Example: $y = 0.01x^2$

d) 1)
$$\approx$$
 81 m

2)
$$\approx 196 \text{ m}$$

$$\approx 256 \text{ m}$$

15. a)
$$A = 8x^2$$

b)
$$A = 6.75x^2$$

c)
$$A = 2.5x^2$$

d)
$$A \approx 0.71x^2$$

Practice 4.2 (cont'd)

16. a) $y \approx 0.045 \sqrt{x}$

b) 1)
$$\approx 0.45 \text{ m/s}$$

2)
$$\approx 1.01 \text{ m/s}$$

3)
$$\approx 1.41 \text{ m/s}$$

17. $y = 0.00762x^2$

вестю» **4.**3

Second-degree and square root situations

Problem

Page 33

Page 32

No, a hard disk whose recordable "width" is 3 cm cannot contain 42 GB of data.

Activity 1

- a. 1) The time it took (in months) since the security was listed on the Stock Exchange for the share value to reach \$45.
 - 2) The time (in months) since the security was listed on the Stock Exchange during which the share value was less than \$45.
- **b.** Yes, since by isolating t^2 in the first equation, $t^2 = 36$, is obtained, which corresponds to the second equation.
- c. 1) 6
 - 2) It takes 6 months for a security listed on the Stock Exchange to attain a value of \$45.
- **d.** The value of the share is less than \$45 for the first 6 months.
- e. 1) The scooping time required to store 1790 L of water in each of the plane's tanks.
 - 2) The scooping time required to store a maximum of 1790 L of water in each of the plane's tanks.
- f. Yes, since by isolating \sqrt{t} in the first equation, $\sqrt{t} = 2$ is obtained, which corresponds to the second equation.
- q. 1) 4
 - 2) It takes 4 s to store 1790 L of water in each of the plane's tanks.
- h. [0, 4]

Activity 2

Page 35

a.
$$y > 2x^2$$

- **b.** 1) They are the ordered pairs for which the selling price of a perfume will be strictly greater than double the square of its manufacturing cost.
 - 2) They are the ordered pairs for which the selling price of a perfume is equal to double the square of its manufacturing cost, which is in this case excluded from the inequality.
- c. 1) No.

2) No.

3) Yes.

4) Yes.

d. 1) Yes.

2) No.

3) No.

- 4) No.
- e. 1) \$[0, 4[
- 2) \$[0, 6[
- 3) \$[0, 12[
- 4) $[0, 5\sqrt{7}]$

Technomath

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a. 1)
$$y \ge 0.5x^2$$

2)
$$y \le 0.25x^2$$

b. 1)
$$24 \ge 0.5(-3)^2$$

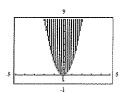
 $24 \ge 4.5$ corresponds to a true inequality; therefore, the ordered pair (-3, 24) belongs to the solution set.

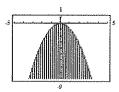
2)
$$15 \le 0.25(4^2)$$

 $15 \le 4$ corresponds to a false inequality; therefore, the ordered pair (4, 15) does not belong to the solution set.

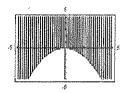
- c. 1) Several answers possible. Example: (-10, 1)
 - 2) Several answers possible. Example: (10, 1)

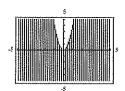






3)





Practice 4.3

Page 39

227.22	•	a)

Х	У
-5	75
i	3
3	27
7	147
12	432

)	х	У
	1	2
	36	12
	81	18
	169	26
	400	40

c)

X	y	
-3	37.8	
-1.5	9.45	
2.5	26.25	
5.9	≈ 146.20	
≈ 10.90	499	

d) [

-	Х	У
	2	6
	8	-12
	18	-18
	50	−30
	98	42

2. a) No solution.

b)
$$-16 \le x \le 0$$



c) No solution.

d)
$$-\frac{3}{2} \le x \le \frac{3}{2}$$



e) 7 < x

f) $x \le -\frac{1}{2}$



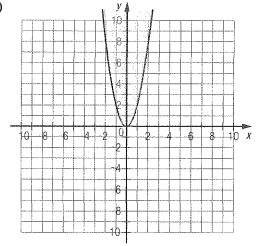
- 3. a) $f(x) < x^2$
- b) $f(x) \le -x^2$
- c) $f(x) \ge -x^2$
- d) $f(x) > x^2$

Practice 4.3 (cont'd)

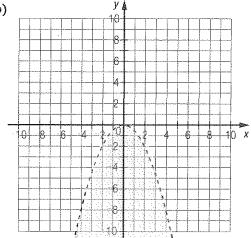
Page 40

- 4. a) $x^2 \ge 9$
- b) $x^2 < 64$
- c) $x^2 \le 17.64$
- d) $x^2 < 82.81$
- 5. a) $y \ge 2x^2$
- c) $y \ge \frac{-x^2}{5}$
- b) $y < \frac{x^2}{2}$ d) $y < -3x^2$

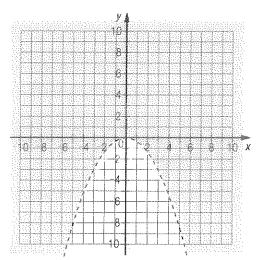
6. a)



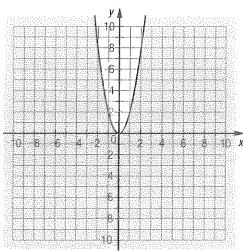
b)



C)



d)



7. Benjamin will have repaid his debt after 14 weeks.

Practice 4.3 (cont'd)

Page 41

- 8. a) 1) $A = c^2$
 - 2) $A = \frac{c^2}{2}$
 - 3) $A = \pi r^2$
 - b) 1) 40.96 cm²
 - 2) $\approx 57.25 \text{ cm}^2$
 - 3) $\approx 1520.53 \text{ cm}^2$
 - c) 1) 18 cm
 - 2) 9.4 cm
 - 3) 16.2 cm
- **9.** It took the missile approximately 47.43 s to reach the flying object.
- **10.** It takes 9 h to learn a score comprising 360 measures.
- 11. The edge must measure 15 cm.

Practice 4.3 (cont'd)

Page 42

- 12. a) c = h
 - b) 44.32 cm
- 13. a) 1) h(x) = -2400x + 33600
 - 2) $g(x) = 4000 \sqrt{x}$
 - 3) $f(x) = 500x^2$
 - b) 1) 500 people are carriers of the bacterium.
 - 2) 8000 people are carriers of the bacterium.
 - 3) 11 313 people are carriers of the bacterium.
 - 4) 2400 people are carriers of the bacterium.
 - c) 1) The number of bacteria is greater than 9000 over the interval of](2.25)², 10.25[months.
 - 2) The number of bacteria is less than 4000 over the interval of $[0, \sqrt{8} \ [\ \cup \]12.3, 14]$ months.
 - 3) The number of bacteria is between 7000 and 11 000 over the interval of $]\sqrt{14}$, $(2.75)^2[\ \cup\]9.41\overline{6}$, $11.08\overline{3}[$ months.

BECTION 4.

Exponential functions

Problem

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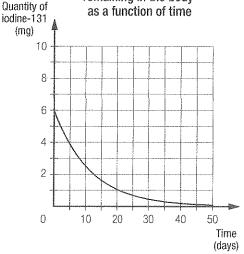
The person must make 1023 moves to rebuild a tower comprising 10 discs on another rod.

Activity 1

- a. There are 6 mg of iodine in the patient's body.
- b. 1) There are 3 mg of iodine in the patient's body.
 - 2) There are 1.5 mg of iodine in the patient's body.
 - 3) There are 0.75 mg of iodine in the patient's body.

€.

Quantity of iodine-131 remaining in the body as a function of time



d. The quantity of iodine will never be zero since it decreases by half of the initial quantity every 8 days.

Activity 2

Page 45

a.

Time (days)	1	2	3	4	5
Number of bacterias in Culture ①	8000	64 000	512 000	4 096 000	32 768 000
Number of bacterias in Culture ②	9000	81 000	729 000	6 561 000	59 049 000

- **b.** $n = 1000(3)^{2t}$
- c. At the end of the first day, Culture ① contains 8 times more bacteria than it did initially and Culture ② contains 9 times more bacteria than it did initially.
- d. Yes, because of the laws of exponents: $2^{3t} = (2^3)^t = 8^t$.
- e. $n = 1000(3)^{2t} = 1000(3^2)^t = 1000(9)^t$

Activity 3

Page 46

- a. 1) $\log_4 64 = 3$
- 2) $32 = 2^5$
- 3) $\log_1 27 = -3$
- 4) $\log_{18} 6 = x$
- 5) $2 = 4^{\frac{1}{2}}$
- 6) $x = 7^3$
- 7) $\log_{\text{base}} m = n$
- 8) $m = (base)^n$
- **b.** 1) The logarithmic expression is the same in the three equalities.
 - 2) Only the base of the logarithmic forms is different in each of the three equalities.
- **c.** 1) Equality ①: ≈ 1.404
 - Equality (2): ≈ 1.404
 - Equality (3): ≈ 1.404
 - 2) The quotients are all equal.

- 3) To divide 2 logarithmic expressions, any base can be chosen, as long as it is the same for the dividend and the divisor.
- d. 1) ≈ 0.477
- $2) \approx 0.602$
- 3) ≈ 0.699
- 4) ≈ 0.778
- 5) ≈ 0.845
- e. 1) The values of the logarithms calculated are the same as those presented in the table when base a is equal to 10.
 - 2) The 60 key on a calculator can be used to calculate the logarithm of a number with base 10.

Activity 4

Page 47

- a. The greater the concentration of hydrogen ions in a solution, the lower the pH.
- b. 1) 10⁻¹² mol/L
- 2) 0.1 mol/L
- 3) 1000 mol/L
- c. 1) 13

2) 2

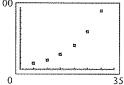
- 3) -2
- d. The function associated with this situation does not have a zero that would have corresponded to the pH of a solution when the concentration of hydrogen ions is nil.
- e. The concentration of an aqueous solution cannot be nil since it is impossible, for any real exponent, that the power of a number bearing an exponent is 0.

Technomath

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a. The value of a represents the initial value of the curve that models this situation. The value of b represents the base.

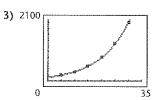




2)



 $y = 122.39(1.1)^x$



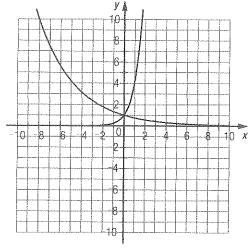
1. a) Yes.

b) No.

c) Yes.

d) Yes.

2. a)



b)	Function	i(x)	j(x)
	1) Initiai value	410	1
	2) Variation	Decreasing over ℝ	Increasing over R
	3) Sign	Positive on R	Positive on R

- 3. a) $y = 5(9)^x$
- b) $y = -3(2)^x$
- c) $v = 9^x$
- d) $y = -7(\frac{64}{27})^x$
- e) $y = 9.61^{x}$
- 4. a) $\log_5 125 = 3$
- b) $\log_3 81 = 4$
- c) $\log_c y = x$
- d) $\log_a 49 = 3$
- e) $\log_{\frac{4}{2}} \frac{3}{2} = -2$
- f) $\log_{t} \frac{r}{s} = d$

5. a) 3

b) 3

c) -1

d) 1

e) 0

- f) ≈ 2.0959
- $q) \approx 1.6309$
- h) ≈ 0.9201

Practice 4.4 (cont'd)

Page 54

- 6. A T, B 2, C 4, D 3
- 7. a) $y = 1.5^x$
- **b**) $y = 0.5^x$
- c) $y = 3(3)^{2x}$

- 8. a) 3
- b) 5
- c) $\frac{1}{2}$
- d) $\frac{1}{4}$

Practice 4.4 (cont'd)

Page 55

- 9. a) $y = -3 \log 2x$ b) $y = -3 \log_4 \frac{x}{2}$ c) $y = 5 \log_2 \frac{x}{4}$ d) $y = -3 \log_7 \frac{x}{2}$
- **10.** a) $f(x) = 45\left(\frac{5}{3}\right)^x$ b) $f(x) = \frac{1}{2}(6)^x$

 - c) $f(x) = -4(0.5)^x$
- d) $f(x) = 1.25(1.5)^x$
- 11. This family will have to allow a budget of \$229.73/week.

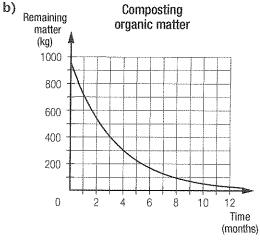
12. a) $p = 10^d$

Practice 4.4 (cont'd)

- b) The opacity of the film is approximately 316.23 u.
- c) No.
- **13.** a) $f(x) = 2000(1.065)^x$ where x represents the time (in years).
 - f(x) represents the value of the investment (in \$).
 - b) 1) The value of this investment will be \$2,268.45.
 - 2) The value of this investment will be \$2,493.18.
 - 3) The value of this investment will be \$2,740.17.
 - c) The value of the investment will be \$2,340 after approximately 2.49 years.
- 14. a) There are 10 000 000 spores on this person's hands before the gel is applied.
 - b) The number of spores will have decreased by half after 2 s.
 - c) The number of spores will be nil after 9 s.

Practice 4.4 (cont'd)

- 15. a) 1) There are 960 km of organic matter.
 - 2) There are approximately 170.86 km of organic matter.
 - 3) There are approximately 30.41 kg of organic matter.



- c) 1) The initial quantity of organic matter will be reduced by half after approximately 2.41 months.
 - 2) The quantity of organic matter will be 105 kg after approximately 7.69 months.
- **16.** a) $f(x) = 100(3)^x$ where x represents the time (in years).
 - f(x) represents the number of frogs.
 - b) There are 24 300 frogs.
 - c) There will be 218 700 frogs.



Exponential and logarithmic situations

Problem

Page 58

The sterilization process must last longer than 11 min for there to be fewer than 1024 germs.

Activity 1

Page 59

a.
$$0.98^t = 0.98$$

- **b.** 1) Only the presence of an exponent other than 1 distinguishes the two sides of the equality.
 - 2) For the two sides of the equality to be equivalent, the exponent t must have a value of 1.
- c. 1) $\frac{10(0.98)^t}{10} = \frac{5}{10}$ $(0.98)^t = 0.5$
 - 2) $\log_{0.98} 0.5 = t$
 - 3) $t = \frac{\log 0.5}{\log 0.98}$
- **d.** The temperature of the wire is approximately 34.31°C when its conductivity is 5 S/m.
- e. $10(0.98)^t \ge 5$ $\log 0.5$
- f. $t \le \frac{\log 0.3}{\log 0.98}$

Activity 2

Page 60

- a. The independent variable is part of the argument of the logarithm.
- b. 1) $\frac{3\log_7 2x}{3} = \frac{6}{3}$ $\log_7 2x = 2$
 - 2) $7^2 = 2x$
 - 3) $\frac{49}{2} = \frac{2x}{2}$ x = 24.5
- **c**.]0, ∞[
- d.]0, 24.5]

Practice 4.5

Page 63

1. a) 6

b) 40

c) -165

d) 0

e) 234

f) 25

- **2.** a) $5^{3x} = 5^3$, therefore, x = 1.
 - **b)** $10^{0.5x} = 10^3$, therefore, x = 6.
 - c) $(\frac{2}{3})^x = (\frac{2}{3})^2$, therefore, x = 2.
 - **d)** $6^{\frac{3}{2}x} = 6^2$, therefore, $x = \frac{4}{3}$.
 - e) $(\frac{2}{3})^{0.2x} = \frac{2}{3}$, therefore, x = 5.
 - f) $7^x = 7^0$, therefore, x = 0.
- 3. a) x = 1
- b) x = 2
- c) $x = -\frac{1}{2}$
- **d)** $x = -\frac{1}{3}$
- 4. a) ≈ 0.36
- b) ≈ 1.996
- c) ≈ 0.61
- d) ≈ -1.58
- e) ≈ -2.41
- f) ≈ 0.18

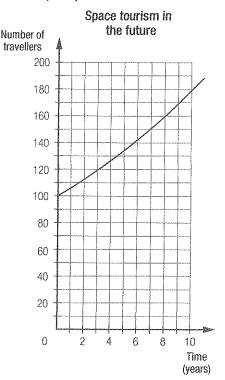
 $g) - \frac{4}{9}$

- h) Impossible in \mathbb{R} .
- 5. a) $x < \frac{5}{3}$
- b) x < -4
- c) $x \ge -2.5$
- **d**) x < 1
- e) $x \le 30$
- f) $x \ge \frac{5}{12}$
- 6. a) 972
- b) $\frac{2}{405}$
- c) ≈ 0.0063
- d) $\frac{1}{36}$
- e) $\frac{1}{4}$
- f) ≈ 0.0379
- 7. a) $0 \le x \le \frac{1}{3}$
- b) $x \ge 625$
- c) $x > \frac{1}{30}$ e) $0 < x \le \frac{1}{128}$
- d) $-\frac{1}{54} < x < 0$ f) $x \ge \left(\frac{27}{8}\right)^{0.75}$

Practice 4.5 (cont'd)

- 8. a) 1) The value of this investment is \$5,550.
 - 2) The value of this investment is \$6,150.50.
 - 3) The value of this investment is \$8,425.29.
 - b) 1) The value of the investment will have doubled after 6.64 years; however, since the interest rate is compounded annually, it will have doubled after 7 years.
 - 2) The value of the investment will have tripled after 10.53 years; however, since the interest rate is compounded annually, it will have tripled after 11 years.
 - 3) The value of the investment will have quadrupled after 13.28 years; however, since the interest rate is compounded annually, it will have quadrupled after 14 years.

b)



- c) 133 people will make this trip during the 5th year.
- d) The number of space tourists will exceed 3000 after approximately 58.37 years.
- 10. a) $y = 6.8(1.01)^x$

x: the time (years)

y: population (billions of inhabitants)

b) Number of inhabitants

> (billions) 16

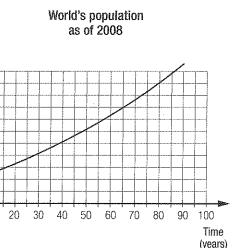
> > 14

12

10 8

6

10



- c) The world's population will be greater than 9.5 billion in 2042.
- 11. a) The interest rate is approximately 10.41%.
 - b) 1) An interest rate of approximately 14.87%.
 - 2) An interest rate of approximately 9.05%.
 - 3) An interest rate of approximately 5.95%.
 - c) The sum will have doubled after approximately 11.9 years, therefore, after 12 years.

Practice 4.5 (cont'd)

- 12. a) 1) The earthquake is approximately 3.16 times less powerful.
 - 2) The earthquake is approximately 31.62 times more powerful.
 - 3) The earthquake is 100 times more powerful.
 - 4) The earthquake is approximately 1258.93 times more powerful.
 - b) 1) The magnitude is approximately 6.9.
 - 2) The magnitude is approximately 2.7.
- 13. a) There will remain approximately 60.75 g of radioactive materials.
 - b) The remaining quantity of radioactive materials is less than 40 q after approximately 9.19 years.
- 14. a) The initial charge decreases by half after approximately 0.15 s.
 - b) The charge of the capacitor is less than 1% of its initial charge after more than 1 s.
 - c) No, since the charge of the capacitor follows the rule of an exponential function which can never equal 0.

Practice 4.5 (cont'd)

Page 66

- 15. a) The pressure is approximately 99.37 kPa.
 - b) The pressure is 100 kPa at an altitude of approximately 0.79 km.
 - c) A pressure of 70 kPa is obtained at a temperature of approximately 137.38 K.
- **16.** a) $Q_r = 12(0.8)^t$
 - b) Approximately 4.92 g of reagents remain.
 - c) The quantity of reagents will have decreased by half after approximately 3.11 s.
 - d) The experiment lasts approximately 31.77 s.

Practice 4.5 (cont'd)

- 17. The number of insects will be greater than 9000 after more than approximately 37.17 days.
- 18. a) 1) 1.5%
- $2) \approx 14.03\%$
- b) 1) 98.5%
- $2) \approx 85.97\%$
- 19. a) The metal rod expands by approximately 2.54 mm.
 - b) The metal rod expands by 2 mm at 20°C.
 - c) The metal rod expands by more than 4 mm at temperatures greater than 2000°C.
 - d) The metal rod can attain a maximum expansion of approximately 3.35 mm.

- 20. a) The wave undergoes an attenuation of -0.97 dB.
 - b) The power of the wave is approximately 3981.07 W.
 - c) The wave undergoes an attenuation of -20 dB.



Chronicle of the past

Page 69

1. a) 182

- b) 1701
- c) 13 835 076
- d) 391

2. a) No.

b) Yes.

c) No.

- d) Yes.
- 3. a) The logarithm of a product is equivalent to the sum of the logarithms of the factors.
 - b) The logarithm of a quotient is equivalent to the difference of the logarithms of the factors.

In the workplace

Page 71

- 1. a) The maximum tension will approach 75 000 N.
 - b) The maximum tension will approach 120 000 N.
 - c) The maximum tension will approach 150 000 N.
- 2. 41 layers of carbon fibres are required.
- 3. a) The pressure increases.
 - b) The speed decreases.
- 4. The speed is approximately 447.21 m/s.

Overview

Page 72

- 1. a) $y = \frac{3}{16}x^2$
- c) $y = -\sqrt{-3x}$
- d) $v = 4 \log x$
- **e)** $y = -2(3)^x$
- f) y = -3[-x] + 3

Overview (cont'd)

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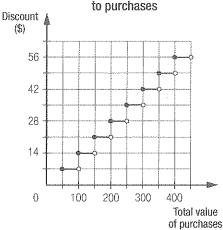
- 2. a) 1) Increasing over $]-\infty$, 0] and decreasing over $[0, \pm \infty]$.
 - 2) Negative over \mathbb{R} .
 - 3) 0
 - 4) 0
 - **b)** 1) Increasing over \mathbb{R} .
 - 2) Positive over \mathbb{R} .
 - 3) None.
 - 4) 7
 - Increasing over]0, ±∞[.
 - 2) Negative over [0, 1] and positive over $[1, +\infty[$.
 - 1 3)
 - 4) None.

- d) 1) Decreasing over $]-\infty$, 0].
 - 2) Positive over $]-\infty$, 0].
 - 3) 0
 - 4) 0

3. A4, B3, C5, D1, E2

4. a)

Discount relative to purchases



b) $y = 7 \frac{1}{50} x$

x: purchase amount

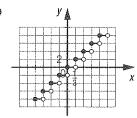
y: discount

- c) The amount of the client's purchase is greater than or equal to \$300 but less than \$350.
- d) The client obtains a discount of \$28.
- 5. a) $y = 8^x$
- **b)** $y = 0.25^x$
- c) $y = \sqrt{5^x}$

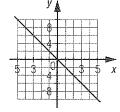
Overview (cont'd)

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6. a) 1)

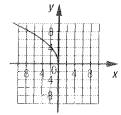


- 2) Domain: \mathbb{R} , range: $\{y \in \mathbb{Z} \mid y = 2x \text{ where } x \in \mathbb{Z}\}.$
- None.
- b) 1)



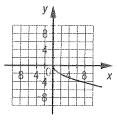
- 2) Domain: \mathbb{R} , range: $]-\infty$, 0].
- 3) Maximum: 0

c) 1)



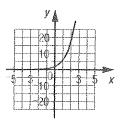
- 2) Domain:]-∞, 0], range: [0, ∞[.
- 3) Minimum: 0

d) 1)



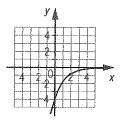
- 2) Domain: [0, ∞[, range:]-∞, 0].
- 3) Minimum: 0

e) 1)



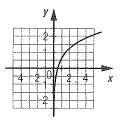
- 2) Domain: \mathbb{R} , range: $]0, -\infty[$.
- 3) None.

f) 1)



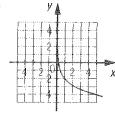
- 2) Domain: \mathbb{R} , range: $]-\infty$, 0[.
- 3) None.

g) 1)



- 2) Domain: $]0, \infty[$, range: \mathbb{R} .
- 3) None.

h) 1)



- 2) Domain:]0, ∞[, range: ℝ.
- 3) None.
- 7. a) $x = \pm 1.5$
- b) x = -16
- c) x = 16
- d) x = 4
- e) x = 4

- f) $x \approx 1.29$
- g) $x \approx -1.63$
- h) x = 0.625
- i) -640
- 8. a) -1 < x < 1
- b) $-0.5 \le x \le 0.5$
- c) x < -36
- d) x < 1156
- e) $x < \frac{3}{2}$ g) $x > \frac{1}{4}$
- f) x < 1h) x > 9
- i) -0.0625 < x < 0
- 9. a) Option C will provide her with a 1.5% commission at the beginning of her career.
 - b) 1) 1.62%
 - 2) 6%
 - $3) \approx 3.54\%$
 - c) Option C (≈ 16.25% commission, whereas she would earn 12.5% with Option A and 10% with Option B).

Overview (cont'd)

- 10. a) The value of the investment will be \$26,500.
 - b) This sum corresponds to 106% of the initial sum.
 - c) The value of the investment will be \$26,522.50.
 - d) This sum corresponds to 106.09% of the initial sum.

		Plan A		Plan B			
Time (months)	Time (years)	Calculation	Value of the investment (\$)	Time (months)	Time (years)	Calculation	Value of the investment (\$)
9 -2-	- 0	-25 000/106%	25.000		- 0 -	= 25 0000 00P	25000
The second secon		6288292		6	0.5	25 000(1.03)	25,750
: 1 2 :		25 000(1.06) ¹	26,500	5342	4.5	- 25-000(1.03)*	26,522.50
				16	15	25 000(1.03) ³	=27,318.18
24	2	25 000(1.06) ²	28,090		Ž	25 000(1.03) ⁴	~28,137.72
				30	25	25 000(1.03) ⁵	~28,981.85
36	3	25 000(1.06) ³	29,775.40	36	3	25 000(1.03)6	=29,851.31
				42	35	25 000(1.03) ⁷	≈30,746.85
48	4	25 000(1.06)4	≈31,561.92	48	4	25 000(1.03) ⁸	≈31,669.25
****	182						
	X	25 000(1.06)		Υ	25 000(1.03) ²

f) Plan B is the most advantageous. According to the table, the value of Plan B is greater than that of Plan A after one year.

Overview (cont'd)

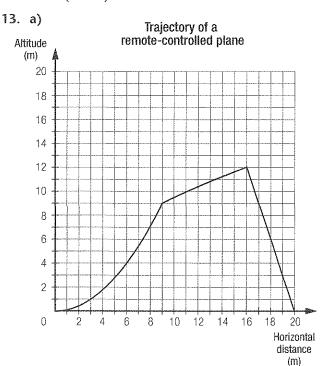
e)

Page 76

- 11. a) The ordered pair (2, 45) indicates that after 2 s, the base of the seat is at a height of 45 m relative to the ground.
 - b) The duration of the ride is 14 s.
 - c) $y = 11.25x^2$
 - d) 1) The seats are 7.2 m from the ground.
 - 2) The seats are approximately 19.01 m from the ground.
 - 3) The seats are 36.45 m from the ground.
 - e) 1) The seats are 4.05 m from the ground after 0.6 s.
 - 2) The seats are 11.25 m from the ground after 1 s.
 - 3) The seats are 28.8 m from the ground after 1.6 s.
- **12.** a) $y = 15\sqrt{x}$
 - x: time elapsed since the end of the season (month)
 - y: percentage of unemployed people
 - **b)** 1) Approximately 25.1% of the people are unemployed.
 - 2) Approximately 28.46% of the people are unemployed.
 - c) 1) 45% of this city's workers are unemployed after 9 months.
 - 2) 3250 of this city's workers are unemployed after approximately 2.78 months.

d) No, since for 100% of the workers to be unemployed using the rule that models this situation, 44 months are required which corresponds to more than one year.

Overview (cont'd)



- b) 1) The plane is at an altitude of approximately 5.44 m.
 - 2) The plane is at an altitude of approximately 10.82 m.
 - 3) The plane is at an altitude of 9 m.
- c) 1) The horizontal distance covered is approximately 6.71 m or approximately 18.33 m.
 - 2) The horizontal distance covered is approximately 13.44 m or approximately 16.33 m.
 - 3) The plane will never reach an altitude of 16 m.
- 14. There will be 28 000 bacteria.

15. a)
$$y = 4\left(\frac{1}{8}\right)^x = 4(8^{-1})^x = 4(2^{-3})^x = 4(2)^{-3x}$$

b) $y = 4\left(\frac{1}{8}\right)^x = 4(8^{-1})^x = 4(2^{-3})^x = 4(2)^{-3x} = 2^2 \times 2^{-3x} = 2^{-3x+2}$

- **16.** a) Mary: $a = 200(1.03)^{6t}$ Eugene: $a = 500(1.07)^t$
 - b) 1) The difference will be approximately \$215.83.
 - 2) The difference will be approximately \$553.90.
 - 3) The difference will be approximately \$5,007.36.
 - c) The value of Mary's assets will be \$1,407 after approximately 66 years.
 - d) The value of Eugene's assets will be less than \$1,609 for approximately 18 years.
 - e) No, during the 9th year, the value of Mary's assets will become greater than that of Eugene's assets.

Overview (cont'd)

Page 78

- 17. The frequency of a la is approximately 220.31 Hz.
- **18. a)** 1) $y = 2^{3x}$ x: time (h) y: number of bacteria 2) y = 1 073 741 824
 - b) The number of bacteria is greater than 100 million after more than approximately 8.86 h.
- 19. a) Final value of the investment (\$)
 40,000
 24,000
 16,000
 8,000
 0
 2 4 6 8 10
 Time (years)

b) The value of the investment will be approximately \$31,366.24.

Overview (cont'd)

Page 79

- 20. a) The pilot is 112.36 m from her ejection point.
 - b) The pilot is 250 m from her ejection point after approximately 3.16 s.
 - c) Yes, the pilot is safe since she is approximately 315.06 m from her ejection point.
- 21. The two cities will have the same number of inhabitants in 2012.

Bank of problems

Page 80

22. • Determine the rule of the orange curve. The rule is that of a second-degree polynomial function of the form $y = ax^2$.

$$400 = a \times 400^2$$

$$a = 0.0025$$

Therefore, the rule is $y = 0.0025x^2$.

 Calculate Titan's vertical distance when its horizontal distance is 1351 million kilometres.

$$y_3 = \frac{1351^2}{225} - \frac{88 \times 1351}{9} \times \frac{48400}{9} \approx 280$$

This value corresponds to the *y*-coordinate of the separation point of the spacecraft.

• Determine the *x*-coordinate of the separation point of the spacecraft.

$$280 = 0.0025x^2$$

$$112\ 000 = x^2$$

$$x \approx 334.66$$

The coordinates of the separation point of the spacecraft are (334. $\bar{6}$, 280).

23. Jairo invested his money at an annual interest rate of approximately 11.87%.

Bank of problems (cont'd)

- 24. The measurement of the initial surface area of the skin will triple after approximately 5.46 h or approximately 5 h 27 min.
- 25. The volume of this area is approximately 143 849.03 m³.

Statistical measures and correlation

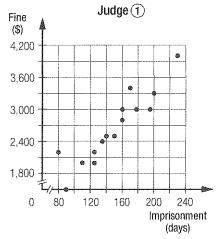


Justice

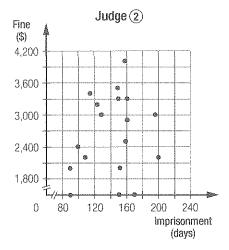
The following is a possible procedure that may be used to analyze the decisions rendered by each of the judges.

- Calculate the mean of the sentences handed down by each judge.
 - Judge ①: The mean of the prison sentences: ≈ 149.87 days
 The mean of the fines: $\approx $2,653.33$
 - Judge ②: The mean of the prison sentences: ≈ 141.67 days The mean of the fines: $\approx $2,779.47$
 - Judge ③: The mean of the prison sentences: \approx 151.25 days The mean of the fines: \approx \$2,487.50
- Determine the mean deviation of each statistical variable.
 - Judge ①: For the imprisonment, the mean deviation is approximately 32.54 days. For the fines, the mean deviation is approximately \$524.22.
 - Judge ②: For the imprisonment, the mean deviation is equal to 24.4 days.

 For the fines, the mean deviation is equal to \$584.24.
 - Judge ③: For the imprisonment, the mean deviation is approximately 49.94 days. For the fines, the mean deviation is approximately \$486.25.
- Describe the correlation between the number of days of imprisonment and the amount of the fine.



This correlation is very strong and positive.



This correlation is very weak and relatively positive.

For Judge ③, the contingency table indicates that the correlation between the two variables is linear and positive.

Conclusion: The judge's choice

In light of the results studied, Judge ②'s decisions appear to be consistent, but the relationship between the prison sentence and the fine imposed on a person who is convicted is very weak. For Judges ① and ③, the relationship between the prison sentence and the amount of the fine appears to be strong. However, by analyzing the mean deviation relative to the sanctions imposed by each of the judges, Judges ① and ② appear to render the most consistent decisions.



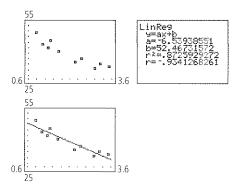
Shaping public opinion

The following procedure may be used to solve this LES.

 Analyze the effect of the investments made in advertising using technology.

Impact on the percentage of alcohol-related accidents.

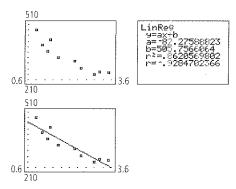
Variable *x* corresponds to the amount invested in prevention (\$ in millions). Variable *y* corresponds to the percentage of alcohol-related accidents.



The linear correlation coefficient is approximately -0.93, which indicates a strong and negative correlation. The equation of the linear regression line is $y \approx 26.5x + 52.5$.

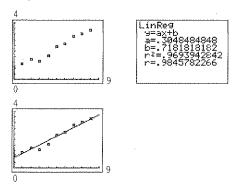
Effect on the number of road-block arrests.

Variable *x* corresponds to the amount invested in prevention (\$ in millions). Variable *y* corresponds to the number of road-block arrests.



The linear correlation coefficient is approximately -0.93, which indicates a strong and negative correlation. The equation of the linear regression line is $y \approx -82.3x + 505.8$.

• Determine the equation of the linear regression line where *x* corresponds to the time elapsed (in years) since 1998 and *y*, to the amount invested in prevention (\$ in millions).



The linear correlation coefficient is approximately 0.99, which indicates a very strong and positive correlation. The equation of the linear regression line is $y \approx 0.3x + 0.72$.

• Determine the possible amount invested in prevention in 2020.

By substituting 22 for variable x in the equation $y \approx 0.3x + 0.72$, it can be determined that the amount invested in prevention is approximately \$7.32 million.

 Calculate the percentage of alcohol-related accidents that could potentially occur and the number of road-block arrests in 2020.

By substituting 7.32 for variable x in the equation $y \approx 26.5x + 52.5$, it can be determined that the percentage of alcohol-related accidents should be approximately 4.92%.

 Calculate the number of road-block arrests in 2020.

By substituting 7.32 for variable x in the equation $y \approx 282.3x + 505.8$, it can be determined that the number of road-block arrests should be nil.

Conclusion

The analysis of this statistical data indicates that the investments made in prevention from 1998 to 2007 had a major effect on certain factors related to alcohol-associated impaired driving. In fact, the percentage of alcohol-related accidents and road-block arrests are decreasing significantly, which indicates that attitudes towards this scourge appear to be changing. If the trend continues, in 2020, the number of alcohol-related accidents will decrease to approximately 5% and the number of road-block arrests could become nil. Investments must continue to be made in prevention campaigns that influence people's driving habits.



Prior learning 1

Page 84

a. 1) Agent A: none

Agent B: \$212,500

2) Agent A: \$196,500

Agent B: \$212,500

3) Agent A: \$203,000

4) Agent A: \$155,000

Agent B: \approx \$202,777.78

Agent B: \$125,000

b. Agent A: \$56,840

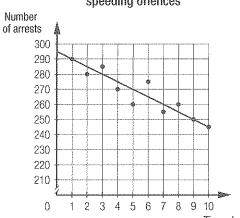
Agent B: \$73,000

Prior learning 2

Page 85

- a. No. For example, the number of arrests decreased by 10 from May to June, whereas it decreased by 20 from October to November.
- b. 1) A first-degree polynomial function.
 - 2) Several answers possible. Example:

Arrests for major speeding offences



Time elapsed since April 1, 2008 (months)

Knowledge in action

Page 88

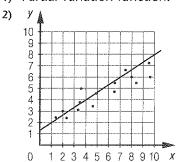
- 1. a) 1) 1 computer.
- 2) 1 computer.
- 3) 1.33 computers.
- b) 1) 275 cm
- 2) 225 cm
- 3) ≈ 238.27 cm

- c) 1) 64
- 2) 64
- 3) ≈ 63.55
- d) 1) 15 years.
 - rs. 2) 15 years.
- 3) 15 years.
- 2. a) 636 graduates.
- b) [25, 30[years.
- c) 30 years.
- d) [30, 35[years.
- e) ≈ 31.67 years.
- 3. a) Distribution (1).
 - b) Distributions (1), (2) and (3).

Knowledge in action (cont'd)

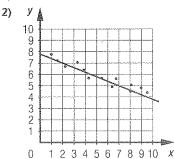
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4. a) 1) Partial variation function.

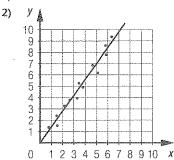


3)
$$y = \frac{2}{3}x + \frac{4}{3}$$

b) 1) Partial variation function.



- 3) $y = -\frac{2}{5}x + \frac{39}{5}$
- c) 1) Direct variation function.

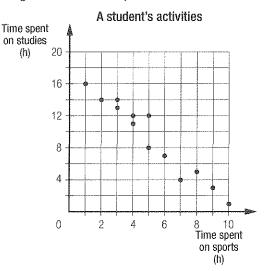


- 3) $y = \frac{7x}{5}$
- 5. a)

Measure	Group ①	Group ②
Median	42.5	45
Range	12	6
Mode	40	45

b) Class ②, since all the results in this class are greater than or equal to 43.

б. т



Knowledge in action (cont'd)

- 7. Several answers possible. Example:
 - a) y = 25x + 150
 - b) 1) 42 L of syrup.
- 2) 2 L of syrup.
- c) 3900 L of maple sap are required.

8. The distance covered by this airplane would be approximately 1040 km.



Statistical measures and graphs

Problem

Page 91

Overall, Shuttle C. is involved in the longest missions.

Activity 1

Page 92

- a. The mean of the distances covered is 305.97 yards.
- b. 1) Steve Allan.
- 2) Bubba Watson.

Golfer	Distance covered by the ball during a drive (yards)	Deviation between the distance covered by the ball and the mean of the distances
Bubba Watson	315.1	9.13
Robert Garrigus	311.0	5.03
John B. Holmes	310.3	4.33
Dustin Johnson	309.7	3.73
Steve Allan	303.2	2.77
Tag Ridings	303.0	2.97
Nick Watney	302.9	3.07
Adam Scott	302.1	3.87
Davis Love III	301.3	4.67
Charles Warren	301.1	4.87

- d. The mean deviation is approximately 4.44 yards.
- e. The value of the distances are clustered for the female golfers, since the mean deviation is less.
- f. Bubba Watson, since his deviation from the mean is greater than that of Lorena Ochoa.

Activity 2 - The evolution of statistics

Page 93

- a. 1) Distribution A: ≈ 10.86 Distribution B: ≈ 22.57
 - 2) Distribution A: 18 Distribution B: 34
- b. Distribution B, since the mean deviation is greater.

Distribution A c. 1)

Dis	stribution A	Dis	stribution B
Data value	Square of the deviations from the mean (data value – mean) ²	Data value	Square of the deviations from the mean (data value – mean) ²
4	≈ 47.02	14	≈ 73.47
5	≈ 34.31	16	≈ 43.18
6	≈ 23.59	21	≈ 2.47
11	≈ 0.02	23	≈ 0.18
15	≈ 17.16	24	≈ 2.04
17	≈ 37.73	26	≈ 11.76
18	≈ 51.02	34	≈ 130.61

- 2) Distribution A: ≈ 30.12 Distribution B: ≈ 37.67
- d. Distribution A: ≈ 5.49 Distribution B: ≈ 6.14
- e. In Distribution A, since the mean deviation is the smallest.

Technomath

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a. 31

- b. A7
- c. All the data values in the table from the first cell A1 to the last cell E9.
- d. 1) 2704

2) ≈ 96.57

3) ≈ 25.11

4) ≈ 29.53

Practice 5.1

Page 96

1. a) 1) Temperature at the beginning of June

Day	Temperature (°C)	Deviation from the mean
Sunday	30	0
Monday	28	2
Tuesday	32	2
Wednesday	33	3
Thursday	27	3
Friday	29	1
Saturday	31	1

2) 1.71 °C

b) 1) Temperature at the beginning of January

Day	Temperature (°C)	Deviation from the mean
Sunday	-20	6.14
Monday	-15	1.14
Tuesday	-22	8.14
Wednesday	-18	4.14
Thursday	-10	3.86
Friday	8	5.86
Saturday	4	9.86

2) 5.59 °C

c) 1) **Enrollment in engineering**

Number of registrations	Deviation from the mean
258	32.14
280	10.14
264	26.14
295	4.86
310	19.86
303	12,86
321	30.86
	registrations 258 280 264 295 310 303

2) 19.55 registrations

Area of different territories d) 1)

Territory	Area (km)²	Deviation from the mean
Vatican	0.44	2 206 170.91
France	547 030	1 659 141.35
Morocco	446 550	1 759 621.35
Argentina	2 766 890	560 718.65
Belgium	30 528	2 175 643.35
Canada	9 984 760	7 778 588.65
Québec	1 667 441	538 730.35

- 2) 2 382 659.23 km²
- 2. L_d

Practice 5.1 (cont'd)

Page 97

- 3. a) 1) ≈ 0.81
- $2) \approx 1.06$
- b) 1) ≈ 1.11
- $2) \approx 1.14$
- 4. No. In Florida, the temperature varies less than it does in Canada.

- 5. a) 40, 60
- b) 20, 40
- c) 30, 70
- d) Several answers possible. Example: 35, 40, 45, 55, 60, 65
- 6. a) The results are more concentrated in Group (1).
 - b) The results are more dispersed in Group (3).

Practice 5.1 (cont'd)

Page 98

- 7. a) Canadiens, Maple Leafs, Avalanche, Senators, Red Wings.
 - b) 1) Canadiens: 6.32. Senators: 6.08.

Maple Leafs: 6. Red Wings: 4.24.

Avalanche: 3.6.

2) Canadiens: 8.64.

Maple Leafs: 6.9. Red Wings: 5.34.

Senators: 7.17.

Avalanche: 4.

- c) The Avalanche was the most consistent team since the mean deviation and the standard deviation are the smallest of the five teams.
- 8. a) Alberta.

b) 2001: 3 479 272.08 2006: 3 703 554.16

Practice 5.1 (cont'd)

Page 99

- 9. a) In the distribution representing the maximum speed reached during the final. The mean deviation in this distribution is approximately 0.81 km/h, whereas the mean deviation in the distribution representing the maximum speed reached during a qualifying run is approximately 1.93 km/h.
 - b) In the distribution representing the maximum speed reached during a qualifying run.
- 10. a) 1) The deviation observed in British Columbia is the lowest in July.
 - 2) The deviation observed in Ontario is the highest in August.
 - b) 1) \$5,151.1 million.
 - 2) \$4,992.51 million.
 - c) Québec and Ontario.

Practice 5.1 (cont'd)

Page 100

11. \approx \$1,425.42

12. 32 students are in this group.

- 13. No. Although the best candidates, meaning those whose results have a deviation from the mean greater than 10 are admitted, all the worst candidates will also be admitted. In fact all those whose results have a deviation from the mean less than 10 will be in this group.
- 14. a) 1) ≈ 1284.59 ≈ 1218.72
- 2) ≈ 1543.01 ≈ 1493.94
- b) The standard deviation.



Qualitative interpretation of a correlation

Problem

Page 101

Person **B** and **D** best express the trend displayed in the table.

Activity 1

Page 102

- a. 1) 118 players.
- 2) 54 players.
- 3) 4 players.
- 4) At the least, no players, at the most, 27 players.
- b. $\approx 12.71\%$
- c. 1) Yes, since by drawing a diagonal from the upper left-hand corner to the lower right-hand corner of the contingency table, it may be noted that the data values are more or less reversed.
 - 2) Yes. According to the data gathered, the greater a player's mass, the slower he is.

Activity 2

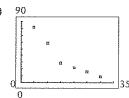
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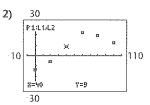
- a. Frances, since she installed an average of approximately 17.38 bundles of shingles in 1 h, whereas Sylvia installed an average of 16.88 bundles.
- **b.** Sylvia had the most consistent performance.
- c. Yes, the greater the number of bundles of shingles installed by Frances, the greater the number of bundles installed by Sylvia as well.
- **d.** Yes, a decreasing function appears to support the scatter plot.
- e. Yes. The steeper the slope of a roof, the fewer the number of bundles of shingles installed by Sylvia in 1 h.
- f. The relation is strong since the points form a curve and since they approach a straight line.

Technomath

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- a. The x-coordinate and y-coordinate of the point on which the cursor is placed..
- b. 1)





Practice 5.2

Page 108

- 1. a) Strong and positive correlation.
 - b) Strong and positive correlation.
 - c) Zero correlation.
- 2. a) 1) Positive correlation.
 - 2) Moderate correlation.
 - b) 1) Positive correlation.
 - 2) Weak correlation.
 - c) 1) Positive correlation.
 - 2) Moderate correlation.

Practice 5.2 (cont'd)

- 3. Scatter plot A.
- 4. a) Weak and negative correlation.
 - b) Strong and positive correlation.
 - c) Strong and negative correlation.
 - d) Moderate and positive correlation.
 - e) Perfect and positive correlation.
 - f) Moderate and negative correlation.
- 5. The height of the students in a school (Table) represents the strongest correlation since the largest numbers are grouped along a diagonal; however, for the time spent on the Internet each week (Table), the largest numbers are located at the extremities of the rows and columns.

Practice 5.2 (cont'd)

Page 110

2)

6. a) Points scored at the start of the 2008-2009 season

Points scored Number of games played	[0, 10[[10, 20[[20, 30[[30, 40[Total
[0, 10[4	0	0	0	4
[10, 20[2	0	0	0	2
[20, 30[4	0	1	0	5
[30, 40[2	5	2	0	9
[40, 50[- in	1	1	3	6
Total	13	6	4	3	26

- b) 17 players.
- c) No players.
- d) 13 players.
- e) $\approx 11.54\%$
- f) No, since most of the data do not follow any of the diagonals in the table.

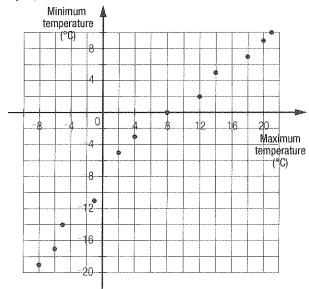
Practice 5.2 (cont'd)

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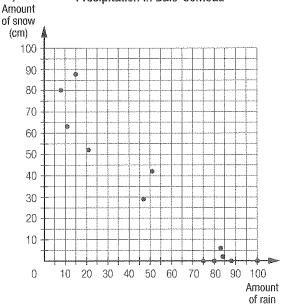
- 7. a) 1) Moderate correlation.
 - 2) Zero correlation.
 - 3) Weak correlation.
 - b) 1) Zero correlation.
 - 2) Strong correlation.
 - 3) Moderate correlation.
 - c) 1) Zero correlation.
 - 2) Zero correlation.
 - 3) Weak correlation.

8. a) 1)

Weather in Baie-Comeau

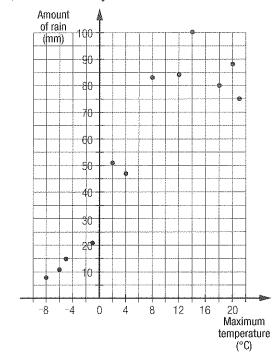


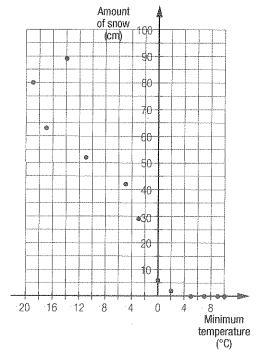
Precipitation in Baie-Comeau



(mm)

3) City of Baie-Comeau





- b) 1) Very strong correlation.
 - 2) Moderate correlation.
 - 3) Weak correlation.
 - 4) Weak correlation.
- 9. No. The correlation is very weak, almost zero.

Practice 5.2 (cont'd)

Page 112

- **10.** Yes, the scatter plot indicates that the correlation is relatively strong.
- 11. a) 1) Strong and positive correlation.
 - 2) Strong and positive correlation.
 - 3) Weak and positive correlation.
 - b) The linear model is the most appropriate since the radius of curvature of an exponential function or a square root is too small.

12. a)

Age Number of cars	[15, 20[[20, 25[[25, 30]	[30, 35[[35, 40[[40, 45[[45 and +	Total
1	2	2		_			_	5
2	1	2	1	1			2	6
3	-	1	-	-	714	2	-	3
4	_	1	2	2	2	_	_	7
5	-	-	_	1	1	1	_	3
6	_	-	_	_	1	0	2	3
7	-	-	-	_		-	2	2
8	-		-	-	-	_	2	2
9	-		-	_		-	1	1
10	-	,,,,,	-	_		_	-	0
11	-		-	_		_	1	1
Total	3	6	3	4	4	4	9	33

b) Moderate and positive correlation.

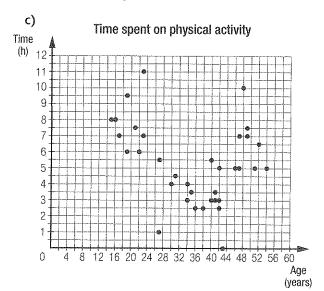
Practice 5.2 (cont'd)

Page 113

- 13. Statement is true since the variables vary inversely and the scatter plot is narrow.
- 14. a) Time spent on physical activity

Time Age (h)	[0, 2[[2, 4[[4, 6[[6, 8[[8, 10[[10, 12[
[15, 25[0	0	0	6	3	1
[25, 35[1	1	6	0	0	0
[35, 45[1	8	2	0	0	0
[45, 55[0	0	5	5	0	1

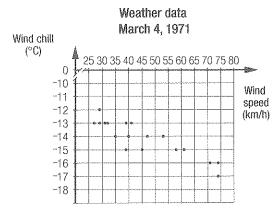
b) Weak and negative correlation.



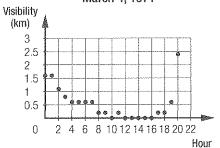
Practice 5.2 (cont'd)

Page 114

15. Several answers possible. Example: The wind speed and the wind chill. The time and the visibility.



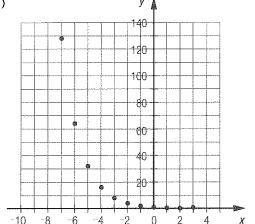
Weather data March 4, 1971



Practice 5.2 (cont'd)

Page 115

16. a) 1)



2) Perfect and negative correlation.

- b) 1) y

 1.4

 1.2

 1

 0.8

 0.6

 0.4

 0.2

 4 6 8 10 12 14 x
 - 2) Strong and positive correlation.
- **17**. Yes. It is a moderate and positive linear correlation.

SECTION

Quantitative interpretation of a correlation

Problem

Page 116

The boiling point of water is approximately 56.96°C when the atmospheric pressure is 30 kPa.

Activity 1

a.

Page 117

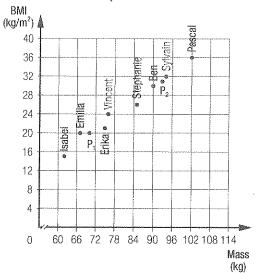
Mass and BMI of

b.

Group of volunteers	Mean mass (kg)	Mean BMI (kg/m²)	Ordered pair comprised of mean mass and mean BMI
Isabel, Emilia, Erika, Vincent	70	20	P ₁ (70, 20)
Stephanie, Ben, Sylvester, Pascal 92.75		31	P ₂ (92.75, 31)

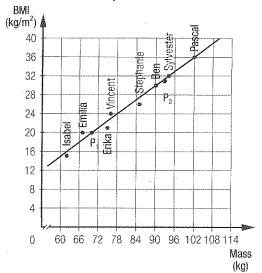
c. 1)

Mass and BMI of Group A volunteers



2)

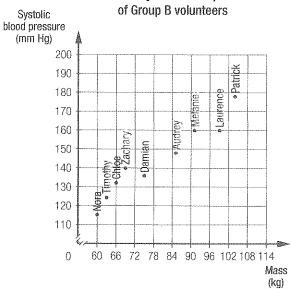
Mass and BMI of Group A volunteers



- **d.** $y = \frac{44x 1260}{91}$
- e. 1) The BMI of a person whose mass is 80 kg could be approximately 24.84 kg/m².
 - 2) The mass of a person could be approximately 109.3 kg.

Activity 1 (cont'd)

Mass and systolic blood pressure



g.

Group of volunteers	Median mass (kg)	Median systolic blood pressure (mm Hg)	Ordered pair comprised of median mass and median systolic blood pressure
Nora, Timothy, Chloe	63	124	M ₁ (63, 124)
Zachary, Damian, Audrey	75	140	M ₂ (75, 140)
Melanie, Laurence, Patrick	99	160	M ₃ (99, 160)

h. 1) 79

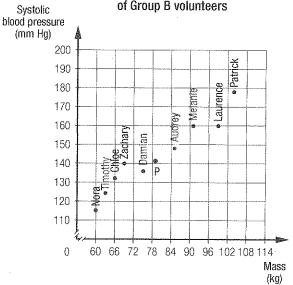
2) $\frac{424}{3}$

.

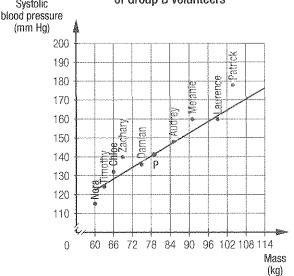
.

1) M

Mass and systolic blood pressure of Group B volunteers



2) Mass and systolic blood pressure Systolic of Group B volunteers



k.
$$y = x + \frac{187}{3}$$

- The systolic blood pressure of a person whose mass is 80 kg could be approximately 142.33 mm Hg.
 - 2) The mass of that person could be approximately 127.67 kg.

Activity 2

Page 120

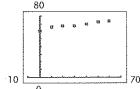
- a. Scatter plot ①: strong and positive correlation.
 Scatter plot ②: moderate and positive correlation.
 Scatter plot ③: moderate and positive correlation.
- b. Contractor 2.
- c. 1) For the rectangle in the graph labelled "Contractor ③."
 - 2) The rectangle is wide.
- **d.** 1) For the rectangle in the graph labelled "Contractor (1)."
 - 2) The rectangle is narrow.
- **e.** 1) Variables other than the living space affect construction costs.
 - 2) The wider the rectangle, the weaker the correlation.
- f. Contractor ② has the most consistent construction costs, since the rectangle surrounding the scatter plot is less inclined.

Technomath

Page 121

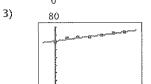
- a. The value of a represents the slope of the line.
- **b.** The value of **b** represents the *y*-intercept of the line.

b. 1)



Med-Med y=ax+b a=.195 b=61.9

2)



Page 125

Practice 5.3

-10

- b) ≈ 0.79
- 1. a) ≈ -0.68 c) ≈ 0.41
- d) ≈ -0.54
- 2. a) 1) ≈ -0.74
 - 2) Several answers possible. Example: $y \approx -x + 11$
 - b) 1) ≈ 0.47
 - 2) Several answers possible. Example: $y \approx 0.93x + 1.3$
 - c) 1) ≈ 0.85
 - 2) Several answers possible. Example: $y \approx x + 0.3$
 - d) 1) ≈ 0.06
 - 2) Several answers possible. Example: $y \approx 0.15x + 1.7$

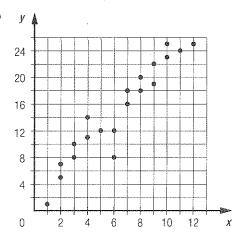
Practice 5.3 (cont'd)

Page 126

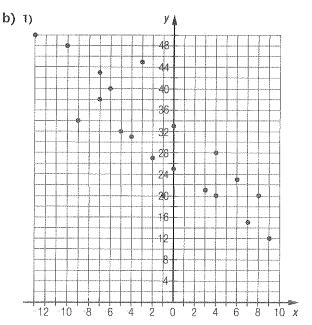
3. 7, 8, 5, 3, 6, 1, 2, 4

Practice 5.3 (cont'd)

- 4. a) $r \approx 1 \frac{1.2}{2.8}$; $r \approx 0.57$
 - b) 1) ≈ -0.43
- $21 \approx 0.70$
- 5. a) Table of values ①: $y \approx 0.83x + 0.62$ Table of values ②: $y \approx -0.59x + 62.16$
 - b) Table of values ③: $y \approx 0.7x 0.34$ Table of values ④: $y \approx -1.5x + 9.14$
- 6. a) 1)



- 2) $r \approx 0.95$
- 3) y = 2.08x + 1.68



- 2) Several answers possible. Example: $r \approx -0.88$
- 3) Several answers possible. Example: y = -1.52x + 28.27

Spring distortion

Practice 5.3 (cont'd)

Page 128

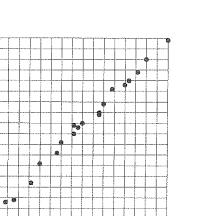
7. a)
Spring

8

3

extension

(cm) 10



40 45 50

Suspended mass

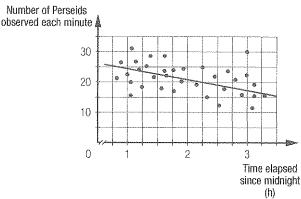
- **b)** Several answers possible. Example: $y = \frac{39}{193}x \frac{959}{4825}$ or y = 0.2x 0.20.
- c) Several answers possible. Example:
 The spring's extension would be approximately
 17.18 cm.

15 20

25 30 35

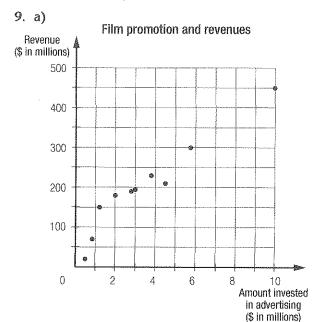
- 8. a) The correlation is moderate and negative.
 - b) Several answers possible. Example:

Observation of Perseids



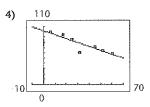
- c) Several answers possible. Example: $y \approx \frac{-10x}{3} + \frac{55}{2}$
- d) 12 or 13 Perseids are observed each minute.

Practice 5.3 (cont'd)



- b) The correlation is moderate and positive.
- c) Several answers possible. Example: y = 40x + 61.86
- d) Several answers possible. Example: The producers can hope to generate \$621.86 million from this film.
- **10.** Several answers possible. Example: The maximum load is approximately 89 258.69 kg.
- **11.** a) Yes. There appears to be a correlation since the scatter plot forms a line.
 - b) Several answers possible. Example: y = -0.5x + 775
 - c) Several answers possible. Example: 300 DVD movies are sold.

3) ≈ -0.88



Problem

Page 130

Several answers possible. Example:

Frozen food having a mass of 320 g is expected to contain 1600 mg of sodium.

Activity 1

Page 131

- a. 1) The correlation is strong and positive.
 - 2) The correlation is very strong and positive.
- b. The correlation coefficient in Screen 2. is much smaller.
- c. The temperature of the water would be 66.98°C.
- d. No. In some cases, there are inconsistent data that must be omitted from the statistical analysis, as is the case in this situation.

Activity 2

Page 132

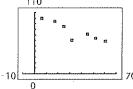
- a. 1) Yes, since the data almost form a straight line.
 - 2) Yes. The monthly quantity of heating oil used is approximately 1600 m³.
 - 3) Yes. The monthly quantity of natural gas used is approximately 3400 m³.
- b. 1) There appears to be a relatively strong relation between these two variables since the diagram presents a scatter plot in which the points nearly form a line.
 - 2) There appears to be a relatively strong relation between these two variables since the diagram presents a scatter plot in which the points nearly form a line.
- c. There appears to be a very strong and positive relation.

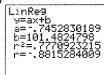
Technomath

Page 133

a. The value of a represents the slope of a regression line. The value of b represents the y-intercept of the regression line. The value of r represents the linear correlation coefficient.







Practice 5.4

Page 136

- 1. a) 1) ≈ 0.92

 - b) 1) ≈ -0.96
 - c) 1) ≈ -0.51
 - d) 1) ≈ 0.76
- 2. a) ≈ 1

 - c) ≈ 384.16 cm
- 2) $y \approx -1.73x + 8.93$

2) $y \approx 2.69x - 2.63$

- 2) $y \approx -0.63x + 5$
- 2) $y \approx 1.39x + 0.18$
- b) $y \approx 11.24x 9.09$
- d) ≈ 23.5 years.

Practice 5.4 (cont'd)

Page 137

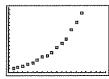
- 3. a) The correlation is moderate and negative.
 - b) Several answers possible. Example: $y \approx -1.075x + 83$
 - c) Several answers possible. Example: The price of a lot might be approximately \$83/m².
 - d) Yes, since the scatter plot follows the direction of a line.
- 4. No, according to the equation of the regression line, 1400 tons of samples yield 127 087 diamonds.

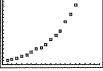
Practice 5.4 (cont'd)

Page 138

- 5. a) After approximately 91.24 days.
 - b) For approximately 187.87 days.
 - c) The duration of the treatment was approximately 139.56 days.
 - d) The pain would be reduced by 100%, meaning that it would disappear.
- 6. a) 1) Table of values (1)

Table of values (2)





- 2) ≈ 0.95
- 3) ≈ 1

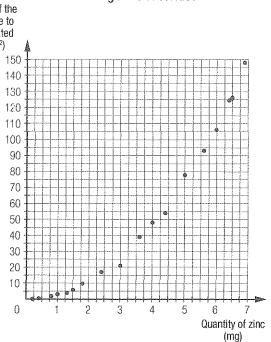
- ≈ 0.98 2)
- 3) ≈ 1
- b) In each case, the exponential model appears to be the most appropriate to represent the situation since the correlation coefficient approaches 1.

Practice 5.4 (cont'd)

Page 139

7. a)
Area of the surface to be plated (cm²)

Plating a metal surface



- b) A second-degree function.
- c) The intensity is almost perfect.
- d) The quantity of zinc required should be 8.25 mg.
- 8. a) The correlation is strong and negative.
 - b) Other variables affect this phenomenon.

Practice 5.4 (cont'd)

Page 140

- 9. a) \$120,400
- **b**) $\approx 57.61 \text{ h}$
- c) $\approx 61.18 \, \text{h}$
- d) \approx \$92,164.8
- e) 117.7 h
- **10.** Several answers possible. Example:
 - a) The length of a person's hair and his or her height.
 - b) The efficacy of an experimental drug.

Practice 5.4 (cont'd)

Page 141

11. a) 15%

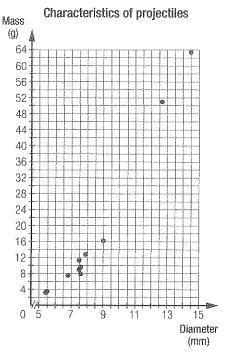
Note: For numbers **b** to **d**, the regression line y = 0.04x + 51.53, where x represents the density (trees/km²) and y, the percentage of pollution, was calculated using the first 5 ordered pairs from the table of values only.

- **b)** 1) $\approx 51.53\%$
- 2) ≈ 22.46%

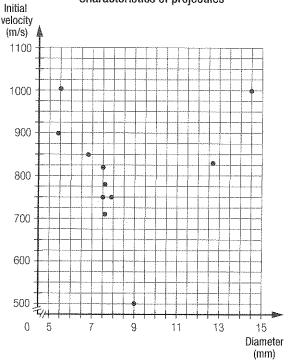
- c) A region that is 40% polluted should have approximately 270 trees/km².
- d) A minimum of 355 trees/km² should be planted in a region that is 30% polluted.

12. a) 1)

2)



Characteristics of projectiles



Characteristics of projectiles

Initial velocity (m/s)

1100

900

800

700

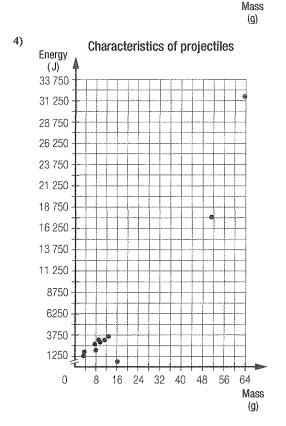
600

500

400

0

16 24 32 40 48



56

- b) There are always two outliers in the scatter plot.
- c) 1) The correlation is strong and positive.
 - 2) The correlation is weak and negative.
 - 3) The correlation is moderate and negative.
 - 4) The correlation is strong and positive.

d) Diameter: ≈ 13 mm
 Initial velocity: ≈ 0 m/s
 Energy: ≈ 10 000 J

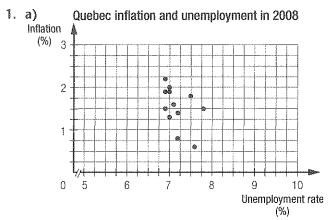


Chronicle of the past

Page 143

- 1. a) The mean number of marbles collected in the centre container is 12.
 - b) The mean deviation is 1.75 marbles.
- 2. a) The height of an adult child is 172.2 cm.
 - b) The mean height of the parents is approximately 158.83 cm.
- 3. The correlation coefficient is approximately 0.67.
- 4. The mean deviation is approximately 19.61.

In the workplace



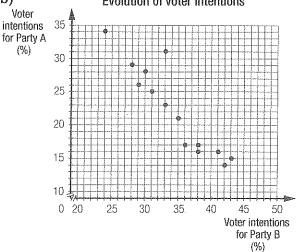
- b) The correlation is weak and negative.
- 2. a) Evolution of voter intentions

	20/60	10/07	11/07	12/07	01/08	02/08	03/08	04/08	05/08	80/90	02/08	80/80	80/60	10/08
Voter intentions for Party A (%)	33	28	24	30	29	31	35	33	38	42	36	43	41	38
Voter intentions for Party B (%)	31	29	34	28	26	25	21	23	16	14	17	15	16	17

b) Voter intentions

(%)

Evolution of voter intentions



- c) -0.92
- d) y = -1.08x + 59.38
- e) Voter intentions for Party B might be 5.38%.

Overview

Page 146

- 1. a) 1) Mean deviation: ≈ 46.01 seats.
 - 2) Standard deviation: ≈ 69.87 seats.
 - b) 1) Mean deviation: ≈ 0.39 h
 - 2) Standard deviation: ≈ 0.49 h

2. a)

Precipitation and crops

Rainfall amounts Wheat (mm) production (kg)		[200, 250]	[250, 300[[300, 350[Total
[750, 1000[1	4	8	10	23
[1000, 1250[3	6	5	7	21
[1250, 1500[7	7	6	4	24
[1500, 1750[11	5	3	1	20
Total	22	22	22	22	26

- b) The correlation is weak and negative.
- c) 44 fields.
- d) 66 fields at the most.

Overview (cont'd)

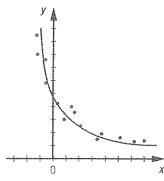
Page 147

- 3. a) Several answers possible. Example: 0.64
 - **b)** Several answers possible. Example:
 - c) Several answers possible. Example: 0.48
- 4. (5), (4), (3), (6), (2), (1)
- 5. No. In the contingency table, most of the data do not follow one of the two diagonals.

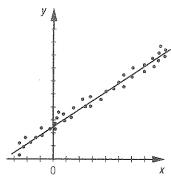
Overview (cont'd)

Page 148

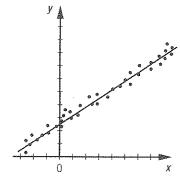
6. a) 1) Several answers possible. Example:



- 2) Exponential function.
- b) 1) Several answers possible. Example:



- 2) Linear function.
- c) 1) Several answers possible. Example:



- 2) Square root function.
- 7. Yes, the biologist is correct, since the correlation coefficient between the variables is -0.5, which indicates that the correlation is weak and that the populations vary inversely.
- 8. a) 100-m race: 0.07 s 200-m race: 0.33 s 4 x 100-m race: 0.37 s
 - b) The 100-m race, since the mean deviation is the smallest of the three.
 - c) Probably that they decrease the mean, since the runners appear to be slower according to the data for the 100-m race.

Overview (cont'd)

Page 149

- 9. Equation 🔘.
- 10. Wendy's mark is approximately 16.
- 11. In 2018.

Overview (cont'd)

Page 150

- 12. Yes. The linear correlation coefficient associated with the situation is approximately 0.98.
- **13.** a) Denmark: y = 0.12x 67.57

France: y = 0.1x - 25.72

Norway: y = 0.1x - 27.54

Sweden: y = 0.08x + 10.57

where x represents the year, and y, the height in centimetres.

- b) In France.
- c) In Denmark.
- d) The height of Swedish men may have been approximately 166.17 cm.
- e) The height of French men might be approximately 178.28 cm.
- f) In approximately 1875.

Overview (cont'd)

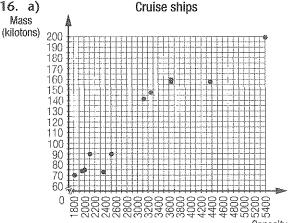
Page 151

- 14. Person A: I think the linear correlation coefficient is very strong because the scatter plot is very narrow.
- 15. a) No, the curve appears to be a good representation of the relationship between these families' annual income and the income allocated for essential needs.
 - b) Yes. The scatter plot indicates a decreasing trend.
 - c) Exponential function.
 - d) $y = 93.32(0.99)^x$
 - e) The percentage could be approximately 7.56%.

Overview (cont'd)

Page 152

16. a) Mass



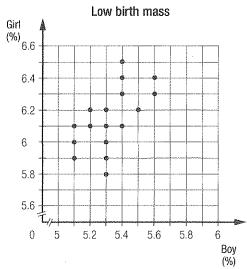
- b) 1) Several answers possible. Example:
 - 2) The correlation is very strong and positive.
- c) Several answers possible. Example: y = 0.04x + 2.27
- d) The mass of this cruise ship could be approximately 242.27 kilotons.
- 17. a) The correlation is strong and positive.
 - b) He can expect approximately 76 victories.

Overview (cont'd)

Page 153

- 18. The heart rate could be 107 beats for each minute.
- 19. a) 1) Boys: 0.13, Girls: 0.14
 - 2) Boys: 0.15, Girls: 0.18
 - b) Boys, since the mean deviation and the standard deviation are less than those of the girls.

c)



d) The correlation is moderate and positive.

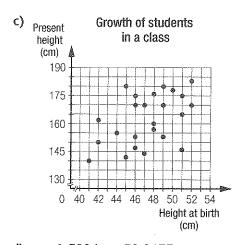
- 20. Yes, it is possible to describe the correlation as very strong and negative. However, the limited amount of data does not ensure that a representative study will be carried out nor that a valid conclusion will be drawn.
- 21. The correlation coefficient is approximately 0.72.

Overview (cont'd)

Page 155

- 22. a) The mean deviation is 11.84 cm.
 - b) Growth of students in a class

Present height Height at (cm) birth (cm)		[149, 158[[158, 167[[167, 176[[176, 185[Total
[41, 44[1	1	dan	0	0	3
[44, 47]	2	2	0	2	2	8
[47, 50[1	2	1	2	2	8
[50, 53[1	0	1	3	2	7
Total	5	5	3	7	6	26



- **d)** y = 1.7826x + 79.2477
- e) The correlation coefficient is approximately 0.42.
- f) His height at birth could have been approximately 64.93 cm.
- 23. According to the correlation coefficient associate with this situation, which is approximately -0.84, this statement is true.

Bank of problems

- 24. The variables for coffee and sugar are the most closely linked since they have a correlation coefficient of approximately 0.89, compared to approximately 0.10 for cocoa and coffee and approximately 0.24 for cocoa and sugar.
- 25. An orchestra should have approximately 137 musicians if the following equation of the regression line is considered: y = 1.4472x + 10.71.

Bank of problems (cont'd)

- 26. None, since at 6 m, there would remain approximately 19.3 ppm of phytoplankton according to the regression line, and since the concentration of phytoplankton decreases by approximately 48 ppm for each degree Celsius lost according to the regression line between the temperature and the quantity of phytoplankton. Therefore, the temperature should be lowered by approximately 10°C.
- **27. Note:** The equation of the regression curve used in this case is $y = 0.45 \sqrt{x}$, where x represents the time (in s) and y the height of the marble (in m). The speed of a marble should be approximately 16.63 m/s.

Right triangles and trigonometric ratios



Designing a plan

The procedure below can be followed to produce the requested plan.

To help students complete this LES, provide them with the following measurements.

Note: A worksheet is provided on page 1 of the Reproducible sheets in "Vision 6."

Calculate the dimensions of the house.

Width: 5 m Length: ≈ 10 m

Height: edges of the roof: \approx 7 m, top of the roof: \approx 8.84 m

· Calculate the dimensions of the bench.

Width: 1 m Length: 1.79 m Height: 1.13 m

· Calculate the dimensions of the tree.

Height: ≈ 3 m Diameter: ≈ 1 m

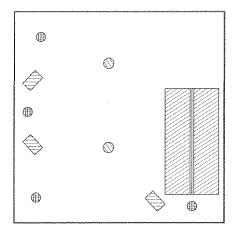
• Calculate the dimensions of the lamp post.

Height: $\approx 4.05 \text{ m}$ Diameter: $\approx 0.91 \text{ m}$

Legend defining the height of each object located on the plan.

[0, 2[m	
[2, 4[m	
[4, 6[m	
[6, 8[m	
[8, 10[m	

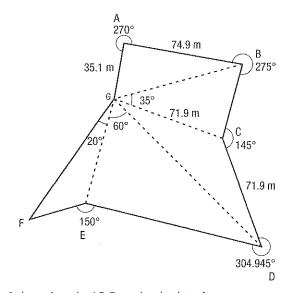
 Choose a ratio that can be used to produce the scale plan, for example 1:357. • Choose the position of each object on the plan.





Specific restrictions

The following procedure may help the urban planning director decide whether to approve or reject the construction of the residence.



Solve triangle ABG and calculate its area.

$$m \angle GAB = 90^{\circ}$$

$$m \angle ABG \approx 25.11^{\circ}$$

$$m \angle BGA \approx 64.89^{\circ}$$

$$m \overline{BG} = \sqrt{35.1^2 + 74.9^2} \approx 82.72 \text{ m}$$

Area of
$$\Delta$$
 ABG = $\frac{35.1 \times 74.9}{2} \approx 1314.5 \ m^2$

Solve triangle BCG and calculate its area.

$$m \angle GBC \approx 360 - 25.11 - 275 \approx 59.89^{\circ}$$

$$m \overline{BC} = 45.62 m$$

Area of
$$\Delta$$
 BCG = $\frac{82.72 \times 71.9 \sin 35}{2} \approx 1705.69 \text{ m}^2$

Solve triangle CDG and calculate its area.

$$m \angle CGD = m \angle CDG \approx 25.06^{\circ}$$

$$m \overline{DG} \approx 130.27 m$$

Area of
$$\Delta$$
 CDG $=\frac{71.9\times71.9~\text{sin}~129.89}{2}\approx1983.26~\text{m}^2$

Solve triangle DEG and calculate its area.

$$m \angle GDE \approx 30^{\circ}$$

$$m \angle DEG \approx 90^{\circ}$$

m
$$\overline{EG} \approx 65.13$$
 m

$$m \overline{DE} \approx 112.82 m$$

Area of
$$\Delta$$
 DEG = $\frac{65.13 \times 112.82}{2} \approx 3673.98 \text{ m}^2$

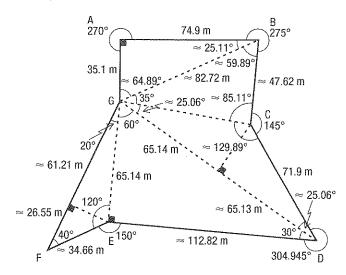
Solve triangle EFG and calculate its area.

$$m \angle GEF = 120^{\circ}$$

$$m \angle EGF = 40^{\circ}$$

Area of
$$\Delta$$
 EFG $= \frac{87.76 \times 65.13 \ sin \ 20}{2} \approx 977.46 \ m^2$

Representation of the situation



Calculate the total area of the lot.

Calculate the maximum available area on this lot for the construction of a residence.

$$9654.89 \div 65 \approx 148.54 \text{ m}^2$$

Area taken up by the residence:

$$12 \times 12 = 144 \text{ m}^2$$

Therefore, the urban planning director should approve the construction of this residence since $144 \text{ m}^2 < 148.54 \text{ m}^2$.



Prior learning 1

Page 160

a. ≈ 0.0067

b. 1)
$$\frac{m \overline{CG}}{m \overline{KO}} = \frac{m \overline{FG}}{m \overline{NO}}$$
 2) $\frac{m \overline{BG}}{m \overline{JO}} = \frac{m \overline{DF}}{m \overline{LN}}$

$$\frac{m \overline{BG}}{m \overline{JO}} = \frac{m \overline{DF}}{m \overline{LN}}$$

c. 1)
$$m \angle P = 117^{\circ}, m \angle J = 50^{\circ}, m \angle I = 13^{\circ}$$

2)
$$\angle$$
 OLN = 65°

3)
$$\angle$$
 LON = 25°

4)
$$\angle$$
 OKL = 50°

d. 1) m
$$\overline{DE} \approx 6.05$$
 cm

2) m
$$\overline{EF} \approx 5.28$$
 cm

Knowledge in action

Page 163

1. (10, 20, 3B, 4A

2. a)
$$x = \frac{70}{3} = 23.\overline{3}$$
 b) $y = 4.5$

b)
$$y = 4.5$$

c)
$$z = 1.04$$

d)
$$a = 7 \text{ or } -7$$
.

3. a)
$$x \approx 3.5 \text{ cm}$$

c) $x \approx 4.26 \text{ cm}$

b)
$$x \approx 1.25 \text{ cm}$$

d)
$$x \approx 4.96$$
 cm

Knowledge in action (cont'd)

Page 164

- 5. Only Reproduction (2) is made to scale.
- 6. a) The sum is always 360°.
 - **b)** The sum of the interior angles of a quadrilateral is 360°. Angles C and D are congruent and right angles; therefore, they each measure 90°. Consequently, there remains a sum of 180° for the last two angles, which are therefore supplementary.
 - c) Yes, since the proof is based on a general case and not on one particular example.

Knowledge in action (cont'd)

- 7. The sum of the interior angles of a quadrilateral is 360°. The opposite angles of a parallelogram are congruent. If one of the angles measures x° and the other, y° , then $2x^{\circ} + 2y^{\circ} = 360^{\circ}$, which can be simplified to $x^{\circ} + y^{\circ} = 180^{\circ}$.
- 8. a) 1) 54°
- 2) 144°
- b) 1) 79.55°
- 2) 169.55°
- c) 1) 45°
- 2) 135°
- d) 1) 23.7°
- 2) 113.7°
- e) 1) 0°
- 90°

- f) 1) Impossible.
- 2) 60°

- 9. a) $\approx 565.25 \text{ km}$
- **b**) $\approx 760.75 \text{ km}$
- c) $\approx 429.25 \text{ km}$
- d) $\approx 535.5 \text{ km}$
- 10. a) The lengths of the sides of Triangle ②: 5 units, 3.75 units and 6.25 units. The lengths of the sides of Triangle ③: 6.25 units, \approx 4.69 units and \approx 7.81 units. The lengths of the sides of Triangle ④: \approx 7.81 units, \approx 5.86 units and \approx 9.77 units. The lengths of the sides of Triangle ⑤: \approx 9.77 units, \approx 7.32 units and \approx 12.21 units.
 - b) ≈ 0.41
 - c) The total area is approximately 88.68 units squared.



Trigonometric ratios

Problem Page 166

The length of the cut line is approximately 1.17 cm.

Activity 1

Page 167

- a. Yes, since two triangles with two congruent corresponding angles are similar (AA).
- b. Triangle ABC: ≈ 0.26 m, triangle DEF: ≈ 1.15 m, triangle GHI: ≈ 2.36 m.

Activity 1 (cont'd)

Page 168

- c. 1) Triangle ABC: 0.052, triangle DEF: \approx 0.052, triangle GHI: \approx 0.052.
 - 2) The same result is obtained.
 - 3) In a right triangle, it is the ratio between the length of the side opposite to an angle and the length of the hypotenuse. $\sin 3^{\circ} \approx 0.052$
 - 4) The value of the ratio between the length of the side opposite to this angle and that of the hypotenuse.
- d. 1) Triangle ABC: \approx 0.998, triangle DEF: \approx 0.999, triangle GHI: \approx 0.999.
 - 2) The same result is obtained.
 - 3) In a right triangle, it is the ratio between the length of the side adjacent to an angle and that of the hypotenuse. $\cos 3^{\circ} \approx 0.999$
 - 4) The value of the ratio between the length of the side adjacent to this angle and that of the hypotenuse.

- e. 1) Triangle ABC: \approx 0.052, triangle DEF: \approx 0.052, triangle GHI: \approx 0.052.
 - 2) The same result is obtained.
 - 3) In a right triangle, it is the ratio between the length of the side opposite to an angle and that of the side adjacent to this same angle. $\tan 3^{\circ} \approx 0.052$
 - 4) The value of the ratio between the length of the side opposite to this angle and that of its adjacent side.

Technomath

Page 169

- a. 1) Respectively, \approx 0.75, \approx 0.42, \approx 0.42 and \approx 0.34.
 - 2) Yes.
- **b.** 1) Respectively, ≈ 0.66 , ≈ 0.91 , ≈ 0.91 and ≈ 0.94 .
 - 2) No. On the contrary, the ratio decreases.
- c. 1) The ratio approaches 1.
 - 2) The ratio approaches 0.

Practice 6.1

Page 171

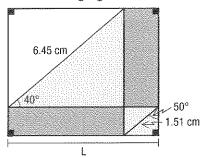
- 1. a) $\sin A \approx 0.72$
- b) $\cos B \approx 0.50$
- c) tan C ≈ 1.45
- d) tan D ≈ 1.87
- e) cos E ≈ 0.71
- f) $\sin F \approx 0.59$

2. a) ∠ H

- **b)** ∠ H
- c) $\angle D$ or $\angle E$.
- **d)** ∠ A

e) ∠ G

- f) ∠ B
- 3. Note: The students must base their work on the following figure:



The width of the logo is approximately 6.1 cm.

4. The length of cable required is approximately 11.76 m.

Practice 6.1 (cont'd)

- 5. a) $\angle C \cong 34^{\circ}$, m $\overline{AB} \approx 2.29$ cm, m $\overline{BC} \approx 3.40$ cm
 - b) \angle D \cong 57°, m $\overline{DE} \approx 5.37$ cm, m $\overline{DF} \approx 2.92$ cm
 - c) \angle H \cong 30°, m $\overline{GI} \approx 2.5$ cm, m $\overline{IH} \approx 4.33$ cm
 - d) \angle J \cong 20°, m \overline{JL} \approx 4.40 cm, m \overline{JK} \approx 4.68 cm
 - e) \angle N \cong 65°, m $\overline{MN} \approx 5.68$ cm, m $\overline{MO} \approx 5.15$ cm
 - f) \angle P \cong 45°, m \overline{PR} \approx 3.3 cm, m \overline{PQ} \approx 4.67 cm

- 6. a) $\cos A = \sin B$
 - b) $\sin B > \cos B$
 - c) tan A < tan B
- 7. a) m $\overline{AC} \approx 7.83$ cm
- b) m $\overline{BC} \approx 5.03$ cm
- c) m $\overline{BD} \approx 3.86$ cm
- d) m $\overline{\text{CD}} \approx 3.24 \text{ cm}$
- 8. a) The lengths of the corresponding sides of similar triangles are proportional.
 - b) Angle D and angle A are corresponding angles; consequently, their lengths are equal. Therefore, the same value is calculated.

Practice 6.1 (cont'd)

Page 173

- 9. a) The volume is approximately 90.74 cm³.
 - b) The volume is approximately 282.37 m³.
 - c) The volume is approximately 251.46 cm³.
 - d) The volume is approximately 3.48 cm³.
- 10. a) 1) The distance separating Venus from the Sun is approximately 108 788 133 km.
 - 2) The distance separating Venus from the Earth is 103 272 175 km.
 - b) The distance separating the Earth from the center of Venus is 41 211 867.09 km.
- 11. a) False, since $\frac{\sin S}{\sin T} = \frac{\frac{s}{r}}{\frac{t}{r}} = \frac{s}{t}$, $\frac{\cos S}{\cos T} = \frac{\frac{t}{r}}{\frac{s}{r}} = \frac{t}{s}$ and $\frac{s}{t} \neq \frac{t}{s}$.

 b) True, since $\tan S = \frac{s}{t}$, $\frac{\cos T}{\cos S} = \frac{\frac{s}{r}}{\frac{t}{r}} = \frac{s}{t}$ and $\frac{s}{t} = \frac{s}{t}$.

 - c) False, since $\frac{s}{r} + \frac{t}{r} = \frac{s+t}{r}$, $\tan S = \frac{s}{t}$ and $\frac{s+t}{r} \neq \frac{s}{t}$.
 - d) True, since $\sin R = \frac{r}{r} = 1$.
 - e) True, since the hypotenuse has the greatest length in any triangle.

Practice 6.1 (cont'd)

Page 174

- 12. a) The distance covered is approximately 26.42 m.
 - b) The distance covered is approximately 26.93 m.
 - c) The distance covered is approximately 17.42 m.
 - d) The distance covered is approximately 49.18 m.
- 13. a) The horizontal distance is approximately 27.22 m.
 - b) The vertical distance is approximately 1.88 m.
- 14. a) At 120.94 cm from the ceiling.
 - b) At 123.96 cm from the ceiling.
 - c) At 91.37 cm from the ceiling.

Practice 6.1 (cont'd)

Page 175

15. The statue of *Mercury on Pegasus* has sunk approximately 18.24 cm, whereas the statue of The Flute-playing Shepherd has sunk approximately 7.17 cm.

- 16. a) The circumference is approximately 37 658.18 km.
 - b) The length of the segment is approximately 8199.57 km.

Practice 6.1 (cont'd)

Page 176

- 17. Yes, the geologists are correct, since the height is decreasing.
 - The height of the falls when Measurements (1) were taken: ≈ 60.002 m,
 - the height of the falls when Measurements (2) were taken: ≈ 59.989 m,
 - the height of the falls when Measurements (3) were taken: ≈ 59.469 m.
- 18. a) The distance is approximately 8.14 m.
 - b) The distance is approximately 6.48 m.
 - c) The distance is approximately 12.73 m.

Practice 6.1 (cont'd)

Page 177

- 19. The distance separating the two airplanes at 6:30 p.m. is approximately 3184.26 km.
- 20. The distance separating the Sun from star 61 Cygni is approximately 1.03×10^{14} km.



Finding unknown measurements

Problem

The data collected is not valid, since the measures of the angles of the triangle formed by the three satellites are approximately 58.87°, 60.56° and 60.56° and since the measure of one of the angles is less than 59°.

Activity 1

- a. 1) The sine of the ball's angle of deviation is approximately 0.042532.
 - 2) The measure of the ball's angle of deviation is between 1 and 1.5°.
 - 3) Yes, since the measure of the angle is less than 2.5°.
- b. 1) Arcsin can be used to calculate the measure of the angle corresponding to a given sine ratio.
 - 2) The measure of the angle is 30°.

Activity 1 (cont'd)

Page 180

- c. 1) The cosine of the ball's angle of deviation is approximately 0.999765.
 - 2) The measure of the ball's angle of deviation is between 1 and 1.5°.
 - 3) Yes, since the measure of the angle is less than 2.5°.
- d. 1) Arccos can be used to calculate the measure of the angle corresponding to a given cosine ratio.
 - 2) The measure of the angle is 60°.
- e. 1) The tangent of the ball's angle of deviation is approximately 0.044931.
 - 2) The measure of the ball's angle of deviation is between 2.5 and 3°.
 - 3) No, since the measure of the angle is greater than 2.5°.
- f. 1) Arctan can be used to calculate the measure of the angle corresponding to a given tangent ratio.
 - 2) The measure of the angle is 45°.
- g. 1) Triangle ①: $m \angle A = 30^{\circ}$, Triangle ②: $m \angle A = 30^{\circ}$, Triangle ③: $m \angle A = 30^{\circ}$.
 - 2) In a right triangle, the length of the side opposite to a 30° angle is equal to half the length of the hypotenuse.

Technomath

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- a. They are all 30°-60°-90° triangles. In each triangle, the shortest side measures half the length of the hypotenuse.
- b. 1)

Screen 3	Screen 4	Screen 5	Screen 6
0.5	0.5	0.5	0.5
≈ 0.87	≈ 0.87	≈ 0.87	≈ 0.86
≈ 0.87	≈ 0.87	≈ 0.87	≈ 0.86
0.5	0.5	0.5	0.5

- 2) In a right triangle, the length of the side opposite to a 30° angle is equal to half the length of the hypotenuse.
- c. 1) Several answers possible.
 - 2) There is only one angle measure, 30°, whose sine is 0.5.
- d. Several answers possible.

Practice 6.2

Page 183

- 1. b), d) and f).
- 2. a) m \angle A \approx 44.96°
- b) m \angle A \approx 60.68°
- c) m \angle A \approx 30.58°
- d) m \angle A \approx 59.92°

- 3. a) m \angle A = 48.89°, m \angle C \cong 41.11°, m $\overline{AB} \approx 6.11$ cm
 - b) m \angle E = 60°, m \angle F = 30°, m $\overline{\text{DF}} \approx 17.32 \text{ cm}$
 - c) m \angle H = 40.02°, m \angle G = 49.98°, m $\overline{\text{HI}} \approx$ 23.82 cm
 - d) m \angle L = 66°, m $\overline{\text{KL}} \approx$ 13.36 mm, m $\overline{\text{JL}} \approx$ 32.84 mm
 - e) m \angle O = 30°, m \overline{MN} = 4.5 cm, m $\overline{MO} \approx 7.79$ cm
 - f) $m \angle Q = 76.66^{\circ}$, $m \angle R = 13.34^{\circ}$, $m \overline{PR} \approx 72.98 \text{ cm}$

Practice 6.2 (cont'd)

Page 184

- 4. a) $\approx 42.07^{\circ}$
- b) $\approx 11.54^{\circ}$
- c) 30°
- d) $\approx 60.46^{\circ}$
- e) $\approx 70.12^{\circ}$ q) $\approx 44.77^{\circ}$
- f) $\approx 27.13^{\circ}$ h) $\approx 74.93^{\circ}$
- i) $\approx 40.36^{\circ}$
- i) ≈ 72.65°
- $k) \approx 34.99^{\circ}$
- 1) 45°
- 5. a) m \angle A = 30°
- **b)** m \angle A = 45°
- c) m \angle A = 45° e) m \angle A = 60°
- d) m \angle A = 60° f) m \angle A = 60°
- 6. a) $\approx 79.61^{\circ}$
- b) $\approx 48.89^{\circ}$
- c) $\approx 64.94^{\circ}$
- d) $\approx 47.92^{\circ}$

7.	Length of side adjacent to angle A (cm)	Length of side opposite to angle A (cm)	Length of hypotenuse (cm)	Length of angle A (°)	Length of angle B (°)
Triangle ①	12	≈ 32.88	35	≈ 69.95	≈ 20.05
Triangle ②	≈ 51.96	30	60	30	60
Triangle ③	11.4	15.5	≈ 19.24	≈ 53.67	≈ 36.33
Triangle 4	45.76	≈ 26.42	52.84	30	60
Triangle (5)	≈ 0,20	≈ 0.40	0.45	63	27
Triangle 6	34.5	46	57.5	≈ 53.13	≈ 36.87

- **8.** The inclination of the sun's rays is approximately 51,34°.
- 9. The distance separating the ship from the church is approximately 1761.96 m.

Practice 6.2 (cont'd)

- 10. $m \angle A \approx 45^{\circ}$, $m \angle B \approx 122.47^{\circ}$, $m \angle C \approx 120.96^{\circ}$, $m \angle D \approx 71.57^{\circ}$
- 11. The measure of the angle of elevation of this light beam is approximately 41.19°.

- **12.** The total length of the dam's braces is approximately 3649.89 m.
- 13. a) For Tourist A, the angle of elevation is approximately 85.18°, whereas for Tourist B, it is approximately 84.32°.
 - b) The measures of the three angles are 90°, approximately 31.89° and approximately 58.11°.

Practice 6.2 (cont'd)

Page 186

- 14. Ramps and meet this construction standard.
- 15. a) The difference is approximately 0.74 m.
 - b) The difference is approximately 9.87°.
- **16.** The length of the pendulum's string is approximately 18.34 cm.

Practice 6.2 (cont'd)

Page 187

- 17. a) The distance separating the two airplanes is approximately 14.97 km.
 - b) Airplane A: ≈ 18.88°. Airplane B: 30°.
 - c) The altitude of Airplane A will be approximately 1300.97 m.
- 18. a) The height of point A will be approximately 6.8 m.
 - b) The height of point A will be approximately 20.4 m.
 - c) The height of point A will be approximately 12.22 m.
 - d) The height of point A will be approximately 22.92 m.

Practice 6.2 (cont'd)

Page 188

- 19. a) $\approx 71.67^{\circ}$
- b) $\approx 70.53^{\circ}$
- c) $\approx 54.56^{\circ}$
- d) $\approx 65.15^{\circ}$
- 20. a) The width of the river is approximately 3 km.
 - b) The measure of the angle of depression is approximately 20.38°.
 - c) The measures of the angle of depression would be approximately 33.02° and approximately 14.57°.

Practice 6.2 (cont'd)

Page 189

- 21. a) The measure of the angle is approximately 57.08°.
 - b) The measure of the angle is approximately 39.77°.
 - c) The three angles measure approximately 65.84°, 57.08° and 57.08°, respectively.
- 22. a) The measure of the angle is approximately 7.21°.
 - b) The measure of the angle is approximately 7.21°.
 - c) The circumference as calculated by Eratosthenes is approximately 39 315.76 km.

eeten 6

Calculating the area of any triangle

Problem

Page 190

The area of this territory is approximately 201.17 km².

Activity 1

Page 191

a. 1) $\frac{b \times h}{2}$

- 2) $\frac{h}{a}$
- **b.** $A = \frac{\overline{b} \times h}{2}$: formula for the area of a triangle. sin $C = \frac{h}{a}$: definition of the sine relation.

 $h = a \sin C$: equivalent expression.

 $A = \frac{b \times a \sin C}{2}$: substitution.

 $A = \frac{ab \sin C}{2}$: equivalent expression.

 $A = \frac{1}{2}ab \sin C$: equivalent expression.

- c. 1) $A = \frac{1}{2}bc \sin A$
- 2) $A = \frac{1}{2} ac \sin B$
- d. 1) $\approx 7.66 \text{ cm}^2$
- 2) $\approx 7.64 \text{ cm}^2$
- e. 1) $\approx 9.4 \text{ cm}^2$
- 2) $\approx 9.4 \text{ cm}^2$
- f. 1) Yes, but the height of the triangle must be determined beforehand using trigonometry.
 - 2) $\approx 7.23 \text{ cm}^2$
- **g.** It is not necessary to know the measure of the height to calculate the area of the triangle.

Technomath

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- a. 1) 5 units and 4 units.
 - 2) The measure of the angle is 40°.
- **b.** 1) $\approx 12.49 \text{ cm}^2$
 - 2) $\approx 10.95 \text{ cm}^2$
 - 3) $\approx 9.94 \text{ cm}^2$

Practice 6.3

- 1. a) $\approx 11.98 \text{ cm}^2$
- **b**) $\approx 9.46 \text{ cm}^2$
- c) $\approx 4.73 \text{ cm}^2$
- **d)** $\approx 11.04 \text{ cm}^2$
- e) $\approx 7.47 \text{ cm}^2$
- f) $\approx 5.39 \text{ cm}^2$
- 2. a) 1) The equality is true.
 - 2) The equality is false.
 - 3) The equality is true.
 - **b)** 1) The sine of an angle and that of its supplement are equal.
 - 2) The cosine of an angle and that of its supplement have opposite signs.
 - 3) The tangent of an angle and that of its supplement have opposite signs.

Practice 6.3 (cont'd)

Page 195

3.

	m BC (cm)	m BD (cm)	m AB (cm)	Area of Δ ABC (cm²)
a)	6	≈ 3.18	≈ 4.45	≈ 13.04
b)	5	≈ 2.65	≈ 4.76	= 10.86
c)	7.5	≈ 3.97	≈ 4.38	≈ 16.3

- 4. a) ≈ 3.38 cm
- b) ≈ 3.72 cm
- c) ≈ 7.76 cm
- d) $\approx 19.2^{\circ}$
- e) $\approx 26.73^{\circ}$
- f) ≈ 4.58 cm
- 5. a) The equality is false.
 - b) The equality is false.
 - c) The equality is false.

Practice 6.3 (cont'd)

Page 196

- 6. a) $\approx 337.43 \text{ cm}^2$
- b) $\approx 15.84 \text{ cm}^2$
- c) $\approx 14.43 \text{ cm}^2$
- d) $\approx 8.97 \text{ cm}^2$
- 7. a) $\approx 6.75 \text{ cm}^2$
- b) $\approx 45.11 \text{ cm}^2$
- c) $\approx 32.14 \text{ cm}^2$
- 8. The measure of angle A is approximately 121.28°.
- 9. a) $\leq 61.82 \text{ m}^2$
- b) $\leq 43.81 \text{ m}^2$

Practice 6.3 (cont'd)

Page 197

- **10.** a) The minimum volume of the cube of ice is approximately 3.89 m³.
 - **b)** The measure of angle θ is approximately 33.69°.
 - c) The minimum amount of ice the sculptor must cut away is 1.62 m³.
- 11. She runs at a speed of 2.12 m/s.
- 12. a) $\approx 25 \text{ cm}$
- b) ≈ 10.72 cm
- 13. a) $\approx 49.99 \text{ cm}^2$
- b) $\approx 68.09 \text{ cm}^2$

Practice 6.3 (cont'd)

Page 198

- 14. Note: The students should read the following statement: While taking a walk in the woods, two campers use a compass and a pedometer. Starting from a point A, they walk 1000 paces in a southeast direction, turn in a south southwest direction, then walk 1500 paces to a point C. They make another turn to go back to their starting point. The measure of the angle formed, contained between segments BC and CA, is 35°. If each pace represents 60 cm, calculate:
 - a) The total distance is approximately 2.15 km.
 - b) The area is approximately 0.3225 km².

- 15. The distance is approximately 49.26 m.
- 16. The area of this triangle is approximately 6.56 units².

Practice 6.3 (cont'd)

Page 199

- 17. a) The Golden Triangle covers approximately 0.25% of the area of the city of Paris.
 - b) The measure of the angle is approximately 61.36°.
- 18. a) $\approx 11.62 \text{ m}$
- b) $\approx 8.16 \, \text{m}$
- c) $\approx 6.64 \text{ m}$
- d) ≈ 15.24 m
- e) $\approx 8.52 \text{ m}$

visi()n

Special features

Chronicle of the past

Page 201

- 1. a) m \angle ADB $\approx 1.80^{\circ}$
 - b) m \angle ADE $\approx 89.1^{\circ}$
 - c) m AD ≈ 399 985 km

2.

Measure of angle AOB subtended at the centre (°)	Length of chord AB (dm)	Measure of angle AOC subtended at the centre (°)	Sine of angle AOC $\left(\frac{m \text{ AC}}{m \text{ AO}}\right)$
0	0.00	0	0
10	0.174	5	0.087
20	0.347	10	0.1735
30	0.518	15	0.259
40	0.684	20	0.342
50	0.845	25	0.4225
60	1	30	0.5
70	1.147	35	0.5735
80	1.286	40	0.643
90	1.414	45	0.707
100	1.532	50	0.766
110	1.638	55	0.819
120	1.732	60	0.866
130	1.813	65	0.9065
140	1.879	70	0.9395
150	1.932	75	0.966
160	1.97	80	0.985
170	1.992	85	0.996
180	2	90	1

3. Arab mathematicians were able to construct a sine table by determining, for each given angle, the measure of half the chord associated with twice the measure of this angle in Hipparchus' chord table.

In the workplace

Page 203

- 1. The dimensions of the rectangular piece Part (1) are approximately 29.36 cm by 11.44 cm.
- 2. The volume of Part ② is approximately 3769.91 cm³.
- 3. a) Depending on the direction in which the sheet of aluminum is placed, 24 or 25 right triangles can be cut.
 - b) The mass of this rectangular sheet of aluminum is approximately 0.12 kg.

Overview

Page 204

- 1. a) ≈ 5.94 cm
- b) ≈ 6.87 cm
- c) ≈ 3.29 cm
- d) $\approx 53.29^{\circ}$
- e) $\approx 55.78^{\circ}$
- f) $\approx 17.06^{\circ}$
- 2. a) $\approx 3.48 \text{ cm}^2$
- b) $\approx 9.84 \text{ cm}^2$
- c) $\approx 6.05 \text{ cm}^2$
- d) ≈ 6.93 cm²
- e) $\approx 6.42 \text{ cm}^2$
- f) $\approx 6.22 \text{ cm}^2$
- 3. a) The area of the illuminated surface is approximately 1784.77 m³.
 - b) The volume of air illuminated by the lighthouse is approximately 11 898.48 m³.

Overview (cont'd)

Page 205

4. a) >

b) =

c) <

- d) =
- 5. a) $\approx 77.77 \text{ cm}^3$
- **b**) $\approx 10.26 \text{ cm}^3$
- c) $\approx 149.94 \text{ cm}^3$
- d) $\approx 36.76 \text{ cm}^3$
- 6. a) $\approx 3.22 \text{ cm}$
- b) $\approx 35.32^{\circ}$
- **c)** 1.95 cm
- d) ≈ 2.39 cm
- e) ≈ 5.28 cm
- f) ≈ 3.69 cm

Overview (cont'd)

Page 206

- 7. The distance between the entry point and the exit point is approximately 8.78 m.
- 8. The length of the shadow is approximately 18.41 m.
- 9. The maximum speed of this aircraft is approximately 396.75 m/s (Mach 1.3) or 1428.3 km/h.

Overview (cont'd)

Page 207

- **10**. a) 1) ≈ 11.22 m
- 2) $\approx 34.29 \text{ m}$
- b) 1) $\approx 129.03 \text{ m}^2$
- 2) $\approx 795.07 \text{ m}^2$
- 11. Option B is the least expensive. (The cost of Option B is approximately \$180.30 and of Option A, approximately \$221.05.)
- **12.** Route , which is approximately 11.62 km, in comparison with Route , which is approximately 13.54 km.

Overview (cont'd)

Page 208

- **13.** The distance between this person and corner B of this temple is approximately 35.02 m.
- 14. a) $\approx 2.6 \, \text{m}$
- b) $\approx 12.48 \text{ m}$
- c) $\approx 3.42 \text{ m}$
- d) $\approx 4.09^{\circ}$
- **15.** a) The angle of elevation of the ramp is approximately 3.58°.
 - b) The volume of this construction is approximately 17.28 m³.
 - c) The area of the surface to be covered is approximately 28.86 m².

Overview (cont'd)

Page 209

- **16. a)** The distance between this person and the top of the falls is approximately 401.12 m.
 - **b)** The measure of angle θ is approximately 7.29°.
- 17. a) The length of the shortest side of the building is approximately 26.11 m.
 - b) The angles measure approximately 90°, 23.54° and 66.46°, respectively.
- **18.** The measure of the angle of elevation is approximately 43.68°.

Overview (cont'd)

Page 210

19. a)

Aiguille du Midi cable car

	Stage 1	Stage 2
Length (m)	2553	2867
Difference in elevation (m)	1279	1470
Station altitude at Chamonix (m)	1038	2317
Station altitude at Aiguille du Mont (m)	2317	3787

- b) 1) The angle of elevation through Stage 1 is approximately 30.06°.
 - 2) The angle of elevation through Stage 2 is approximately 30.85°.
- c) The length of this cable is approximately 5419.87 m.
- **20. a)** The height of this monument is approximately 126.75 m.
 - b) Tourist B is 73.18 m from the monument.
 - c) The angle of elevation for Tourist A is approximately 11.58°, whereas for Tourist B, it is approximately 31.59°.

Overview (cont'd)

Page 211

- 21. a) The height of the moai is approximately 9.88 m.
 - b) The height of the moai's hat is approximately 2.12 m.
- **22. a)** The top of the coach's head is approximately 4.85 m above the water's surface.
 - **b)** The distance *d* is approximately 2.23 m.
- 23. The altitude of point B is approximately 1203.23 m.

Bank of problems

Page 212

- 24. Point A is approximately 4.09 m from the ground.
- **25.** There is a difference of approximately 2.67 m² between the area of Zone **A** and the area of Zone **B**.

Bank of problems (cont'd)

- 26. The initial volume of this pyramid was 4 841 932.42 m³.
- **27.** Point E is located approximately 96.74 cm above ground.



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4	Appropriate	validation of the steps in the solution	***************************************			1
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2	Correct appl	lication of concepts and processes suited to the situation				4
₩ 3	Proper imple	ementation of mathematical reasoning suited to the situation				3
4	Proper orgai	nization of the steps in a proof suited to the situation				2
5	Correct justi	fication of the steps in a proof suited to the situation				Acres 4
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<u> </u>		rpretation of a mathematical message involving at least two semiotic representation				5
3	Production of	of a message appropriate to the communication context				3
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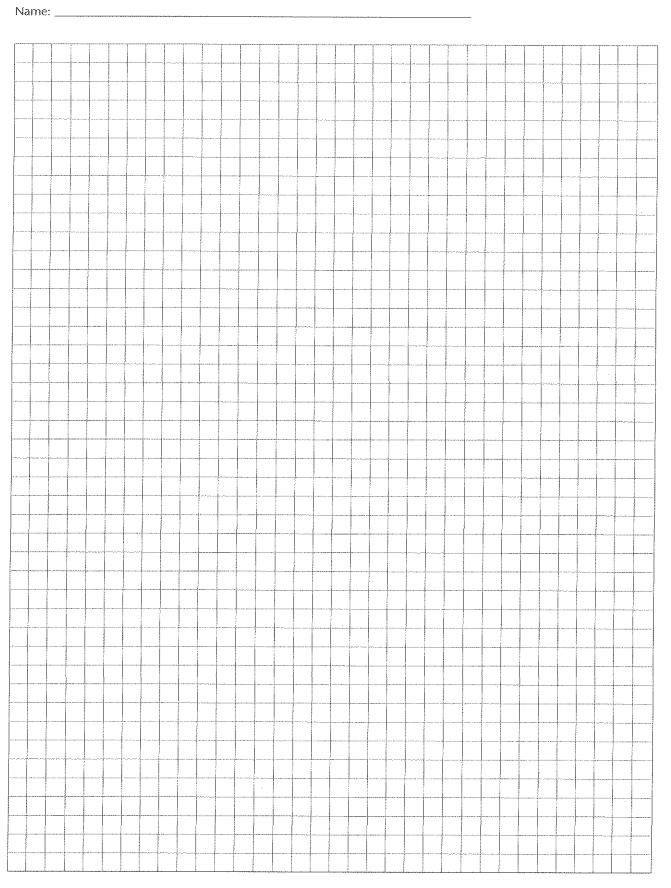
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	2)	I was engaged during each step of this LES.			
	3)	I considered all the ideas presented by my team members.			
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